The Physics of Calorimetry

Dan Green
LC3

Outline

- Units and orders of magnitude
- Ionization and dE/dx, Multiple Scattering
- Photon interactions
  - Photoelectric effect
  - Compton scattering
  - Pair Production
- Electron interactions
  - Bremsstrahlung
  - Critical energy

References

- ***General References - A***

Mechanics, Electricity and Magnetism and Quantum Mechanics

[1] The Feynman Lectures in Physics, R. Feynman, R. Leighton, M. Sands, Addison-Wesley Publishing Co., Inc. (1963).

[2] Classical Mechanics, H. Goldstein, Addison-Wesley Publishing Co., Inc. (1950).

[3] Classical Electricity and Magnetism, W.K.H. Panofsky and M. Phillips, Addison-Wesley Publishing Co., Inc. (1962).

[4] Quantum Mechanics, E. Merzbacher, John Wiley & Sons, Inc. (1961).

- ***General References - B***

Textbooks on Particle Detectors

[1] Detectors for Particle Radiation, K. Kleinknecht, Cambridge University Press (1987).

[2] Experimental Techniques in High Energy Physics, T. Ferbel, Addison-Wesley Publishing Co., Inc. (1987).

[3] Instrumentation in High Energy Physics, Ed. F. Sauli, World Scientific (1992).

[4] Instrumentation in Elementary Particle Physics, J.C. Anjos, D. Hartill, F. Sauli, M. Sheaf, Rio de Janeiro, 1990, World Scientific Publishing Co. (1992).

[5] Instrumentation in Elementary Particle Physics, C.W. Fabjan, J.E. Pilcher, Trieste 1987, World Scientific Publishing Co. (1988).

[6] "Particle Detectors" C.W. Fabjan, H.F. Fisher, Repts. Progr. Phys. 43, 1003 (1980).

Fundamental Constants

$$\hbar c = 0.2\, GeV\ fm,\ 1\ GeV\ = 10^9\ eV$$

$$= 2000\, eV\ \overset{o}{A}$$

$$1\,\overset{o}{A} = 10^{-8} cm,\ \ 1\ fm = 10^{-13}\, cm$$

$$= 10\, nm.$$

Me = 0.51 MeV
Mp = 938 MeV

α = 1/137
λ = 1/m
a = λ /α
kT = 1/40 eV

Quantity	Symbol, equation	Value	Uncert. (ppm)
speed of light in vacuum	c	299 792 458 m s^{-1}	exact*
Planck constant	h	6.626 075 5(40)$\times 10^{-34}$ J s	0.60
Planck constant, reduced	$\hbar \equiv h/2\pi$	1.054 572 66(63)$\times 10^{-34}$ J s	0.60
		= 6.582 122 0(20)$\times 10^{-22}$ MeV s	0.30
electron charge magnitude	e	1.602 177 33(49)$\times 10^{-19}$ C = 4.803 206 8(15)$\times 10^{-10}$ esu	0.30, 0.30
conversion constant	$\hbar c$	197.327 053(59) MeV fm	0.30
conversion constant	$(\hbar c)^2$	0.389 379 66(23) GeV^2 mbarn	0.59
electron mass	m_e	0.510 999 06(15) MeV/c^2 = 9.109 389 7(54)$\times 10^{-31}$ kg	0.30, 0.59
proton mass	m_p	938.272 31(28) MeV/c^2 = 1.672 623 1(10)$\times 10^{-27}$ kg	0.30, 0.59
		= 1.007 276 470(12) u = 1836.152 701(37) m_e	0.012, 0.020
deuteron mass	m_d	1875.613 39(57) MeV/c^2	0.30
unified atomic mass unit (u)	(mass ^{12}C atom)/12 = (1 g)/(N_A mol)	931.494 32(28) MeV/c^2 = 1.660 540 2(10)$\times 10^{-27}$ kg	0.30, 0.59
permittivity of free space	ϵ_0 } $\epsilon_0\mu_0 = 1/c^2$	8.854 187 817 ... $\times 10^{-12}$ F m^{-1}	exact
permeability of free space	μ_0	$4\pi \times 10^{-7}$ N A^{-2} = 12.566 370 614 ... $\times 10^{-7}$ N A^{-2}	exact
fine-structure constant	$\alpha = e^2/4\pi\epsilon_0\hbar c$	1/137.035 989 5(61)†	0.045
classical electron radius	$r_e = e^2/4\pi\epsilon_0 m_e c^2$	2.817 940 92(38)$\times 10^{-15}$ m	0.13
electron Compton wavelength	$\lambda\!\!\!^{-}_e = \hbar/m_e c = r_e\alpha^{-1}$	3.861 593 23(35)$\times 10^{-13}$ m	0.089
Bohr radius ($m_{\text{nucleus}} = \infty$)	$a_\infty = 4\pi\epsilon_0\hbar^2/m_e e^2 = r_e\alpha^{-2}$	0.529 177 249(24)$\times 10^{-10}$ m	0.045
wavelength of 1 eV/c particle	hc/e	1.239 842 44(37)$\times 10^{-6}$ m	0.30
Rydberg energy	$hcR_\infty = m_e e^4/2(4\pi\epsilon_0)^2\hbar^2 = m_e c^2\alpha^2/2$	13.605 698 1(40) eV	0.30
Thomson cross section	$\sigma_T = 8\pi r_e^2/3$	0.665 246 16(18) barn	0.27
Bohr magneton	$\mu_B = e\hbar/2m_e$	5.788 382 63(52)$\times 10^{-11}$ MeV T^{-1}	0.089
nuclear magneton	$\mu_N = e\hbar/2m_p$	3.152 451 66(28)$\times 10^{-14}$ MeV T^{-1}	0.089
electron cyclotron freq./field	$\omega^e_{\text{cycl}}/B = e/m_e$	1.758 819 62(53)$\times 10^{11}$ rad s^{-1} T^{-1}	0.30
proton cyclotron freq./field	$\omega^p_{\text{cycl}}/B = e/m_p$	9.578 830 9(29)$\times 10^{7}$ rad s^{-1} T^{-1}	0.30
gravitational constant	G_N	6.672 59(85)$\times 10^{-11}$ m^3 kg^{-1} s^{-2}	128
		= 6.707 11(86)$\times 10^{-39}$ $\hbar c$ (GeV/c^2)$^{-2}$	128
standard grav. accel., sea level	g	9.806 65 m s^{-2}	exact
Avogadro constant	N_A	6.022 136 7(36)$\times 10^{23}$ mol^{-1}	0.59
Boltzmann constant	k	1.380 658(12)$\times 10^{-23}$ J K^{-1}	8.5
		= 8.617 385(73)$\times 10^{-5}$ eV K^{-1}	8.4
molar volume, ideal gas at STP	$N_A k$(273.15 K)/(101 325 Pa)	22.414 10(19)$\times 10^{-3}$ m^3 mol^{-1}	8.4
Wien displacement law constant	$b = \lambda_{\max}T$	2.897 756(24)$\times 10^{-3}$ m K	8.4
Stefan-Boltzmann constant	$\sigma = \pi^2 k^4/60\hbar^3 c^2$	5.670 51(19)$\times 10^{-8}$ W m^{-2} K^{-4}	34
Fermi coupling constant‡	$G_F/(\hbar c)^3$	1.166 39(2)$\times 10^{-5}$ GeV^{-2}	20
weak mixing angle	$\sin^2\hat{\theta}(M_Z)$ ($\overline{\text{MS}}$)	0.2319(5)	2200
$W^\pm$ boson mass	m_W	80.22(26) GeV/c^2	3200
Z^0 boson mass	m_Z	91.187(7) GeV/c^2	77
strong coupling constant	$\alpha_s(m_Z)$	0.116(5)	43000

π = 3.141 592 653 589 793 238 $\quad$ e = 2.718 281 828 459 045 235 $\quad$ γ = 0.577 215 664 901 532 861

1 in $\equiv$ 0.0254 m	1 G $\equiv 10^{-4}$ T	1 eV = 1.602 177 33(49) $\times 10^{-19}$ J	kT at 300 K = [38.681 49(33)]$^{-1}$ eV
1 Å $\equiv$ 10 nm	1 dyne $\equiv 10^{-5}$ N	1 eV/c^2 = 1.782 662 70(54) $\times 10^{-36}$ kg	0° C $\equiv$ 273.15 K
1 barn $\equiv 10^{-28}$ m^2	1 erg $\equiv 10^{-7}$ J	2.997 924 58 $\times 10^9$ esu = 1 C	1 atmosphere $\equiv$ 760 torr $\equiv$ 101 325 Pa

* The meter is defined to be the length of path traveled by light in vacuum in 1/299 792 458 s. See B.W. Petley, Nature **303**, 373 (1983).
† At $Q^2 = 0$. At $Q^2 \approx m_W^2$ the value is approximately 1/128.
‡ See discussion in Sec. 26 "Standard Model of electroweak interactions."

Properties of Materials

$Z/A \sim \frac{1}{2}$
$V \sim A \sim a^3$
$\sigma_I \sim A^{2/3}$
$[N_o\rho/A]\langle L\rangle\sigma = 1$

[# targets(nuclei)/volume]
longitudinal distance (transverse distance^2)
$\rho\langle L\rangle = A/\sigma$
$\sim A^{1/3}$
$dE/d(\rho x) \sim 1.5$ MeV/(gm/cm^2)
$\rho X_o \sim 1/Z$

ATOMIC AND NUCLEAR PROPERTIES OF MATERIALS

. Table revised June 1994. Gases are evaluated at 20°C, 1 atm, (in parentheses) or at STP [square brackets].

Material	Z	A	Nuclear[a] total cross section σ_T [barn]	Nuclear[b] inelastic cross section σ_I [barn]	Nuclear[c] collision length λ_T [g/cm^2]	Nuclear[c] interaction length λ_I [g/cm^2]	$dE/dx\|_{min}$[d] $\left[\frac{\text{MeV}}{\text{g/cm}^2}\right]$ () is for gas	Radiation length[e] X_0 [g/cm^2] () is for gas	Radiation length[e] X_0 [cm] () is for gas	Density[f] [g/cm^3] () is for gas [g/ℓ]	Refractive index n[f] () is $(n-1)\times 10^6$ for gas
H_2 gas	1	1.01	0.0387	0.033	43.3	50.8	(4.103)	61.28	865	(0.0838)[0.090]	[140]
H_2 (B.C., 26K)	1	1.01	0.0387	0.033	43.3	50.8	4.045	61.28	865	0.0708	1.112
D_2	1	2.01	0.073	0.061	45.7	54.7	(2.052)	122.6	757	0.162[0.177]	1.128
He	2	4.00	0.133	0.102	49.9	65.1	(1.937)	94.32	755	0.125[0.178]	1.024[35]
Li	3	6.94	0.211	0.157	54.6	73.4	1.639	82.76	155	0.534	—
Be	4	9.01	0.268	0.199	55.8	75.2	1.594	65.19	35.3	1.848	—
C	6	12.01	0.331	0.231	60.2	86.3	1.745	42.70	18.8	2.265[g]	—
N_2	7	14.01	0.379	0.265	61.4	87.8	(1.825)	37.99	47.0	0.808[1.25]	1.205[300]
O_2	8	16.00	0.420	0.292	63.2	91.0	(1.801)	34.24	30.0	1.14[1.43]	1.22[266]
Ne	10	20.18	0.507	0.347	66.1	96.6	(1.724)	28.94	24.0	1.207[0.900]	1.092[67]
Al	13	26.98	0.634	0.421	70.6	106.4	1.615	24.01	8.9	2.70	—
Si	14	28.09	0.660	0.440	70.6	106.0	1.664	21.82	9.36	2.33	—
Ar	18	39.95	0.868	0.566	76.4	117.2	(1.519)	19.55	14.0	1.40[1.782]	1.233[283]
Ti	22	47.88	0.995	0.637	79.9	124.9	1.476	16.17	3.56	4.54	—
Fe	26	55.85	1.120	0.703	82.8	131.9	1.451	13.84	1.76	7.87	—
Cu	29	63.55	1.232	0.782	85.6	134.9	1.403	12.86	1.43	8.96	—
Ge	32	72.59	1.365	0.858	88.3	140.5	1.371	12.25	2.30	5.323	—
Sn	50	118.69	1.967	1.21	100.2	163	1.264	8.82	1.21	7.31	—
Xe	54	131.29	2.120	1.29	102.8	169	(1.255)	8.48	2.77	3.057[5.858]	[705]
W	74	183.85	2.767	1.65	110.3	185	1.145	6.76	0.35	19.3	—
Pt	78	195.08	2.861	1.708	113.3	189.7	1.129	6.54	0.305	21.45	—
Pb	82	207.19	2.960	1.77	116.2	194	1.123	6.37	0.56	11.35	—
U	92	238.03	3.378	1.98	117.0	199	1.082	6.00	≈0.32	≈18.95	—
Air, (20°C, 1 atm.), [STP]					62.0	90.0	(1.815)	36.66	[30420]	(1.205)[1.29]	(273)[293]
H_2O					60.1	84.9	1.991	36.08	36.1	1.00	1.33
CO_2					62.4	90.5	(1.819)	36.2	[18310]	[1.977]	[410]
Shielding concrete[h]					67.4	99.9	1.711	26.7	10.7	2.5	—
Borosilicate glass (Pyrex)[ℓ]					66.2	97.6	1.695	28.3	12.7	2.23	1.474
SiO_2 (fused quartz)[m]					67.0	99.2	1.697	27.05	11.7	2.32[m]	1.458
Methane (CH_4)					54.7	74.0	(2.417)	46.5	[64850]	0.423[0.717]	[444]
Ethane (C_2H_6)					55.73	75.71	(2.304)	45.66	[34035]	0.509(1.356)[n]	(1.038)[n]
Propane (C_3H_8)					—	—	(2.262)	—	—	(1.879)	
Isobutane ($(CH_3)_2CHCH_3$)					56.3	77.4	(2.239)	45.2	[16930]	[2.67]	[1900]
Octane, liquid ($CH_3(CH_2)_6CH_3$)					—	—	2.123	—	—	0.703	
Paraffin wax ($CH_3(CH_2)_nCH_3$, $\langle n\rangle \approx 25$)					—	—	2.087	—	—	0.93	
Nylon, type 6					—	—	1.974	—	—	1.14	
Polycarbonate (Lexan)					—	—	1.886	—	—	1.200	
Polyethylene terephthlate (Mylar) ($C_5H_4O_2$)					60.2	85.7	1.848	39.95	28.7	1.39	—
Polyethylene (monomer $CH_2 = CH_2$)					56.9	78.8	2.076	44.8	≈47.9	0.92–0.95	—
Polyimide film (Kapton)					—	—	1.820	—	—	1.420	
Polymethylmethacralate (Lucite, Plexiglas) (monomer ($CH_2 = C(CH_3)CO_2CH_3$))					59.2	83.6	1.929	40.55	≈34.4	1.16–1.20	≈1.49
Polystyrene, scintillator (monomer $C_6H_5CH{=}CH_2$)					58.4	82.0	1.936	43.8	42.4	1.032	1.581
Polytetrafluoroethylene (Teflon) (monomer $CF_2 = CF_2$)					—	—	1.671	—	—	2.20	
Polyvinyltolulene, scintillator (monomer 2-$CH_3C_6H_4CH{=}CH_2$)					—	—	1.956	—	—	1.032	
Barium fluoride (BaF_2)					92.1	146	1.303	9.91	2.05	4.89	1.56
Bismuth germanate (BGO) ($Bi_4Ge_3O_{12}$)					97.4	156	1.251	7.98	1.12	7.1	2.15
Cesium iodide (CsI)					—	—	1.243	—	—	4.51	
Lithium fluoride (LiF)					62.00	88.24	1.614	39.25	14.91	2.632	1.392
Sodium fluoride (NaF)					66.78	97.57	1.69	29.87	11.68	2.558	1.336
Sodium iodide (NaI)					94.8	152	1.305	9.49	2.59	3.67	1.775
Silica Aerogel[o]					65.5	95.7	1.83	29.85	≈150	0.1–0.3	1.0+0.25ρ
NEMA G10 plate[p]					62.6	90.2	1.87	33.0	19.4	1.7	—

Properties of Materials

Table 1: Physical properties of some materials used in calorimeters.

