Curved Track Segment Finding Using Tiny Triplet Finder (TTF) (abstract & summary)

Jinyuan Wu, M. Wang, E. Gottschalk and Z. Shi

***Abstract*— We describe the applications of a track segment recognition scheme called the Tiny Triplet Finder (TTF) that involves the grouping of three hits satisfying a constraint forming of a track segment. The TTF was originally developed solving straight track segment finding problem, however, it is also suitable in many curved track segment finding problems. The examples discussed in this document are among popular detector layouts in high-energy/nuclear physics experiments. Although it is not practical to find a universal recipe for arbitrary detector layouts, the method of the TTF application is illustrated via the discussion of the examples. Generally speaking, whenever the data item to be found in a pattern recognition problem contains two free parameters, and if the constraint connecting the measurements and the two free parameters has an approximate shift invariant property, the Tiny Triplet Finder can be used.**

***Index Terms*—Trigger, Pattern Recognition, Tiny Triplet Finder, TTF, FPGA Firmware**

I. Introduction

TRACK segment finding is often the first step in many trigger systems executing complicate processes for high-energy physics experiments. To identify and to confirm a straight-line segment in a plane with 2 parameters, for example, at least 3 hits that satisfy a constraint are needed. The 3 hits are grouped together to form a data item called "triplet". Straightforward software implementation of such a function would require O(n^3) execution time, where n is number of hits per plane, in order to examine all possible combinations of three hits using three layers of nested loop. In FPGA hardware implementation, this execution time must be reduced to O(n), to match the time required to fetch the data. The execution time is reduced by "unrolling" two layers of loops, which consumes a significant amount of silicon resources in FPGA devices. The number of logic elements needed in many typical triplet finding implementations is O(N^2) where N is the number of bins that each plane is divided into.

An algorithm, the Tiny Triplet Finder (TTF) was developed for triplet finding. The logic element usage of the TTF implemented in FPGA devices is O(N log(N)) which is significantly smaller than O(N^2) when N is large.

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J. Wu, M. Wang, E. Gottschalk and Z. Shi are with Fermi National Accelerator Laboratory, Batavia, IL 60510 USA (phone: 630-840-8911; fax: 630-840-2950; e-mail: jywu168@ fnal.gov).

The TTF was initially developed to solve a straight track segment fining problem. In this article, we will discuss the application of the TTF in curved track segment finding.

The difficulties of curved track segment finding come in two aspects. The first is the nonlinear nature of the track. The second is extra number of parameters. In general, a curved track in a plane has 3 parameters that require at least 4 hits to satisfy a constraint. Grouping 4 data items is more complicate than grouping 3 or triplet finding.

In high-energy physics experiments, sometimes tracks come from a know point, either the collision axis or a point-like target. In these cases, a curved track projected to a plane has 2 parameters, and the TTF can be used to find the track segments.

II. Cylindrical Detectors

Consider a cylindrically symmetric colliding detector with 3 (or more) angular measurement layers with radii R_1, R_2 and R_3 in the end-view as shown in Fig. 1.

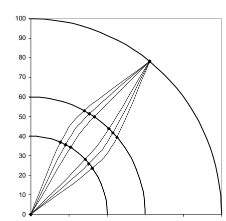

Fig. 1. Circular track roads passing a hit on the outer-most detector layer in a cylindrical detector are drawn.

A circular track come from the collision beam axis can be specified with 2 parameters, the initial angle ϕ_0 and the radius of curvature R_0:

$$r = 2R_0 \sin(\phi - \phi_0)$$

For a hit on the outer-most layer with radius R_3 and angle ϕ_3, there are many possible tracks or "roads" passing through it. Each road maps a hit in each of the inner layers. A coincident of two hits corresponding to same road on the two inner layers signifies a triplet with 3 hits: (R_3,ϕ_3), (R_2,ϕ_2) and (R_1,ϕ_1) from a possible circular track.

Define the angular offsets between the inner layer hits and the outer-most layer hit:

$$a_2 = \phi_2 - \phi_3$$
$$a_1 = \phi_1 - \phi_3$$

A constraint between the two inner layer hits belonging to same road can be found:

$$\frac{\sin(a_2)}{R_3\cos(a_2)-R_2}=\frac{\sin(a_1)}{R_3\cos(\alpha_1)-R_1}$$

Because of the axial symmetry of the detector, the equation above depends only on the angular offsets, rather than the hit angles.