	Z	ρ g.cm^{-3}	I/Z eV	$(1/\rho)dT/dx$ MeV/g.cm^{-3}	ε MeV	X_0 cm	λ_{int} cm
C	6	2.2	12.3	1.85	103	~ 19	38.1
Al	13	2.7	12.3	1.63	47	8.9	39.4
Fe	26	7.87	10.7	1.49	24	1.76	16.8
Cu	29	8.96		1.40	~ 20	1.43	15.1
W	74	19.3		1.14	~ 8.1	0.35	9.6
Pb	82	11.35	10.0	1.14	6.9	0.56	17.1
U	92	18.7	9.56	1.10	6.2	0.32	10.5

Ionization energy
~ Eo = 13.6 eV
critical energy ~ 1/Z
Xo ~ (A/Z)(1/Z) ~ 1/Z
$\langle L \rangle \sim A^{1/3}/\rho$

Energy, Size, Coupling

Electromagnetic and Strong Coupling

$$a \ = e^2 / \hbar c \sim 1/137$$

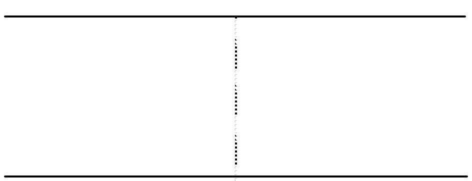
$$a_s = g_s^2 / \hbar c \sim (1/10 - 1)\ (Table\ 1.1)$$

$$E_o = -mc^2 a^2 / 2 \quad (Table\ 1.1,\ Rydberg)$$

$$m_e c^2 = 0.51 MeV, E_o = 13.6\ eV$$

$a_o \sim \lambda/\alpha = 0.56$ Angstroms
$E_n = E_o/n^2$

$a_n = a_o/n^2$



Amplitude ~ α

Binding Energy ~ α^2

Atoms - Ionization Energy and Size

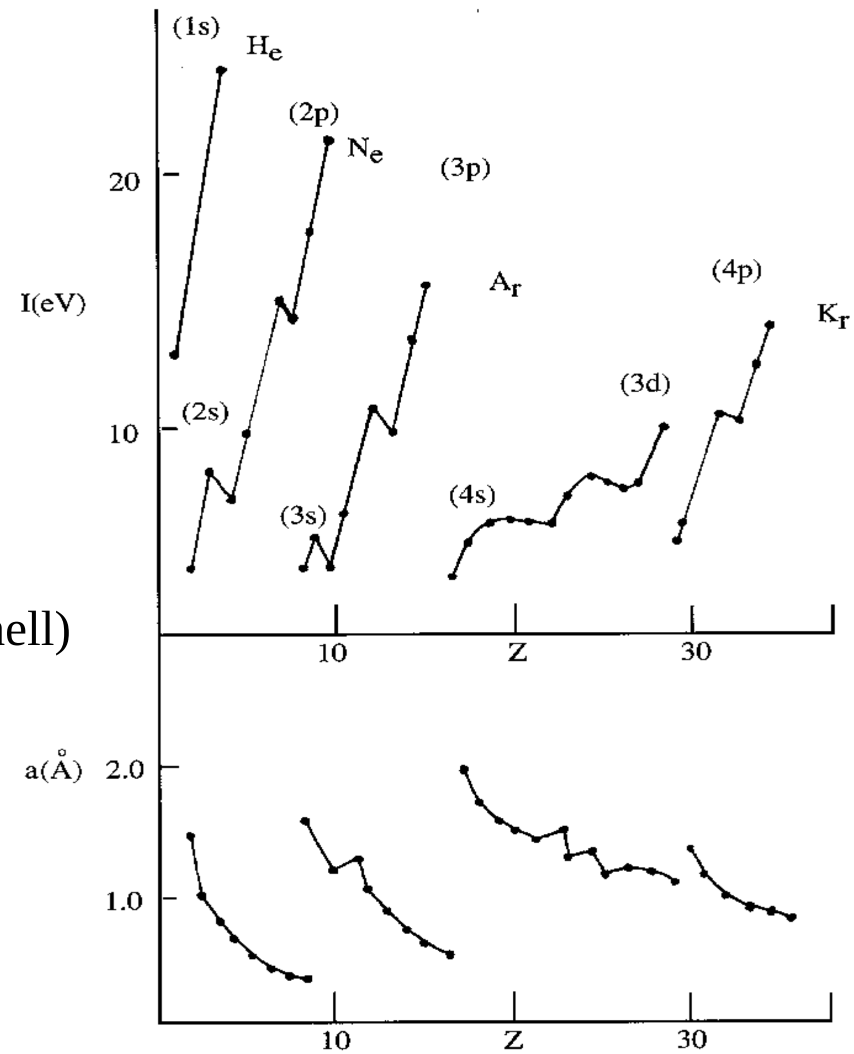
n = 1, l = 0 ml = 1 spin = 2
n = 2, l = 0, 2 states
l = 1, 6 states [(2l+1)2]

metals → noble gases

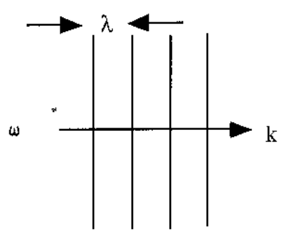
centrifugal repulsive potential
$U \sim L^2/r^2$
Lower l fills first

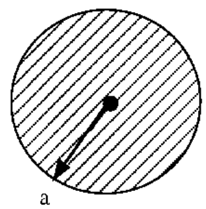
Metals "see" 1 e of charge (closed shell)

Lightly bound, $E \sim 1/n^2$, $a \sim n^2$



Cross Sections





$$N(x) = N(0)\exp(-N_o r x s \ / \ A)$$

$$\langle L \rangle^{-1} = [N_o r \, s \ / \ A](cm)^{-1}$$

$$\langle L r \rangle^{-1} = [N_o \, s \ / \ A](gm / cm^2)^{-1}$$

$$s_{atom} \sim p \, a_o^2 \sim 3x10^8 \, b, a_o \sim 1 \overset{o}{A}$$

$$s_{nuc} \sim p \, a_N^2 \sim 31 mb, a_N \sim 1 \, fm$$

$$(\hbar c)^2 = 0.4 \, GeV^2 \, mb$$

$$1mb = 10^{-27} \, cm^2, 1b = 10^{-24} \, cm^2$$

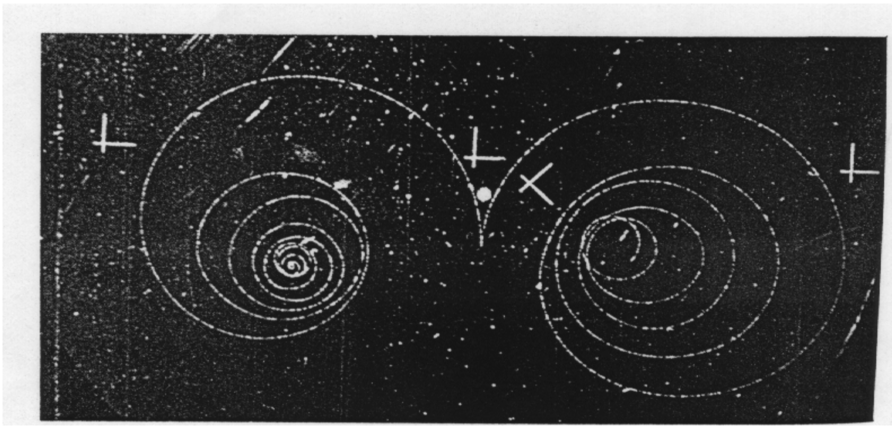
$$s_N \sim p \, a_N^2$$

$$\sim A^{2/3}$$

$$\langle L \rangle \sim A^{1/3} \qquad (Eq. \, 1.8)$$

$$l_I \sim (35 \, gm / cm^2) A^{1/3}$$

Nuclear vs EM Cross Sections



$$s_N \sim A^{2/3} \bar{\lambda}_p^2$$

$$s_B \sim (Za)^2 a \bar{\lambda}_e^2$$

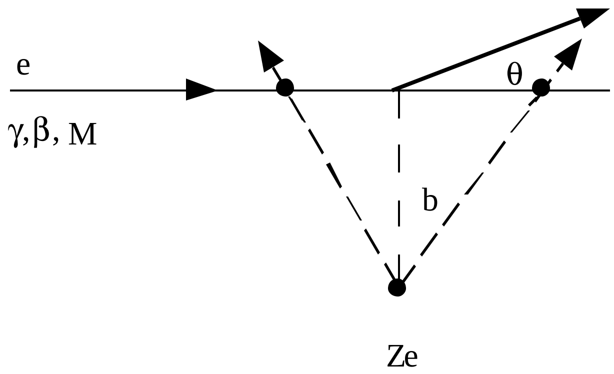
$$l_1 / X_o \sim (Z/A)/[5.1A^{2/3}]$$

see Properties
cross at Z ~ 3
at high Z, Xo <<λ, Why
basis of calorimetric particle ID

Ionization and dE/dx

- Coulomb Collisions
- Multiple Scattering
- "Radiation" Length
- Recoil Energy Distribution
- dE/dx
- Minimum Ionizing Particles
- Range and Momentum

Coulomb Scattering



$$dP \sim d\vec{b} = ds$$

Can’t aim

$$ds \sim bdbdf = \frac{ds}{d\Omega} d\Omega,\ d\Omega = \sin q\, dq df$$

$$\frac{ds}{d\Omega} = \frac{b}{\sin q}\left(\frac{db}{dq}\right)$$

Just relabelling b → θ

$$F(b) = Ze^2 / b^2$$

$$\Delta t = 2b / v$$

$$\frac{ds_R}{d\Omega} \sim \left(\frac{Za}{Mv^2}\right)^2 / q^4$$

Rutherford scattering off the nucleus

$$\Delta p_T \sim F(b)\Delta t,\ \vec{F} \equiv d\vec{p} / dt$$

$$q \sim \Delta p_T / p,\ \Delta p_T = 2Za / bv$$

$$q_R \sim 2Za / pvb$$

$$s \sim \int_{q_{\min}}^{\infty} \left(\frac{ds}{d\Omega}\right) 2p q dq = \int_{o}^{a_o} 2p b db =$$

$$\sim p a_o^2 \sim 1/q^2{}_{min} \sim \int_{q_{\min}}^{\infty} dq / q^3$$

Δp_T → $2Z\alpha/bc$

independent of incident particle mass, velocity

→ MIP

Multiple Scattering

Mean square scattering angle

$$\left\langle q^2 \right\rangle \equiv \frac{\int q^2 \frac{ds}{d\Omega} d\Omega}{\int \frac{ds}{d\Omega} d\Omega} \sim \frac{\int (dq/q)}{\int (dq/q^3)}, \quad Eq.5.4$$

$$\sim 2q^2_{\min} \left[\ln(q_{\max}/q_{\min}) \right]$$

$$\left\langle q^2_{MS} \right\rangle = N \left\langle q^2 \right\rangle$$

$$N = \left(N_o r\, s \;/\; A \right) dx, \;(Section\ 1)$$

$$= dx / \left\langle L \right\rangle$$

$$\left\langle q^2_{MS} \right\rangle = \left[\frac{N_o r\, dx}{A} \right] 2p \left[\frac{2Za}{pbc} \right]^2 \left[\ln(\;) \right]$$

Multiple Scattering - II

$$\left\langle q^2_{MS} \right\rangle = \frac{dx}{X_o} m^2 / \left(ab^2 p^2\right)$$

$$X_o^{-1} = \frac{16}{3}\left(\frac{Nr}{A}\right)\left(Z^2 a\right)\left(a^2 / m^2\right)[ln()]$$

Define for now, see brem later

$$E_s \equiv \sqrt{\frac{4p}{a}}(mc^2) = 21MeV$$

Radiation length

$$\sqrt{\left\langle q^2_{MS} \right\rangle} = \frac{E_s}{pb}\sqrt{\frac{dx}{X_o}}$$

Stochastic process – e.g. diffusion goes as √x

$$q_{MS} = \left(\Delta p_T\right)_{MS} / p, \left(\Delta p_T\right)_{MS} = \frac{E_s}{b}\sqrt{\frac{dx}{X_o}} = \frac{E_s}{b}\sqrt{t}$$

Recoil Energy Distribution

$$\Delta p_T \sim 2a / bv,\ Z = 1$$

$$\Delta e \sim \Delta p_T^2 / 2m$$

$$\Delta e \sim 2a^2 / b^2 v^2 m$$

Incoherent

$$ds = d\vec{b} = bdbdf = (ds / dT)dT$$

$$\frac{ds_d}{dT} = 2pb\left(\frac{db}{dT}\right) = \left[\frac{2pa^2}{b^2 c^2 T^2 m}\right]$$

Relabel b → T
Energy given to recoil
(n.b. e here not the nucleus since ~ 1/m)

ionize the e, kick out of the atom – I

Energy Loss in Scattering

$$de \sim \int 2pbdb\ (de / d\vec{b})$$

$$\sim \frac{4pa^2}{mv^2}\left[\ln\left(b_{\max} / b_{\min}\right)\right]$$

$$\frac{dE_1}{d(rx)} \sim \left(\frac{N_o Z}{A}\right) de$$

$$\frac{dE_1}{d(rx)} \sim 4p\left(\frac{N_o Z}{A}\right)\left(a^2 \bar{\lambda}_e\right)\left(\frac{1}{b^2}\right)[\ln(\ )]$$

$$\ln(b_{\max} / b_{\min}) \sim [\ln(T_{\max} / \langle I \rangle)]$$

$$T_{\max} = 2m(bg)^2$$

$$\sim 2m \sim 1\,MeV\ (MIP)$$

$$\langle I \rangle \sim 10\,eV$$

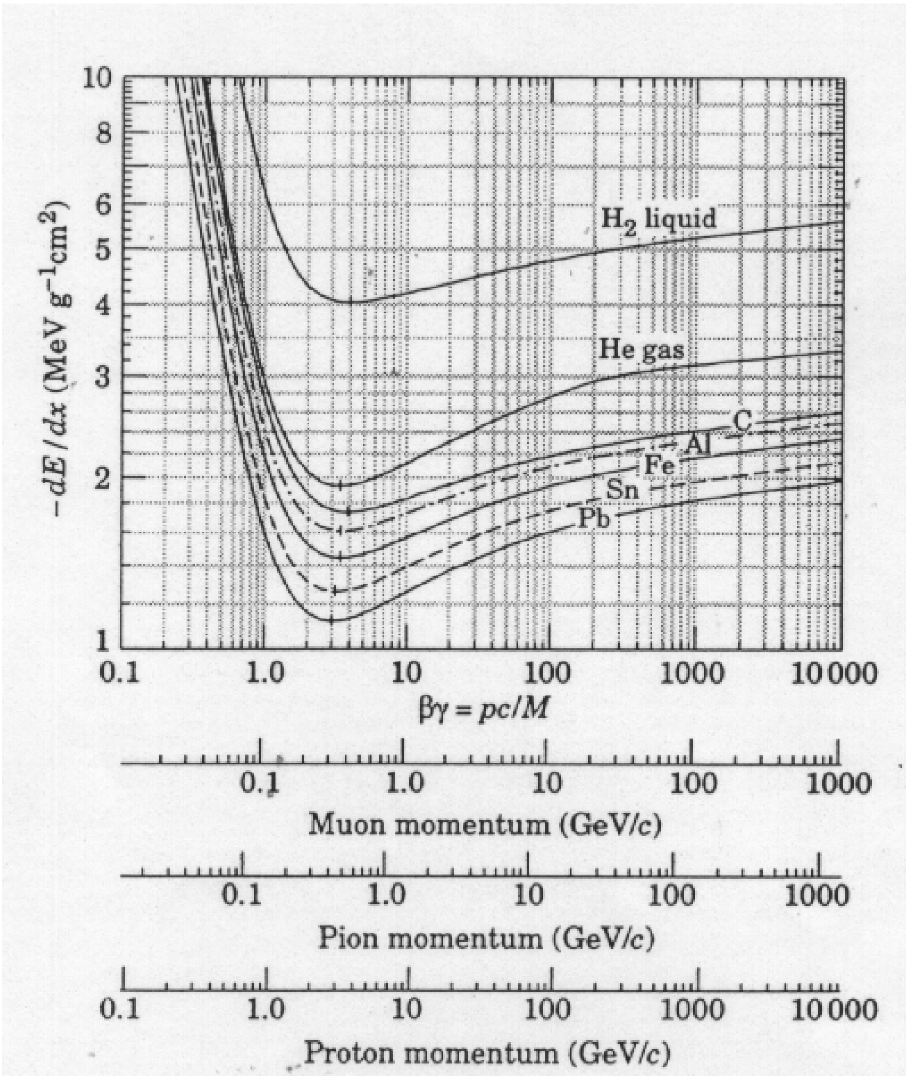
$$\ln(T_{\max} / \langle I \rangle) = 11.5$$

$d\varepsilon \rightarrow 4\pi\alpha^2/(mc^2)(1/\beta^2)(11.5)$

$\sim 3.0[Z/A]\ [MeV/gm/cm^2](1/\beta^2)$ numerically

medical uses of the Bragg peak

dE/dx and Momentum - MIP



$$\frac{dE_I}{dx} \sim \frac{1}{b^2} \sim \frac{M^2}{p^2}$$

$$T = \frac{p^2}{2M}$$

$$\frac{dE_1}{dx} \sim \frac{1}{p^2} \sim \frac{1}{T} \sim \frac{dT}{dx}$$

Range and Momentum

Ultra-relativistic

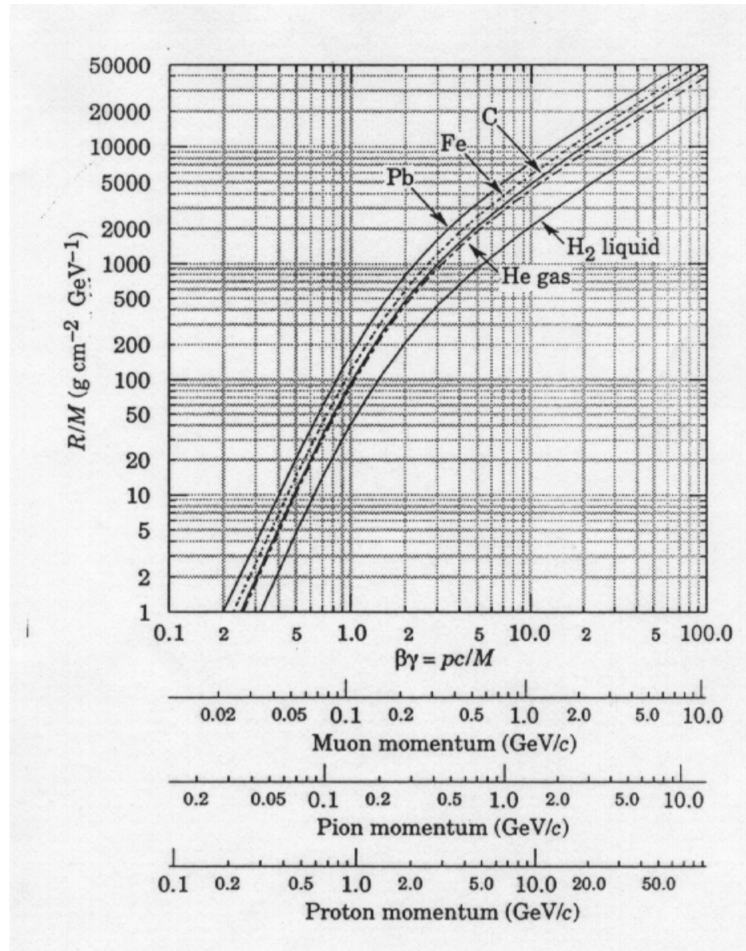
and

Non - relativistic

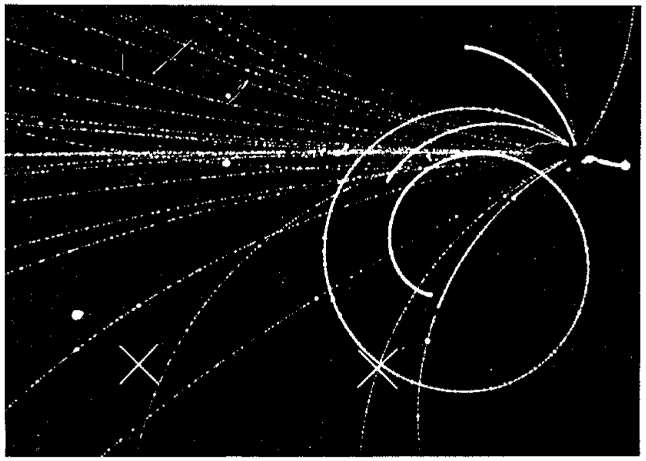
$$\left(\frac{dE_I}{d(rx)}\right)_{MIP} \quad \underline{R} \sim T_o \sim e_o \sim g$$

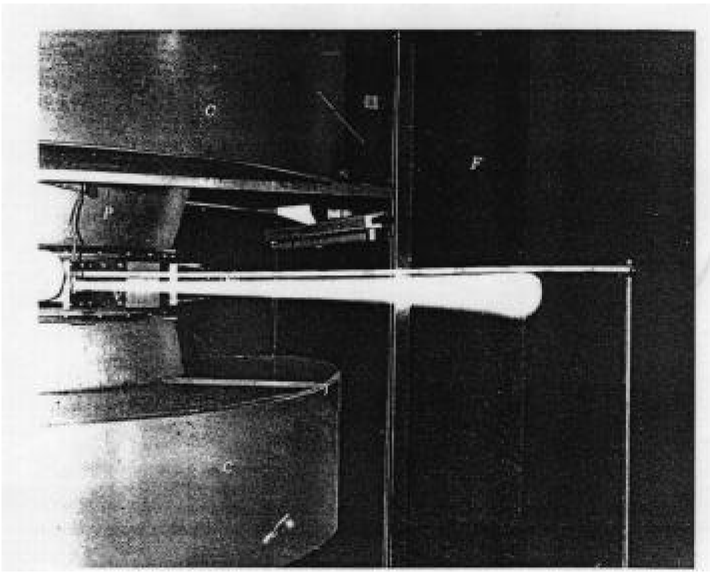
$$\int_{T_o}^{o} T\,dT = \int_{o}^{R} dx$$

$$\underline{R} \sim T_o^{\ 2} \sim p_o^{\ 4} \sim b_o^{\ 4}$$



Range and Incident Charge





if $\Delta p_T \sim F(b)\Delta t \sim z\alpha$
then $dE/dx \sim z^2(Z/A)$
i.e. 1, 4, 9, bubble density

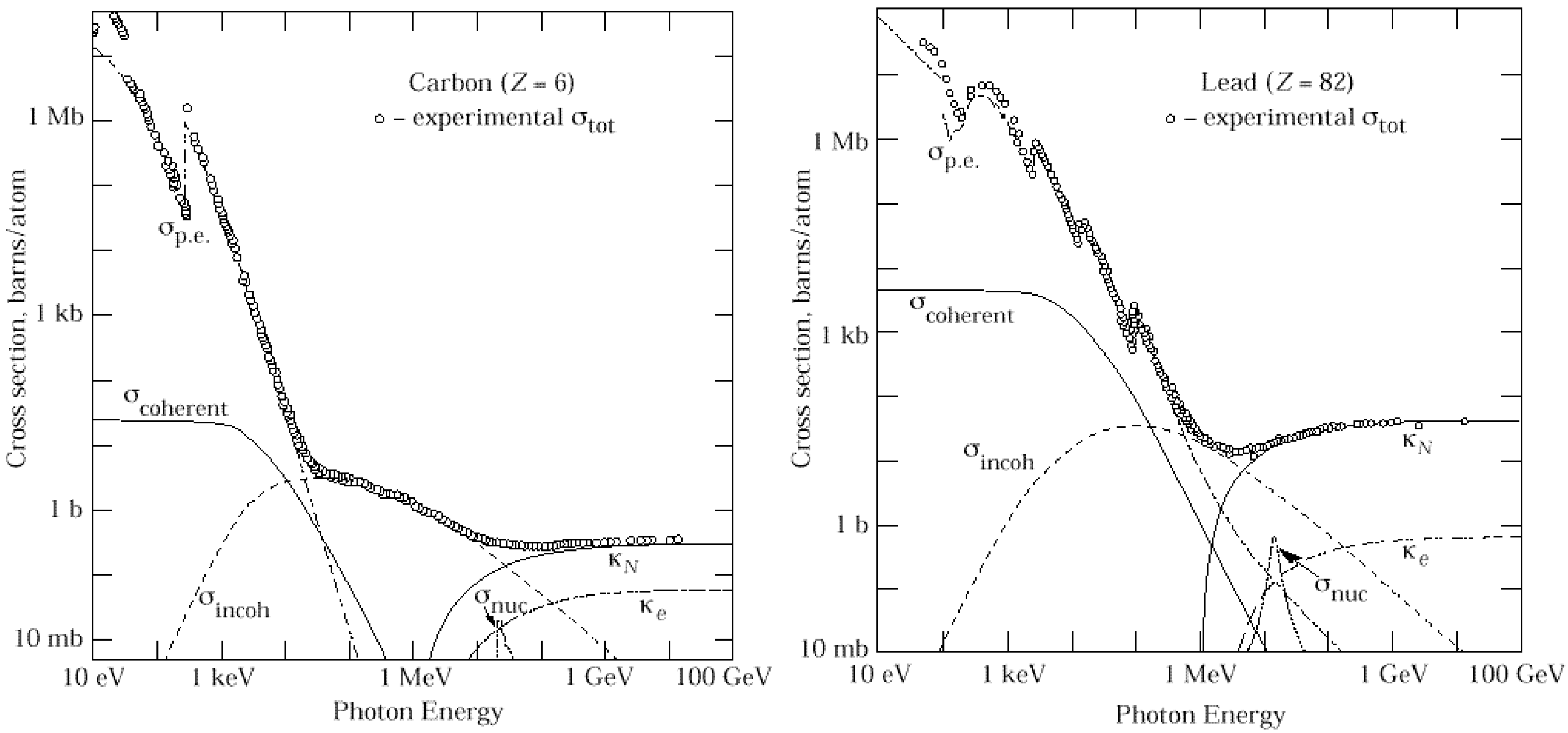
slow down toward the end of range
multiple scattering has $\theta \sim 1/p\beta$

Photon Interactions

- Cross section as a function of photon energy
- Photoelectric Effect
- Thompson Scattering
- Relativistic Photon Scattering
- Compton Effect
- Pair Production by Photons

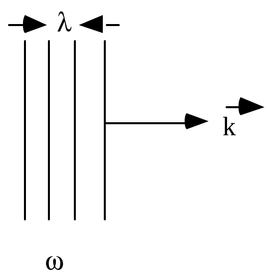
Photon Cross Sections

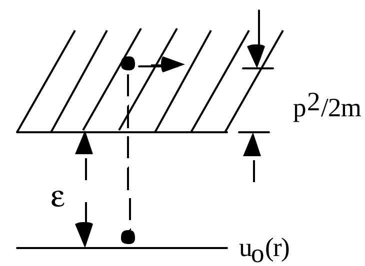
There are 3 regimes and a strong Z dependence



Photoelectric Effect

Photoeffect dominates for photon energy < (10-100) keV





The inner e "see" the full electron charge Z

$$u_o(r) \sim \frac{1}{\sqrt{pa^3}} e^{-r/a}$$

$$a_o = ƛ / a \,,\; a \sim a_o / Z$$

$$E_n = -\left[\frac{mc^2}{2}(Z a)^2\right]/n^2$$

$$= -13.6 eV\, Z^2 \,/ n^2$$

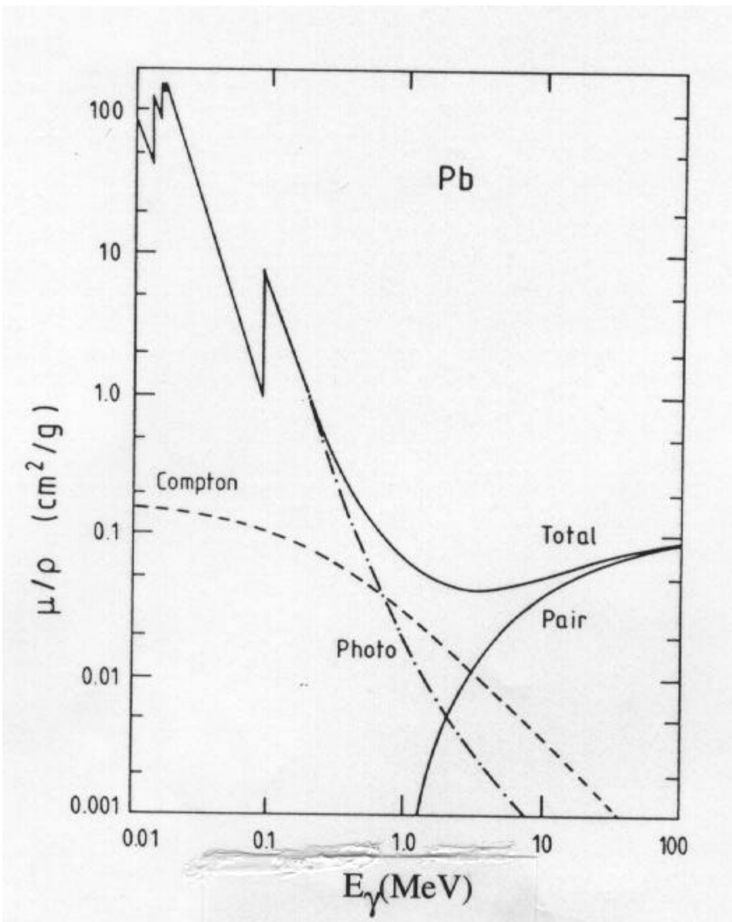
$$s_{PE} \sim a ƛ^2 \left[\frac{mc^2}{\hbar w}\right] \left[\, l_{DB} / a\right]^5$$

$$\sim 1/w^{7/2}$$

$$\hbar w \;\sim p^2 / 2m \quad (Eq.\, 2.2)$$

Photoelectric Effect - II

Inverse mean free path 1/[ρ<L>]



$$\sigma_{PE} \sim \frac{32\pi}{3}\sqrt{2}(Z\alpha)^4 Z\left(\frac{m}{\hbar\omega}\right)^{7/2}(\alpha\bar{\lambda})^2$$

$$\sigma_T \sim \frac{8\pi}{3}Z(\alpha\bar{\lambda})^2$$

$$\sigma_{PE}/\sigma_T \sim 4\sqrt{2}\,(Z\alpha)^4\left(m_e c^2/\hbar\omega\right)^{7/2}.$$

$\sigma \sim Z^5$

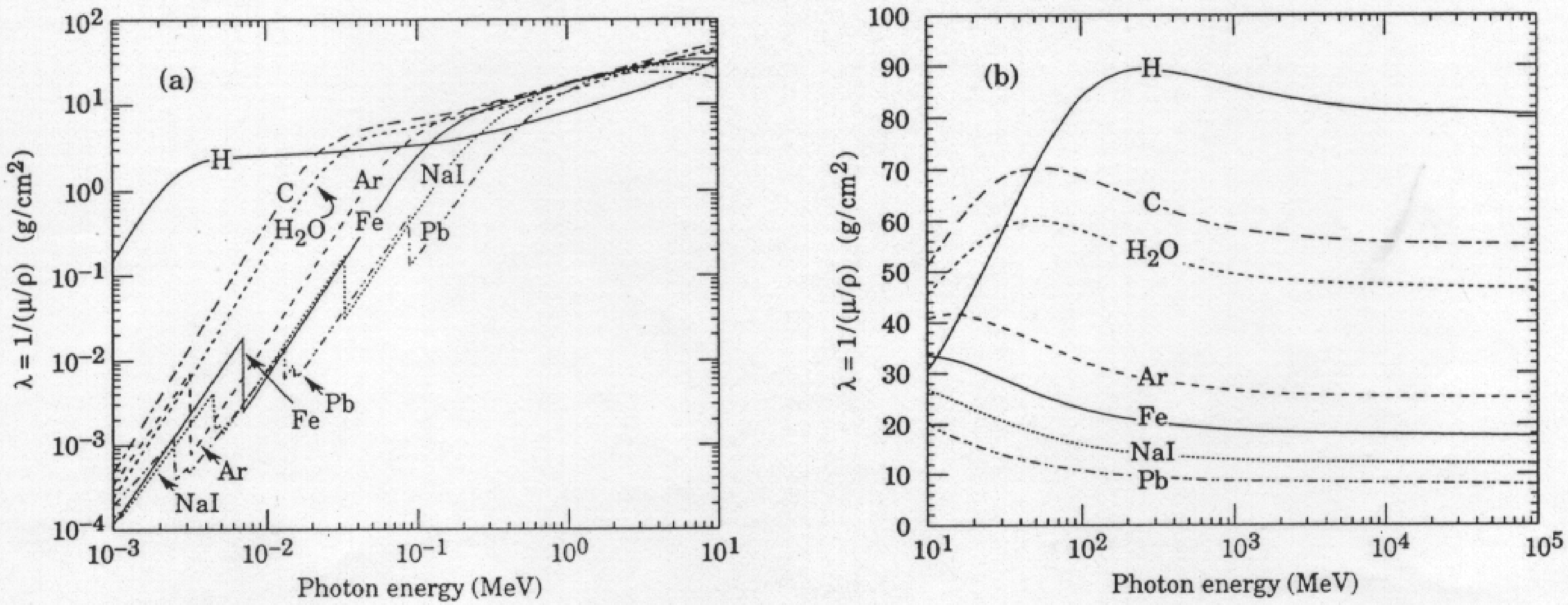
Compare Thompson (γ - e non-relativistic)