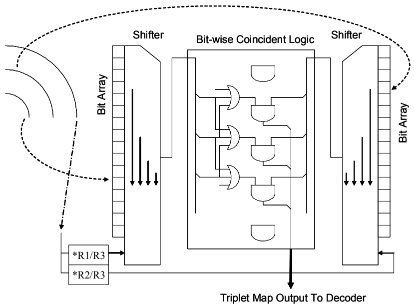

Fig. 2. The tiny triplet finder for cylindrical detectors can be implemented as shown. It contains two bit arrays, two barrel shifters and a bit-wise coincident logic block.

An example of the FPGA implementation of the tiny triplet finder for the cylindrical detector configuration is shown in Fig. 2. The details regarding the TTF operation will be discussed in full text.

Several other detector configurations in which TTF can be used for track segment finding are discussed in the next sections.

III. Planar Detector with Point-Like Target

Consider three detector planes at z = 20, 40 and 60 cm as shown in Fig. 4. Assume the target is point-like and defined as the origin.

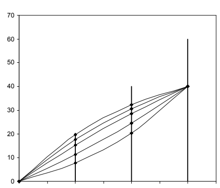

Fig. 4. Circular tracks passing a hit on the outer-most detector layer in a planar detector are drawn.

The x- and y-axes are defined so that the z-x plane is the bend view and the z-y the non-bend view. The track equations with some approximations can be written:

$$y = kz$$

$$x = bz + cz^2$$

In the bend view, consider tracks passing through a given point (x_3, z_3) on the outer-most plane. The constraint between the two hits on the two inner planes is:

$$\left(\frac{x_1}{z_1}\right)=\left(\frac{z_3-z_1}{z_3-z_2}\right)\left(\frac{x_2}{z_2}\right)-\left(\frac{z_2-z_1}{z_3-z_2}\right)\left(\frac{x_3}{z_3}\right)$$

The constraint has a shift invariant property, i.e., the coefficients of x_1 and x_2 does not depend on x_3. This is an example suitable for track segment finding using TTF.

IV. Triplet Finding for Strip Detectors

With same detector area and measurement pitch, the channel count of silicon strip detector is significantly smaller than silicon pixel detector. However, extra considerations must be made for the strip detectors.

A. The x- and y-view planes

Consider the detector layout as shown in Fig. 7. If a plane with x-measurement strips and a plane y- measurement strips are placed close together, it is possible to determine both coordinate if there is only one hit strip in each view. With more than one fired strips in each view, ghost hits are produced at the wrong intersection points.

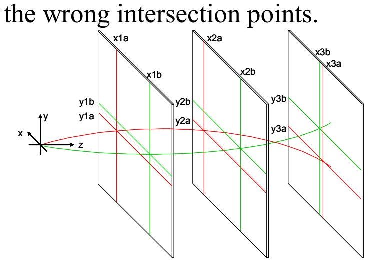

Fig. 7.

To eliminate the ghost hits information either from the third plane of strips with different view (such as u or v) or from other detector planes should be used.

B. The u- and v-view planes

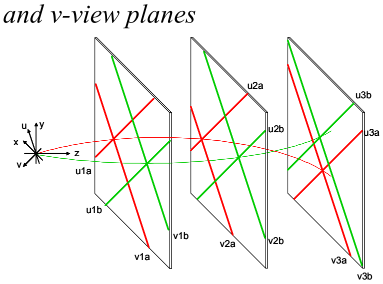

Fig. 8.

Consider a detector with u- and v- planes as shown in Fig. 8. It can be shown that most tracks can be correctly identified.

C. The combination of x-, y-, u- and v-view planes

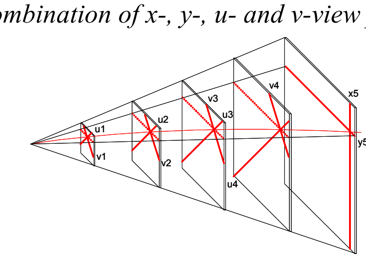

Fig. 9.

Consider a detector configuration as shown in Fig. 9. The planes 1-4 make u- and v-coordinates of the tracks. The plane 5 provides y- and x- measurements.

V. Conclusion

Generally speaking, the triplet finding is a process that groups three data items together with two free parameters (and therefore one constraint). If an approximate shift invariant property exists as in our examples, the Tiny Triplet Finder can be used for the track segment finding process.