If (Zα) ~ 1 – Pb then the cross sections cross at ~ m_e = 0.51 MeV

Photon Mean Free Path

PHOTON AND ELECTRON ATTENUATION

Photon Attenuation Length



Dominated by the photoeffect

Z^5 and $1/\omega^{7/2}$

Thompson/Compton ~ constant ρ<L>
Energy independent

Pair production goes as Xo ~ 1/Z
Energy independent

Photon - e Scattering

Dipole – Larmor, Non-relativistic

$$\frac{d\,\underline{P}}{d\Omega} = \frac{e^2}{4pc^3}|a|^2 \sin^2 q$$

$$a = eE_o / m$$

$$s_{KN} \sim \frac{3}{8} s_T \left(\frac{m}{\sqrt{s}}\right)^2 [1 + ln(\ \ )] \sim a^2/s$$

$$\frac{ds_T}{d\Omega} = \left(e^2/mc^2\right)^2 \sin^2 q \equiv \left\langle d\,\underline{P}/d\Omega\right\rangle / \left\langle\left|\vec{S}\right|\right\rangle$$

$$s_T = \frac{8p}{3}(a\lambda\!\!\!^{-})^2$$

Noρ<L>Zσ_T/A ~ 1

$$s_T = \frac{8}{3}\left[pa_0^2\right]a^4$$

ρ<L> ~ (A/Z) ~ const

$$a_o = \lambda\!\!\!^{-}/a \quad (Section\ \ 1)$$

see photon cross section figure

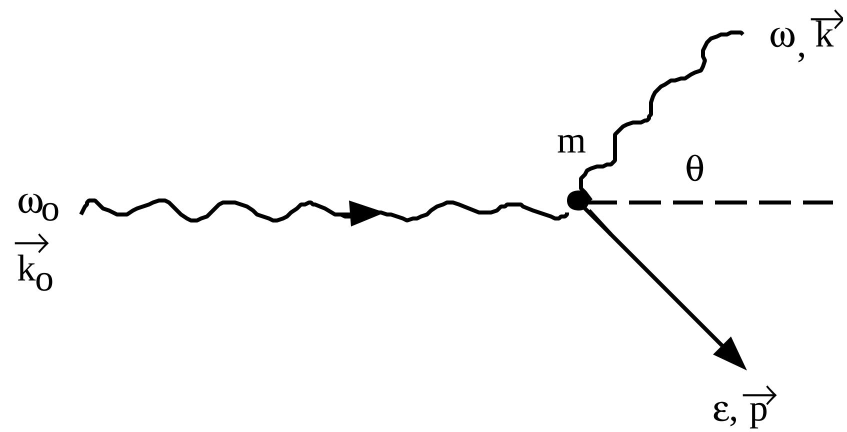
$$s_T / pa_0^2 \sim a^4 \sim 10^{-8}$$

Reduced by α^4 w.r.t. atomic geometric cross section

Compton Scattering - Kinematics

$$w_o + m = e + w$$

$$\vec{k}_o = \vec{p} + \vec{k}$$



$$\left(\frac{1}{w} - \frac{1}{w_o}\right) = \frac{1}{m}(1 - \cos q)$$

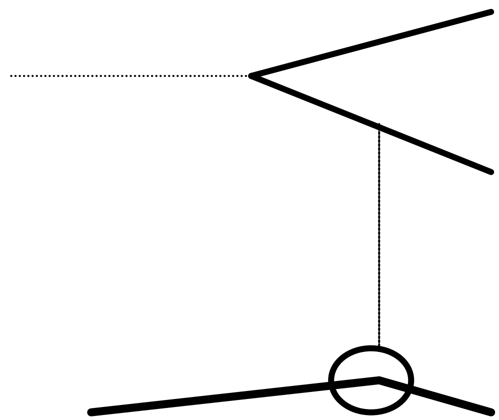
$$l - l_o = 2p\bar{\lambda}(1 - \cos q)$$

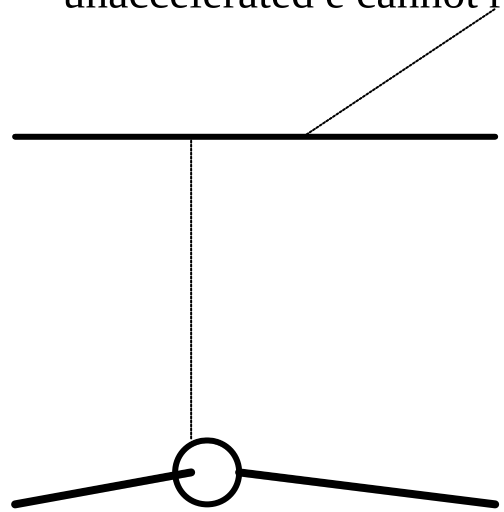
dynamics → e takes off most of the energy

Pair Production

isolated photon cannot "decay" into an e+e- pair

unaccelerated e cannot radiate a photon





g **+ Z --> e^+e^- + Z**

e + Z --> e + Z + g

Pair Production and Bremsstrahlung are topologically similar.

Defer the cross section discussion until radiation by charged particles in accelerated motion.

Electron Interactions

- Radiation in relativistic motion
- Linear and Circular acceleration
- Angular distributions
- Bremsstrahlung as the scattering off virtual photons of the target by the projectile
- Radiation length
- Critical energy

Relativistic Radiation

$$\underline{P} = \frac{2}{3}\frac{e^2}{c^3} A_m A^m$$
$$= \frac{2}{3}(e^2/c^3)\gamma^6 \left[|\vec{a}|^2 - |\vec{\beta} \times \vec{a}|^2 \right]$$

Linear

$$(\underline{P})_o = \frac{2}{3}\frac{e^2}{c^3}\gamma^4 |\vec{a}|^2$$

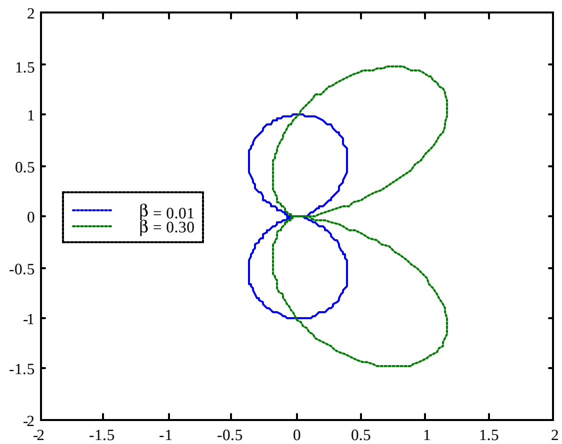
Circular

$$A_m = \left(\frac{\gamma\beta}{m}\right)\frac{d}{dx}\left[e\left(\vec{\beta},1\right)\right], \quad dx = \beta c dt$$
$$(\underline{P})_{LIN} = \frac{2}{3}\frac{e^2}{m^2 c^3}\left(\frac{de}{dx}\right)^2$$

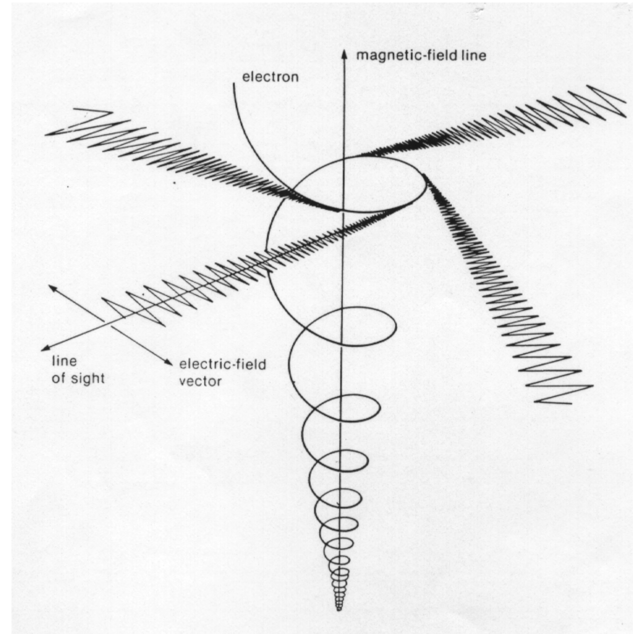
generalize Larmor

→ strong γ dependence
→ radiation is important at high energies

Angular Distributions



dipole tips forward at high γ



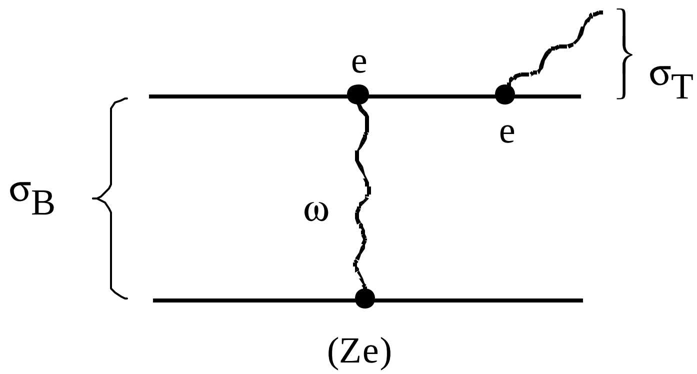
Linear Acceleration **Synchrotron Radiation - Circular**

complex in general but

$<\theta> \sim 1/\gamma$ - "searchlight effect"

→ calorimetry is approximately 1 dimensional

Bremsstrahlung



in a Coulomb collision we can decompose the fields of a charged particle into a distribution of "virtual quanta"

$$\frac{dN_g(w)}{dw} \sim \frac{a}{b^2}\left(\frac{1}{w}\right)[\ln(\ )]$$

$$= \frac{2a}{p}\left(\frac{1}{b^2}\right)\left(\frac{1}{w}\right)[\ln(\ )]$$

The Thompson scattering of the virtual quanta of the target by the projectile

but coherent!

Recall $d\varepsilon \sim \alpha/\beta^2[\ln(\ )](Z^2)$

$$\frac{ds_B}{dw} \sim Z^2 \frac{dN_g}{dw} s_T$$

$$\frac{ds_B}{dw} \sim \frac{(Z^2 a)}{w}(a\lambda\!\!\!^-)^2[\ln(\ )]$$

coherent because the nucleus ~ 1 fm is small on the scale of the projectile deBroglie wavelength. Thus no phase change over nucleus, amplitude ~ Z, cross section ~ Z^2

Radiation Length

$$dE \sim \int_o^E (\hbar w)\left(\frac{N_o r\,dx}{A}\right)\left(\frac{ds_B}{dw}\right)dw$$

$$\frac{1}{E}\left(\frac{dE}{r\,dx}\right) \equiv \frac{1}{X_o}, E(x) = E(o)e^{-rx/X_o}$$

By definition Xo is the bremsstrahlung mean free path

Look at the photon cross section table

$$s_B \equiv [A/(N_o rX_o)]$$

$$X_o = \langle L \rangle$$

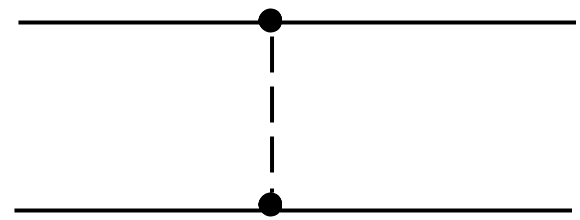
It is the projectile because that acceleration cause the radiation

$$X_o^{-1} = \frac{16}{3}\left(\frac{N_o}{A}\right)(Z^2 a)(a\bar{\lambda}_p)^2[\ln(\ )]$$

$$\sim (Z/A)Z \sim Z$$

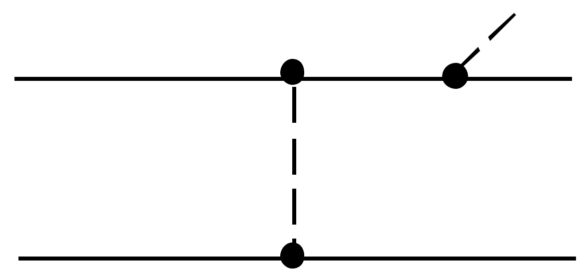
Critical Energy - Ionization and Bremsstrahlung

$$\frac{\frac{dE_B}{d(rx)}}{\frac{dE_I}{d(rx)}} \sim \frac{\frac{16}{3}\left(\frac{N_o}{A}\right)(Z^2 a)(a\bar{\lambda}_P)^2\left[\ln(\ )\right]E}{4p\left(\frac{N_o}{A}\right)(Z)(a^2\bar{\lambda}_T)\left[(\ln')\right]}$$



$Z\alpha^2$

radiation rises with E (relativity)
($Z\alpha$) coherence vs coupling
ionization transfers energy to the atomic e
radiation accelerates the projectile



$Z^2\alpha^3$

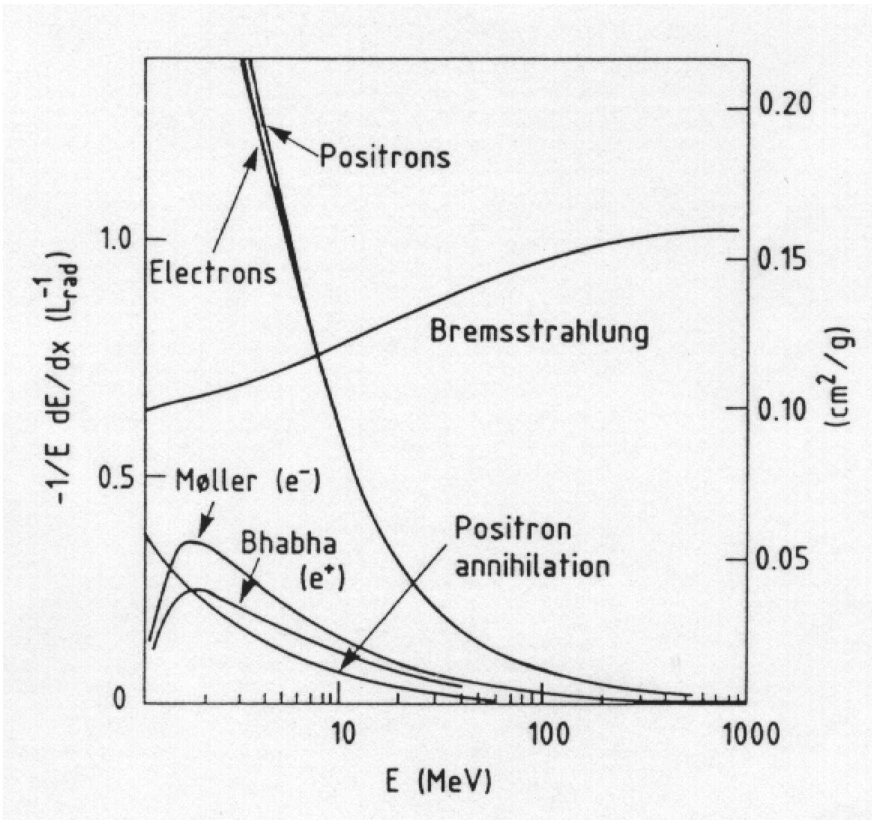
$$E_c \sim \frac{3p}{4}\left(\frac{m_P}{m_T}\right)\left[\frac{m_P c^2}{Za}\right]$$

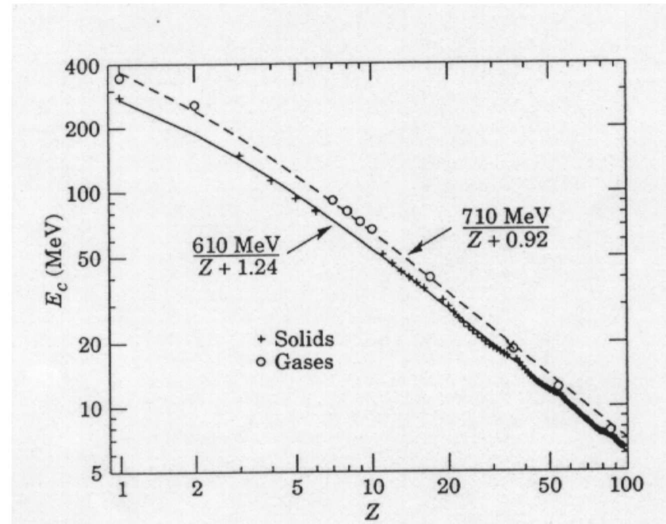
$$\sim 165\, MeV / Z$$

incoherent ionization – 2 vertices
coherent bremss – 3 vertice

Critical Energy

Pb





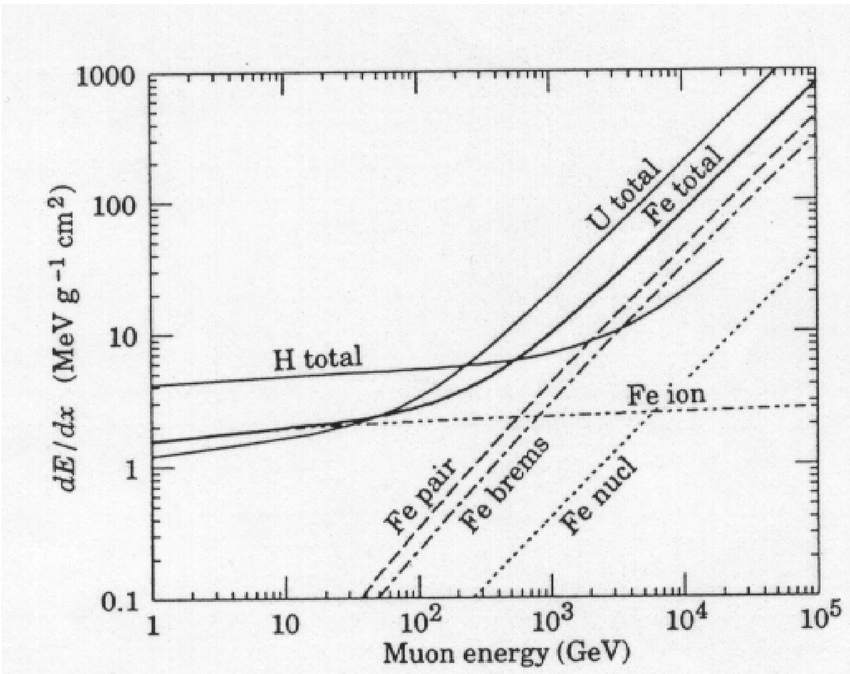
~ 1/Z behavior of Ec

EM cascade stops multiplying at Ec and begins to die out by ionization and photoeffect.

1/E[dE/dx] ~ 1/Xo

Ec ~ 7 MeV

Critical Energy for Muons



Same forces on μ and e
But accel ~ 1/m
And radiation ~ a^2
So much less muon radiation

Ec ~ (m_P^2)

So if 24 MeV for e on Fe
Then ~ (200)*(200) or ~ 1 TeV
For muon

Another process
e + Z → e + Z + "γ" → e + Z + e+e-
$\sigma_{pair} \sim \sigma_B$

EM Calorimetry

- Basic parameters
- The EM Cascade
- Energy Resolution
- Sampling Fluctuations
- Nobel Liquids - Pulse Formation
- Crystals
- Transverse Size
- Leakage
- Calibration

Basic Parameters

$$(dE_B / E) \sim (r\, dx) / X_o$$
$$X_o \sim [180\, gm / cm^2][A / Z^2]$$
$$t = x / X_o$$

Radiation Length
[Basic EM Length Unit]

$$(dE / dx) \sim -E_C / X_o$$
$$E_C \sim [550 (MeV)] / Z$$
$$y = E / E_C$$

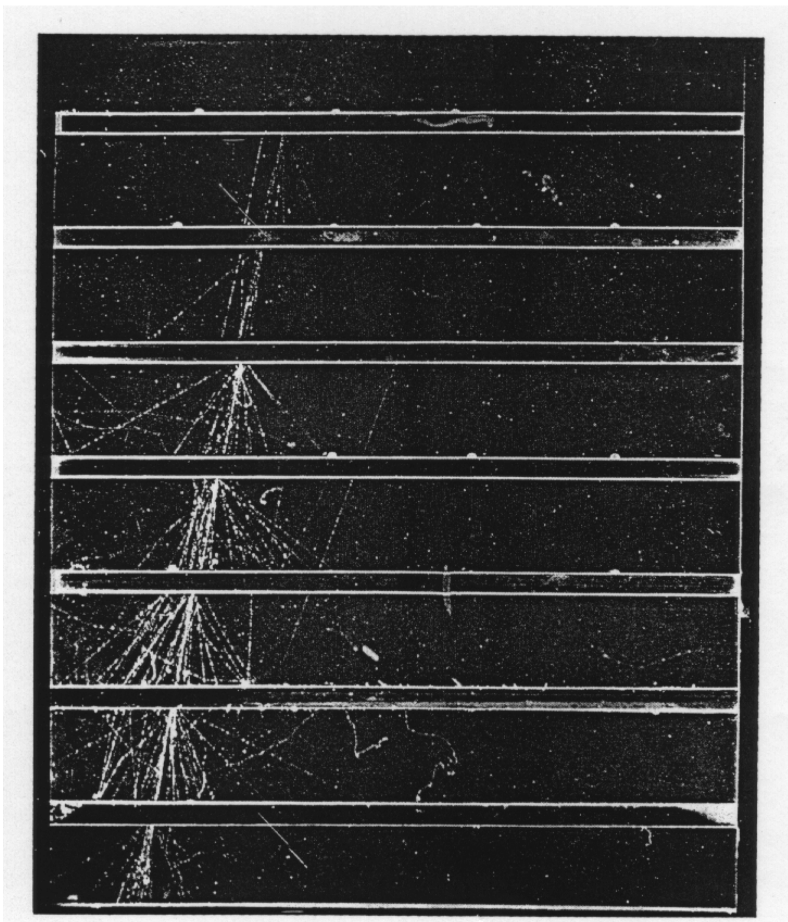
Critical Energy
[EM Energy Unit]

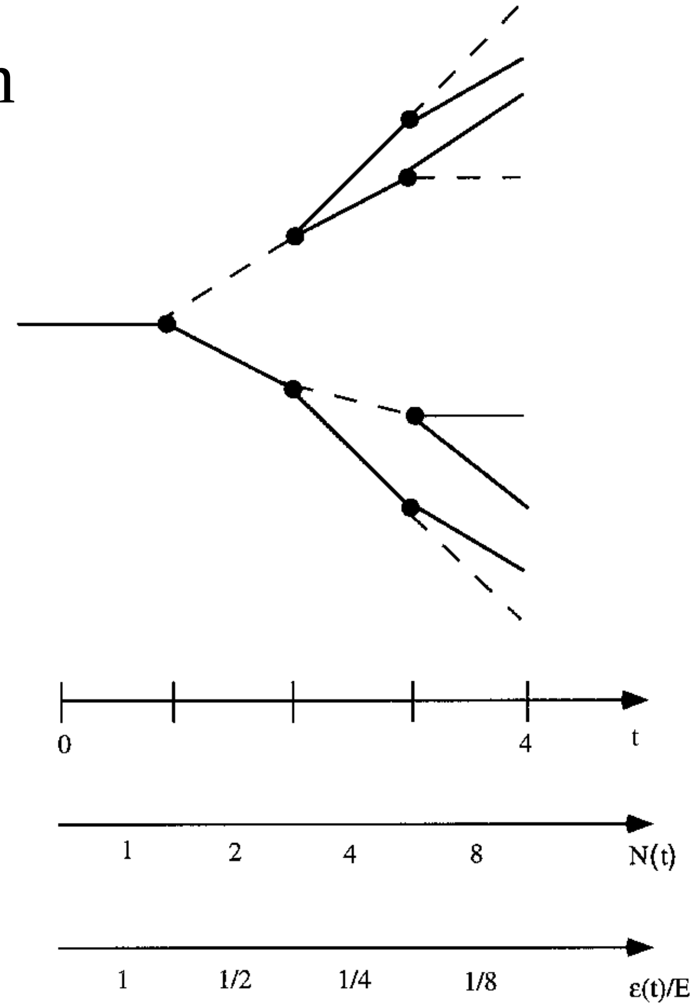
$$dE_I / dx \sim [3(gm / cm^2)][Z / A]$$

Ionization energy loss
[After shower ceases to grow]

EM Cascade

Bremss + Pair → multiplication





Simple EM Cascade Model

$$e(t) = E/2^t = E/N(t)$$

$$N(t) = 2^t$$

$$E_C = E/2^{t_{max}} = e(t_{\max}) = yE$$

$$t_{max} \sim \ln(y)$$

$$N_{max} \sim E/E_C = N(t_{\max}) = y$$

$$L \sim X_o \sum_{i=1}^{t_{max}} N(t)$$

$$\frac{L}{X_o} \sim \int_o^{N_{\max}} N(t)dt \sim \int_o^{t_{\max}} 2^t\,dt$$

$$\sim (E/E_C)/ln^2 = N_{max}^{ln2}$$

Geometric growth of cascade

Ignore Fluctuations:

But for initial interaction point
<L> = Xo
σ = Xo

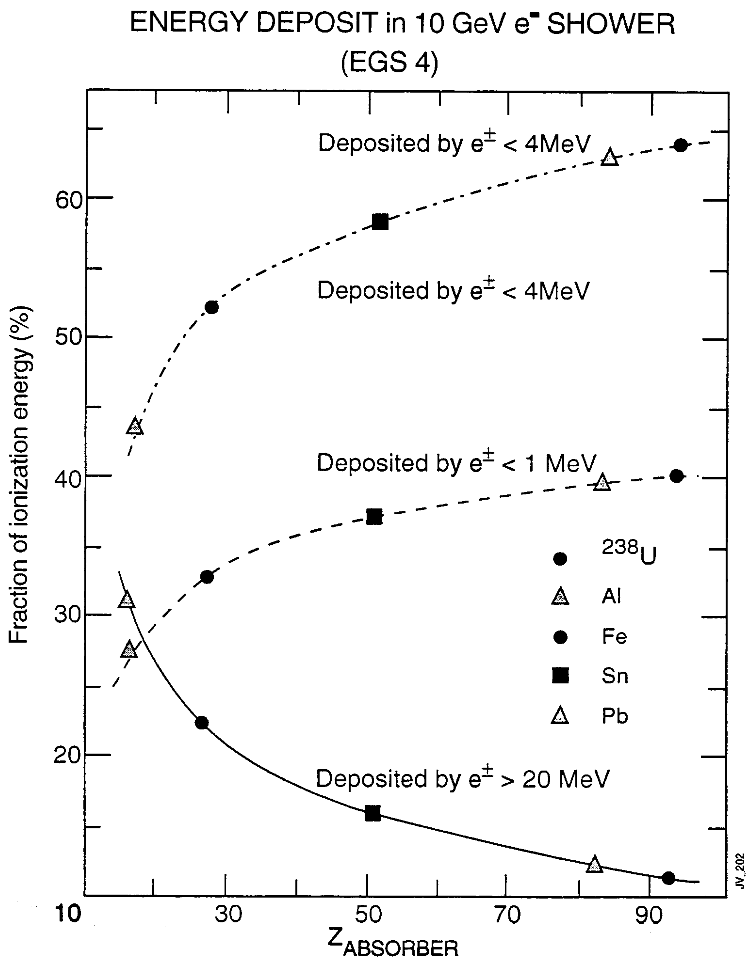
And energy sharing of daughters
- goes from 0 to full parent
- energy, = ½ on average

e.g. 1 GeV e in Pb
$N_{\max} \sim 140$

L = total path length
of all tracks in the shower

n. b. calorimetry is linear - non trivial
ears, eyes, dynamic range

EM Showers - Energy Deposit



After shower maximum there is
No more particle multiplication
But
Ec ~ 7 MeV in Pb
e – ionize
photons Compton scatter
or photoeffect ($\gamma \rightarrow$ e)

ultimately the shower dies off
$\rightarrow$ energy deposit is due to soft e

Stochastic Term - dE/E

$$N_{\max} = E / E_C$$

$$\frac{dE}{E} \sim 1/\sqrt{E} \sim dN / N$$

E,g, 1 GeV e in Pb
dE/E ~ 8 % due to
statistical fluctuations

Profile

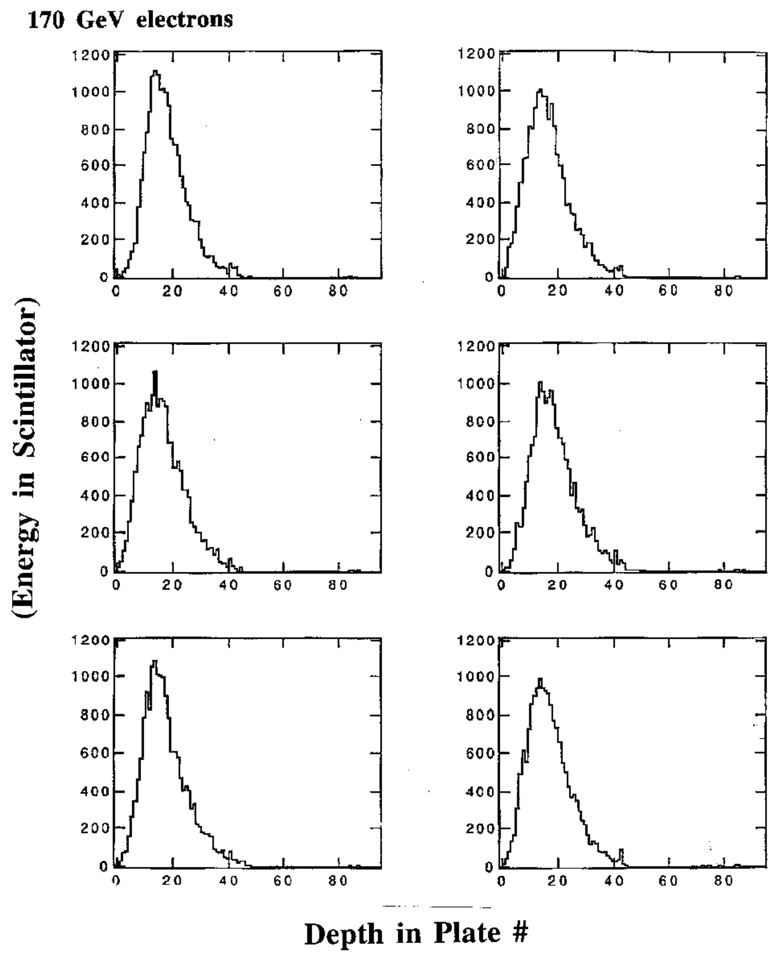
$$\frac{dE/E}{du} = \left[u^a e^{-u}\right] / \Gamma(a+1)$$

$$u = bt,\ b \sim 1/2$$

$$t_{\max} \sim \ln y \sim (a-1)/b$$

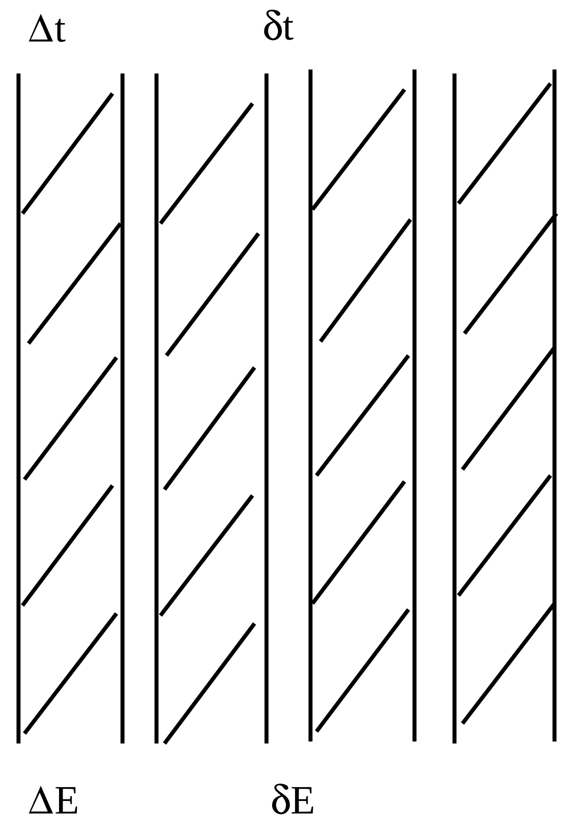
1 GeV e in Pb
t_{max} ~ 5

Individual showers ~ profile smeared by
+- Xo due to first interaction point fluctuations



Sampling Calorimetry

Shower develops in high Z inert plates
Shower is sampled in low Z detectors
(scintillator, gas chambers, Si, noble liquids, crystals)



$$N_S = L/\Delta t$$ number sampled

$$\sim (E/E_C)/\Delta t$$

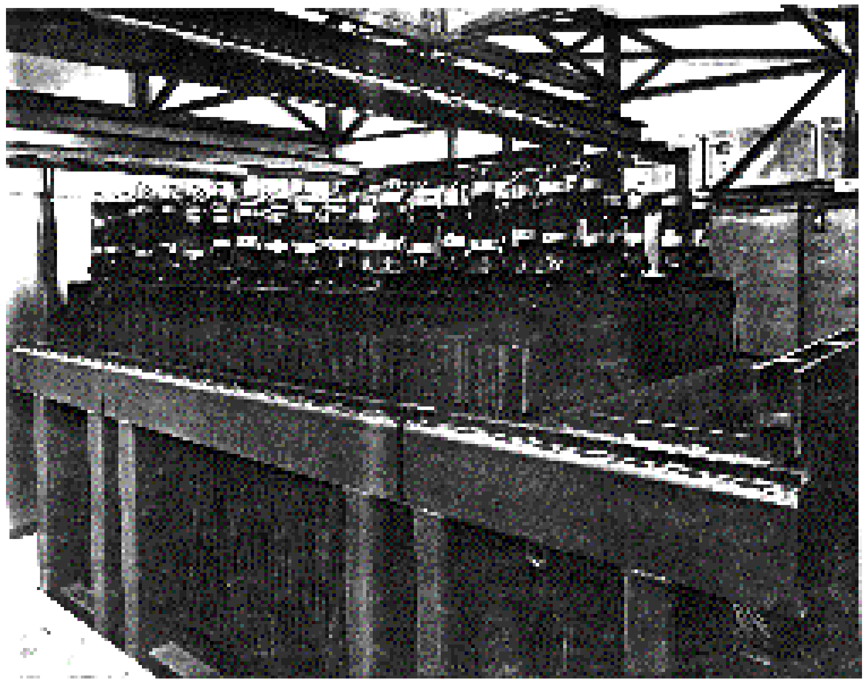
$$\left(\frac{dE}{E}\right)_{samp} = 1/\sqrt{N_S} \equiv a_{samp}/\sqrt{E}$$

$$a_{samp} \sim \sqrt{E_C \Delta t}$$

$$\sim \sqrt{\Delta E}$$

e.g. Pb with 0.5 Xo thick samples.
dE/E ~ 6%/√ E

Scint-WLS Calor



Sampling - II

$$\Delta t \to \Delta t / \langle \cos q_{MS} \rangle$$

$$q_{MS} \sim (E_S / E_C)\sqrt{\Delta t \sim 1} \quad (Section\,5)$$

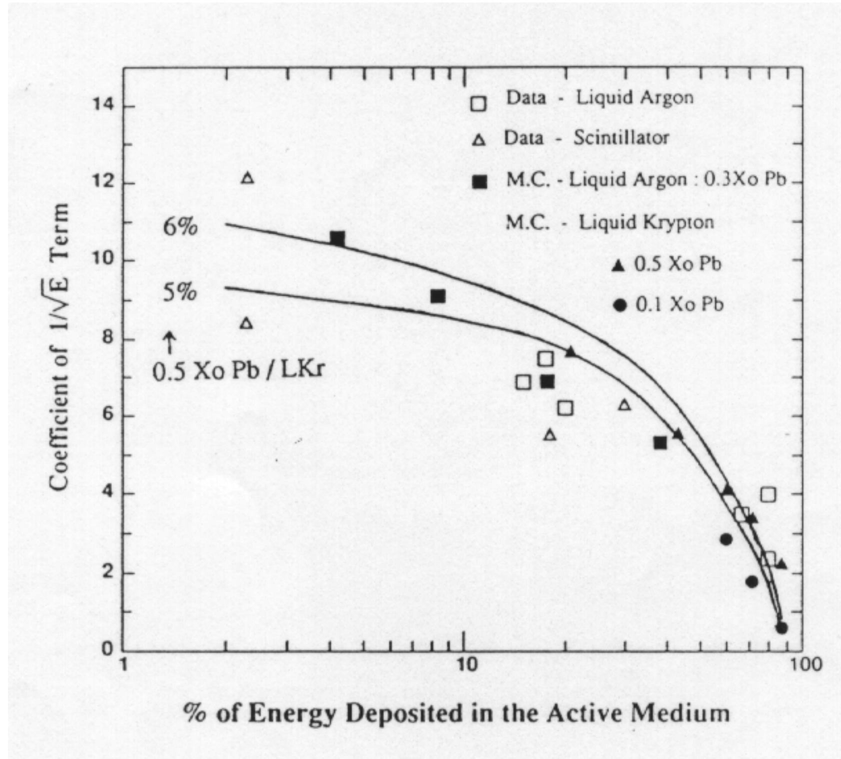
$$\sim (E_S / E_C p)$$

Shower is not simply 1 – d
Energy is deposited by soft tracks
Multiple scattering is large, $\theta_{MS} \sim 1$
→ effective plate thickness is increased

$$a_{samp} \sim \left[(1-W)\left(\frac{\Delta E + dE}{\langle \cos q_{MS} \rangle} \right)^{\left(\frac{1-W}{2}\right)} \right]$$

$$W = dE / (dE + \Delta E)$$

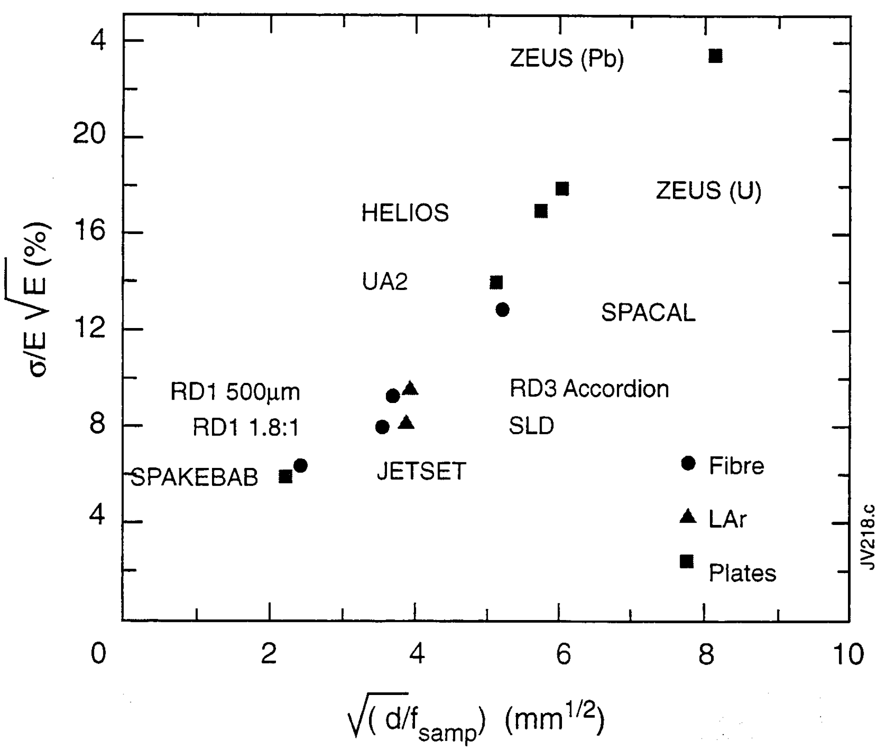
If sampling fraction becomes large
The assumption of development solely
In the samples breaks down



A 6%/√ E stochastic term is possible
With fine sampling and a large
Sampling fraction

Sampling Fraction and dE/E

Data points on precision EM calorimeters



Scintillating Fibers can be fine grained
And have a large sampling fraction, f
Good resolution is achievable with SciFi

Nobel Liquids for Calorimeters

Ionization in noble liquids
Typical is parallel plate
Ionization chamber
e.g. D0, ATLAS, ….

		LAr	LKr	LXe
Density	g/cm^3	1.39	2.45	3.06
Radiation Length	cm	14.3	4.76	2.77
Moliere Radius	cm	7.3	4.7	4.1
Fano Factor		0.11	0.06	0.05
Scintillation Properties				
Photons/MeV		-	$1.9\ 10^4$	$2.6.10^4$
Decay Const. Fast	ns	6.5	2	2
Slow	ns	1100	85	22
% light in fast component		8	1	77
λ peak nm		130	150	175
Refractive Index @ 170nm		1.29	1.41	1.60
Ionization Properties				
W value	eV	23.3	20.5	15.6
Drift vel (10kV/cm)	cm/μs	0.5	0.5	0.3
Dielectric Constant		1.51	1.66	1.95
Temperature at triple point	K	84	116	161

Charge collection time
Is defined by the drift velocity

Pulse Formation

$Q = CV$

$U = CV^2/2 = Q^2/2C \sim QV$

Parallel plate gap

U = stored energy

$$dU = Q_o\, dQ(t)/C = Fdx$$
$$= [q(t)E][\langle v_d \rangle dt]$$

Field E does work on the
Charge q(t) in the gap
Which moved with drift velocity v_d

$$dQ(t) = q(t)\langle v_d \rangle dt[E/V_o] = \left[q(t)mE^2/V_o\right]dt$$
$$\frac{dQ(t)}{dt} \equiv I(t) = q(t)mE^2/V_o$$

Charge Q is induced on the electrodes. $v_d = \mu E$. $I \sim q$.

$\mu = v_d / E$

Pulse Formation - II

Line Charge (muon)

$$q(t) = q_s\left(1-\frac{t}{t_d}\right), \; t<t_d$$
$$= 0 \qquad\qquad , \; t>t_d$$
$$t_d = d/\langle v_d\rangle$$

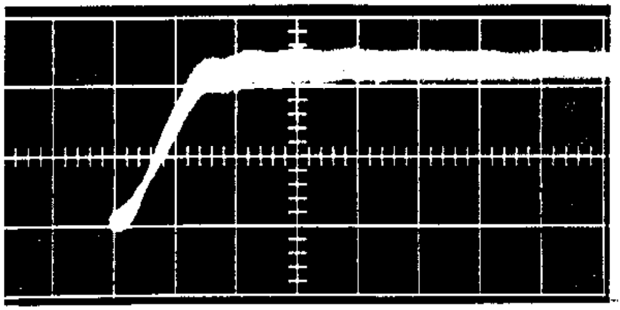
Point Ionization (α source)

$$dQ = \frac{q(t)E}{V_o}\,(\langle v_d\rangle dt)$$
$$q(t) = q_s \; for \; t<t'_d, = 0, t>t'_d, t'_d \equiv x_o/\langle v_d\rangle$$

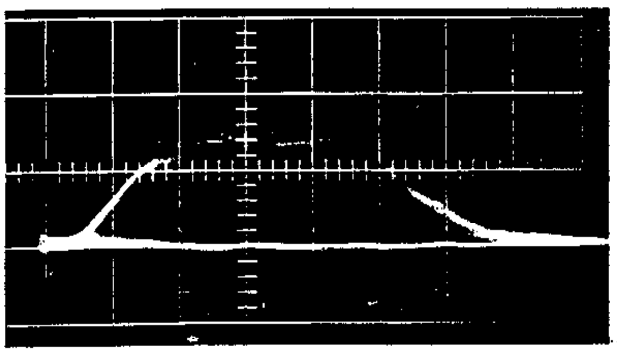
$$I(t) = (q_s/t_d)(1-t/t_d)$$
$$\int I(t)dt \equiv Q(t)$$
$$= q_s[y-y^2/2], \; y=t/t_d$$

$$I(t) = \frac{q_s}{t_d}, \; t<t'_d = 0, \; t>t'_d$$
$$Q(t) = q_s t/t_d$$
$$\leq q_s\,(x_o/d)$$

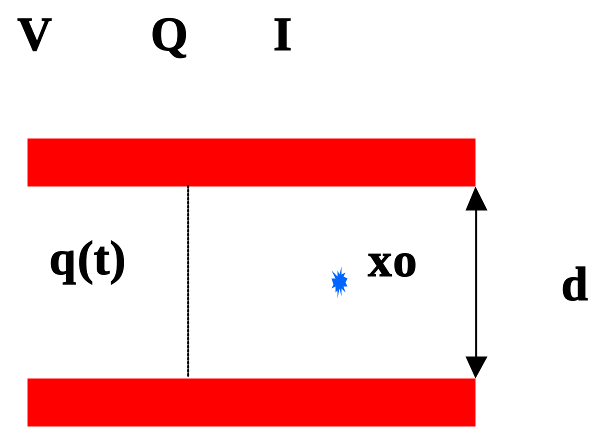
Pulse Formation



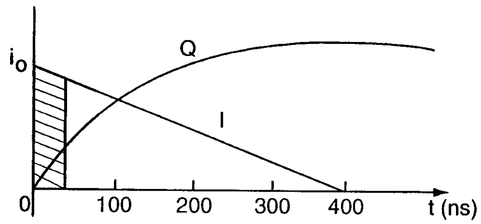
(a)



(b)

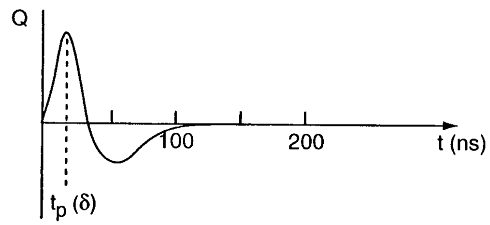


LA Pulse Shaping

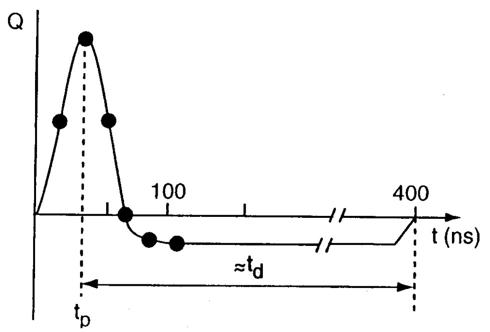


I ~

Q ~ t^2



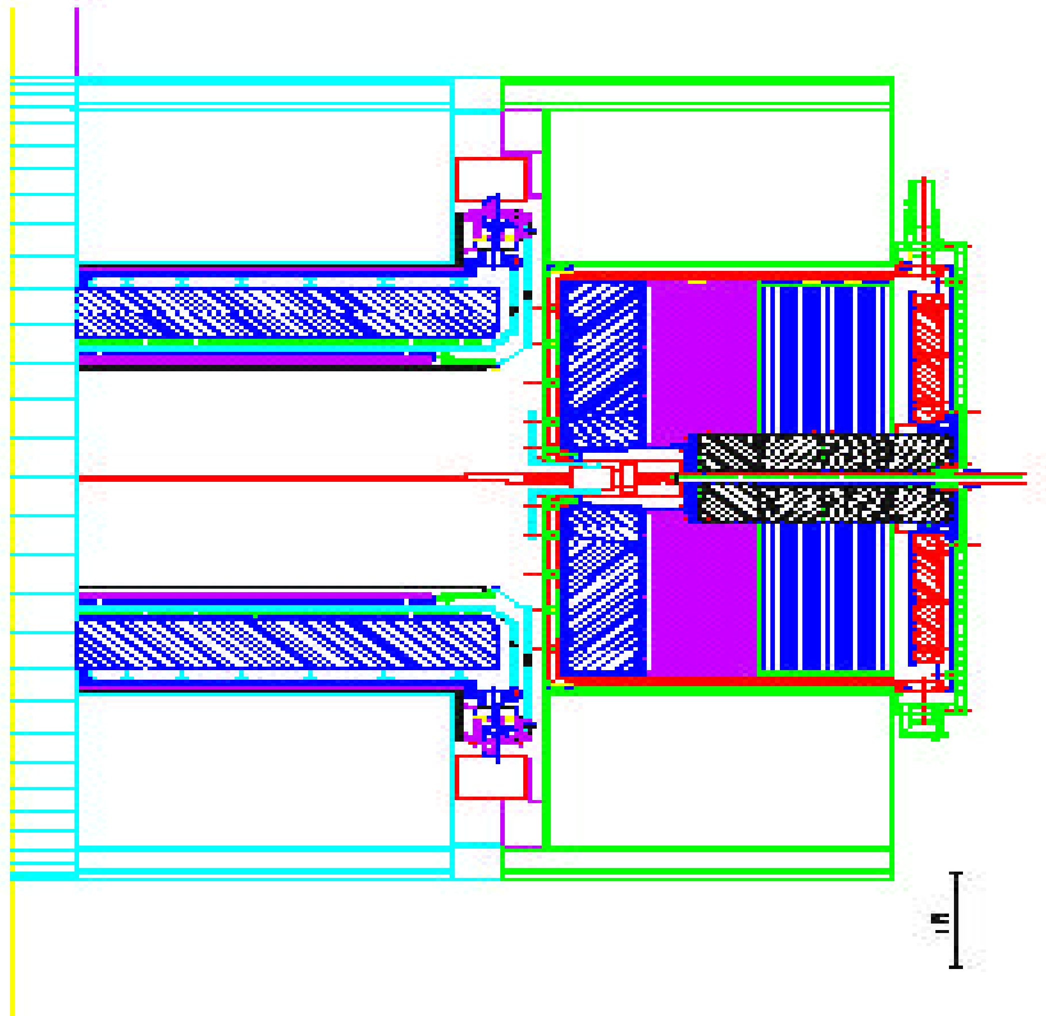
Bipolar pulse shaping
Risetime due to source/cable
Capacity



Fast rise time pulse shaped
Followed by long drift time

If d ~ 1 mm, then τ ~ 200 nsec

ATLAS - LA

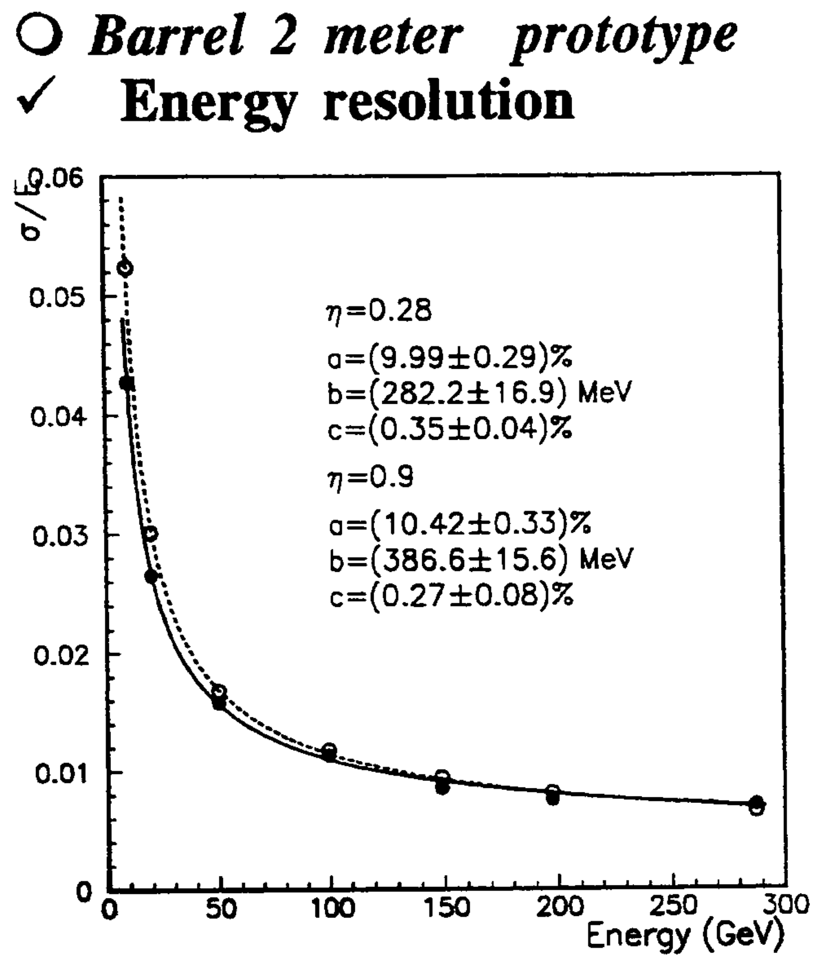


Energy Resolution - LA

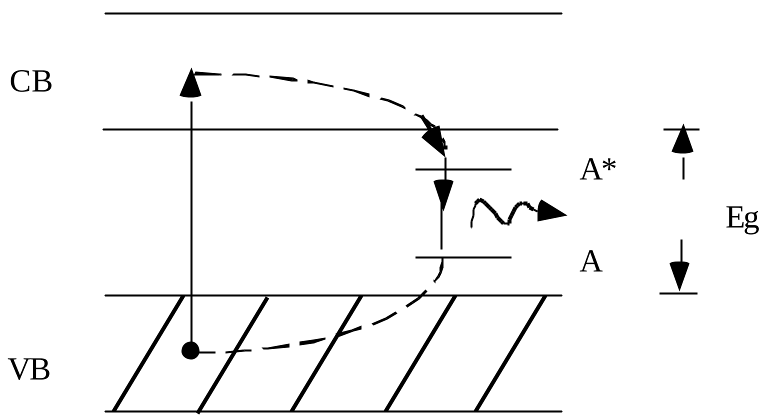
ATLAS

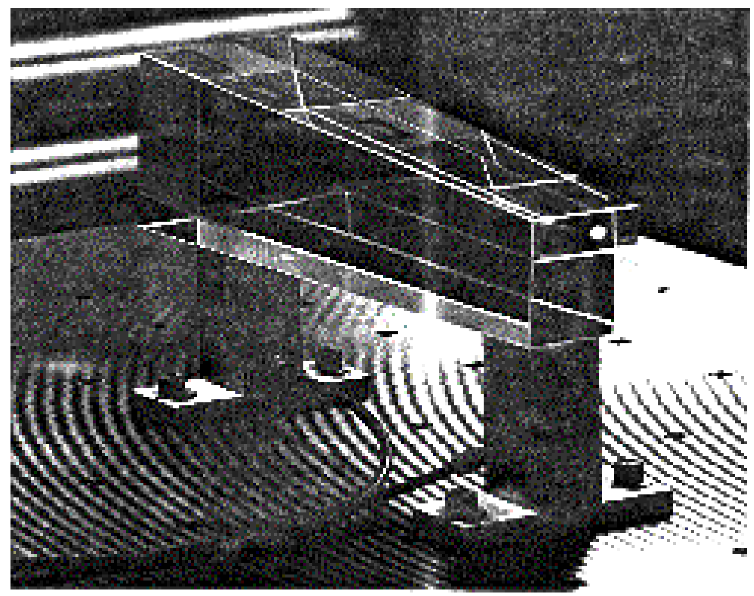
dE/E is ~ 1.2 % at 100 GeV

Constant term, dE/E = b, due to
Non-uniformity of the medium
Controlled here to ~ 0.3 %
An issue with large volume
Detectors, controlling uniformity



Xtals - Fully Active



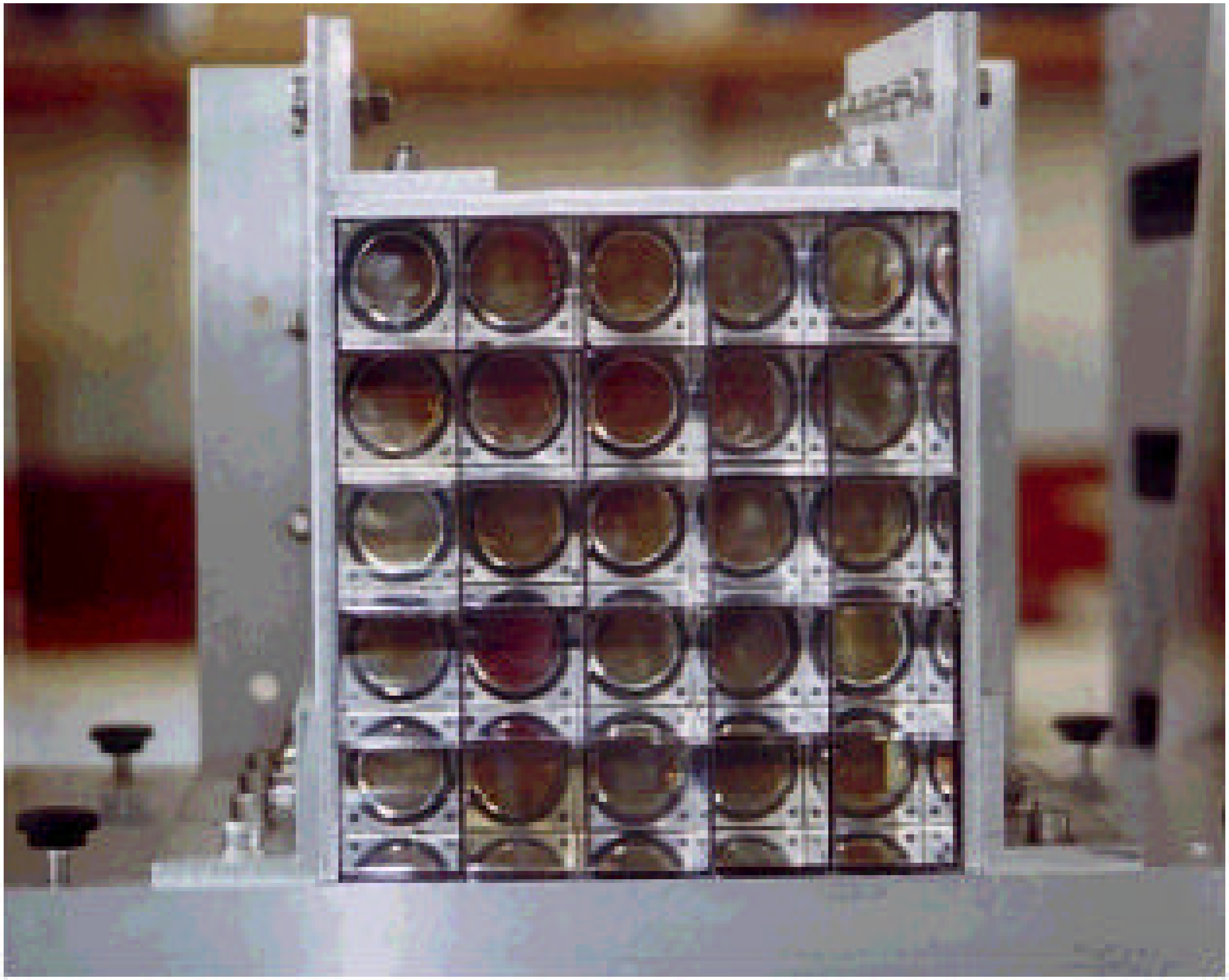


Xtal is transparent to it's emissions

Uses activators, e,g, Thallium in NaU

Trap on activator quickly
For fast light output.

PbWO4 Array - Test Beam



Energy Resolution - Xtals

e.g. PbWO4 – CMS

fully active devices have no sampling fluctuations. However, there is noise and photon statistics, and collection non-uniformity.

dE/E ~ 0.7 % at 100 GeV even though stochastic coefficient is only ~ 2.3 %

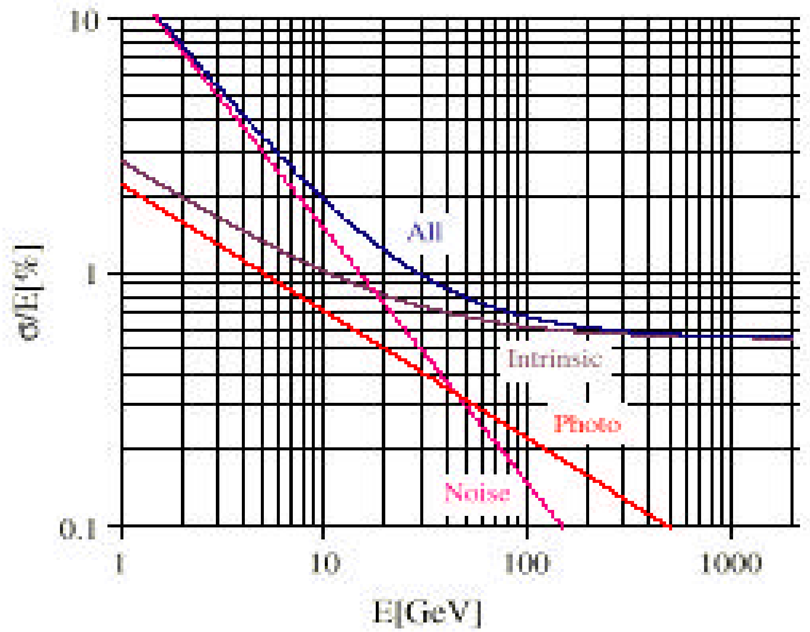
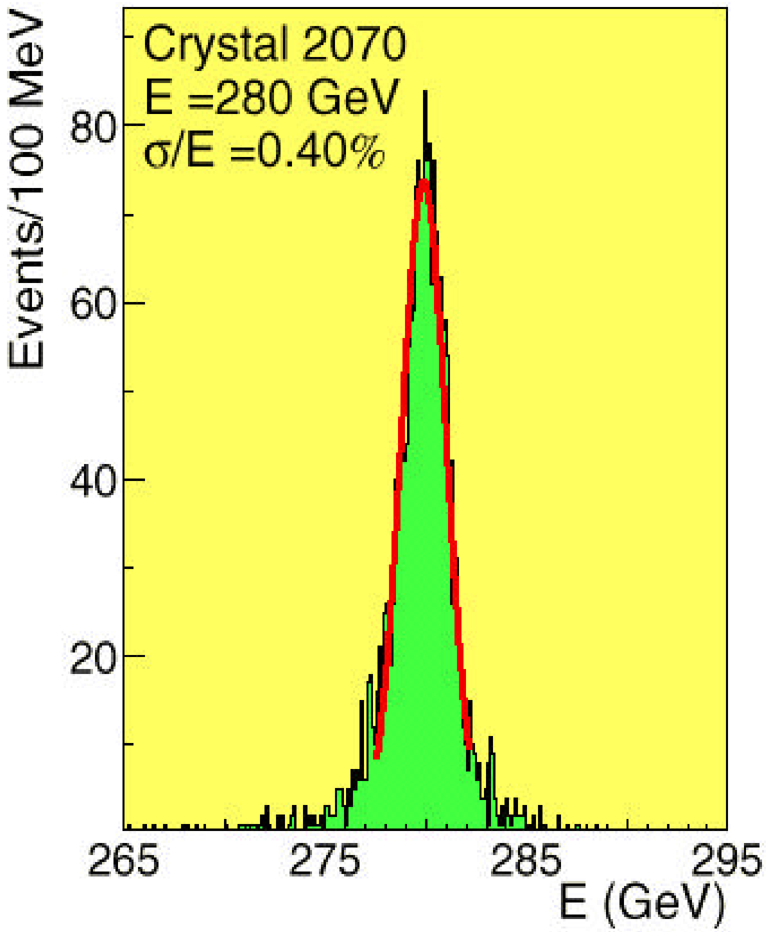
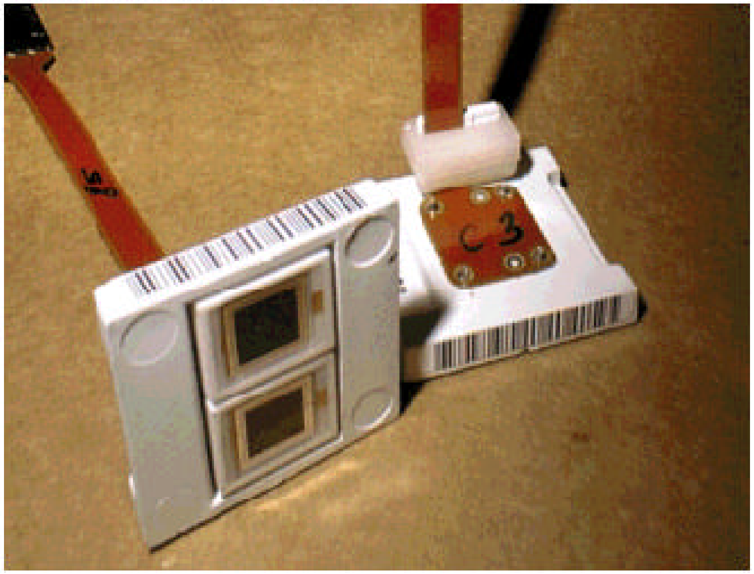


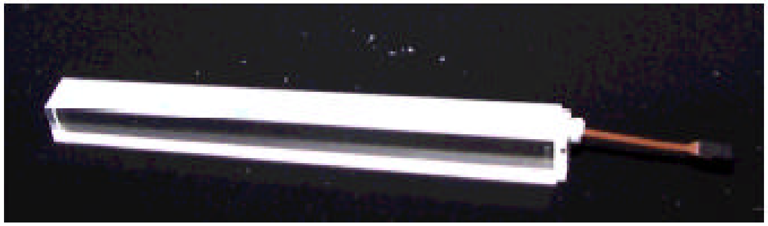
Fig. 1.3: Different contributions to the energy resolution of the $PbWO_4$ calorimeter.

ECAL



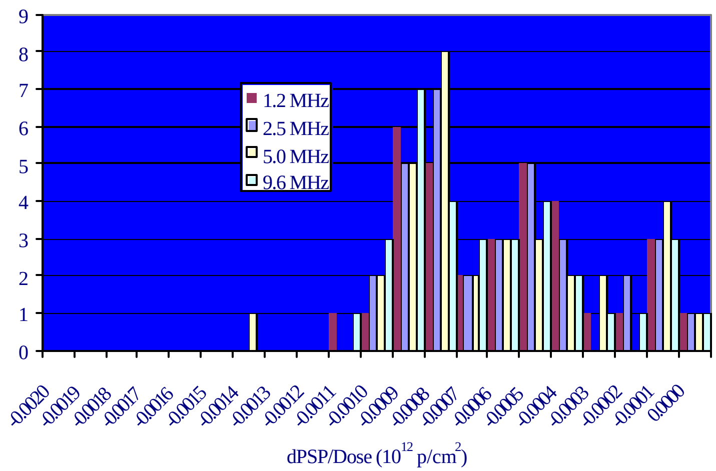
Two APDs 5 x 5 mm surface mounted in a supporting structure (capsule) glued at the rear of the crystal

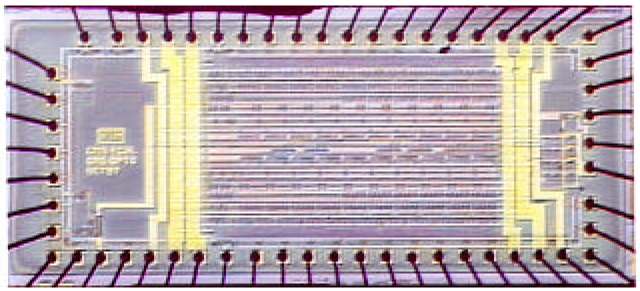




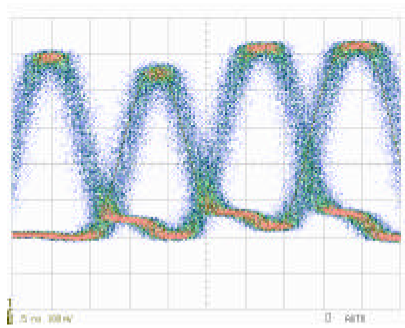
1.4 Electromagnetic Calorimeter (ECAL)

- **The ECAL group has determined that Hamamatsu will be the vendor of choice. The cost and performance of the APD is within CMS specifications.**





- **2 original wafers (packaged externally, tested at CERN)**
  - **Functionally OK, but quite low yields. ESD suspected...**
- **4 wafers probed 11/98 at Honeywell**
  - **Good yields - ESD (ElectroStatic Discahrge) will be fixed with improved protection pads**



Xtals and Calorimetry

Dense – compact – fast – good photon yield – stable

Crystal		NaI(Tl)	CsI(Tl)	CsI	BaF_2	BGO	CeF_3	$PbWO_4$
Density	g.cm^{-2}	3.67	4.51	4.51	4.89	7.13	6.16	8.28
Rad. length	cm	2.59	1.85	1.85	2.06	1.12	1.68	0.89
Moliére radius	cm	4.5	3.8	3.8	3.4	2.4	2.6	2.2
Int. length	cm	41.4	36.5	36.5	29.9	22.0	25.9	22.4
Decay Time	ns	250	1000	35	630	300	10-30	<20>
				6	0.9			
Peak emission	nm	410	565	420	300	480	310-340	425
				310	220			
Rel. Light Yield	%	100	45	5.6	21	9	10	0.7
				2.3	2.7			
d(LY)/dT	%/^{0}C	˜ 0	0.3	- 0.6	- 2	- 1.6	0.15	-1.9
					˜ 0			
Refractive Index		1.85	1.80	1.80	1.56	2.20	1.68	2.16

Transverse Size - Moliere

Physics

$$\langle p_T \rangle \sim m_e$$

$$\langle q \rangle \sim m_e / e(t)$$

$$\langle q \rangle_{SM} \sim m_e / E_C$$

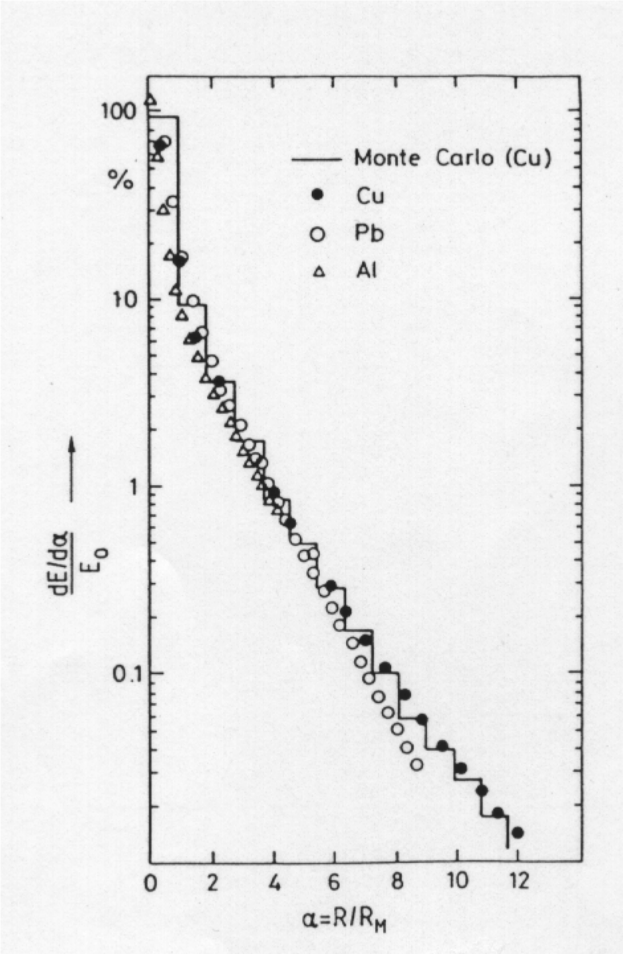
Multiple Scattering

$$\langle p_T \rangle_{MS} \sim E_S \sqrt{t}, \sqrt{t} \equiv 1$$

$$\langle q \rangle_{SM}^{MS} \sim E_S / E_C$$

$$r_M \sim E_S X_o / E_C = \langle q \rangle_{SM}^{MS} X_o$$

$$r_M \sim \left[7(gm/cm^2)\right](A/Z)$$



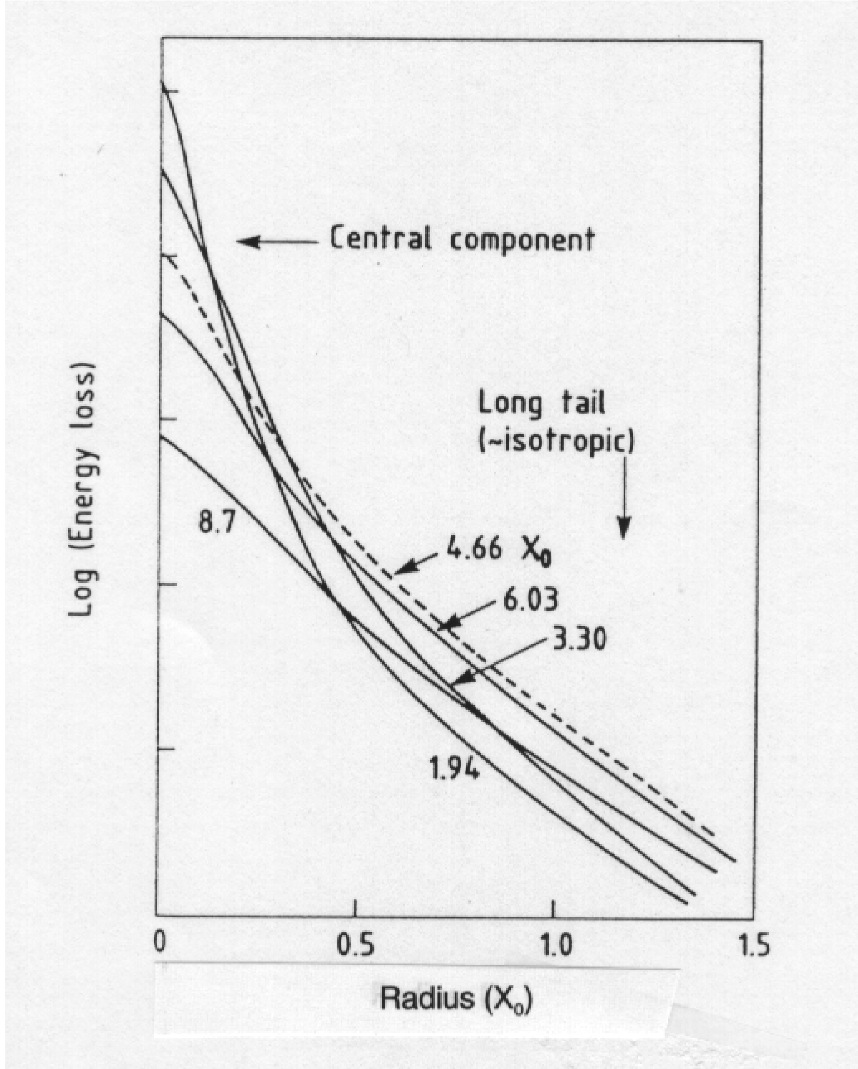
EM shower is well localized transversely
Find e position using energy centroid to
A fraction of the Moliere radius – (2-5cm)
In crystals

Transverse Size and Depth

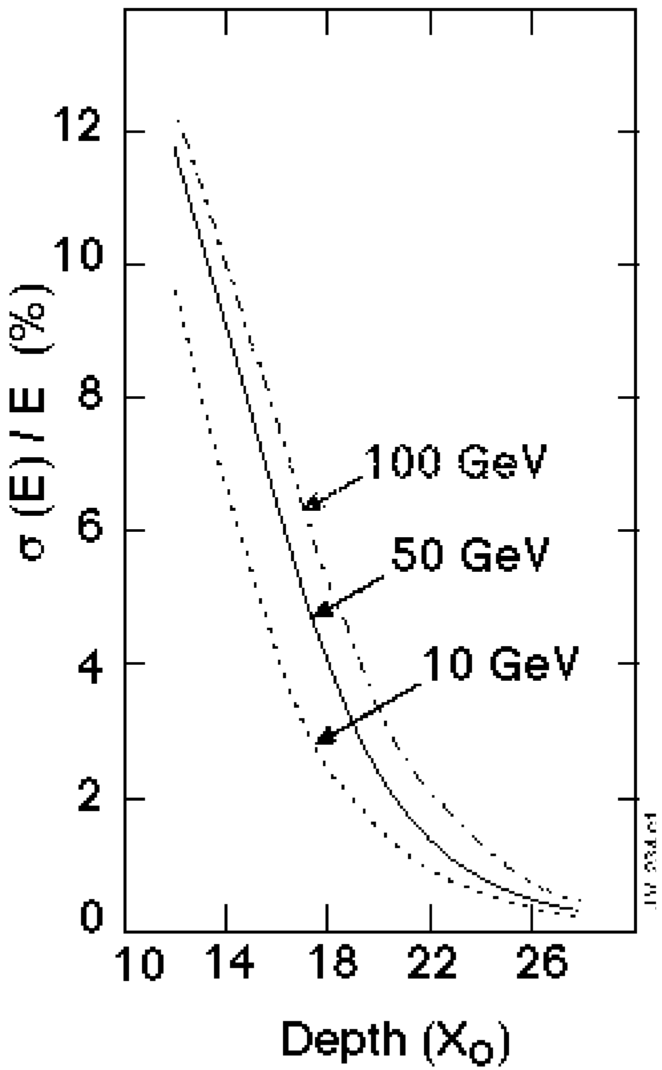
n.b logarithmic plot

shower easily contained in 1 Xo ~ 0.56 cm in Pb

the shower widens as the depth increases since <$\theta(t)$> ~ $E_s/\varepsilon(t)$



Leakage Energy and Depth

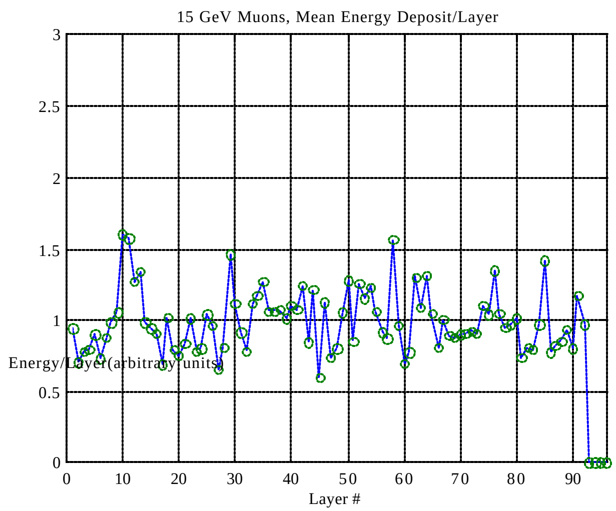


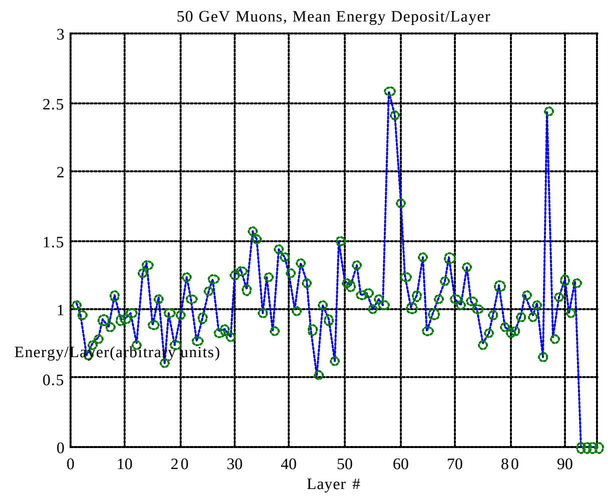
Any EM calorimeter is of finite length
Recall t_{max} ~ y
→ fluctuations hurt because energy lost in leakage fluctuates

gets worse with E
@ 20 Xo total depth → 2% @ 50 GeV
this dE/E would dominate all other error sources for CMS → deeper crystals

Muon Calibration

Calibrate the EM calorimeter in a test beam
With e of variable energy.





Use muons from cosmics or $\pi \rightarrow \mu\nu$ from beam
Puts a M.I.P. in each sample
e.g. 96 individually read out samples.
n.b. 15, 50 GeV muons give ~ dE/dx
recall the relativistic rise in
ionization energy is a small effect

Hadronic Calorimetry

- Basic Parameters
- Hadronic Cascade
- Profiles
- Individual Cascades and Neutral Clusters
- Sampling Fluctuations
- Non-Compensation
- Transverse Size
- Energy Leakage
- Calibration
- Radiation Damage
- Neutrons
- Standard Model and Detectors

Basic Parameters

$l_I \sim [35(gm/cm^2)]A^{1/3}$

$v = x / l_I$

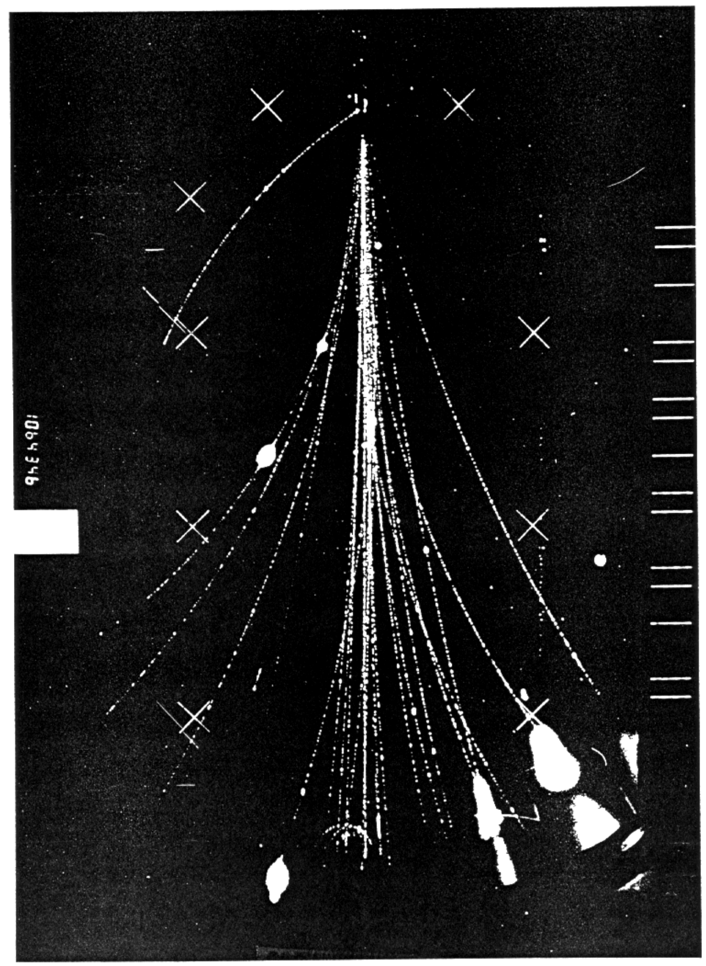
p^+, p^-, p^0

$\langle p_T \rangle_{EM} \sim m_e$

$\langle p_T \rangle_h \sim 0.4 GeV$

Note large multiplicity
And small angle production
→ limited P_T

200 GeV π p Interaction



Basic Parameters - II

Threshold for π multiplication
π p → π π p

$$E_{TH} \sim 2m_p \;= 0.28\,GeV$$

$$N \sim \langle N \rangle \sim \ln E$$

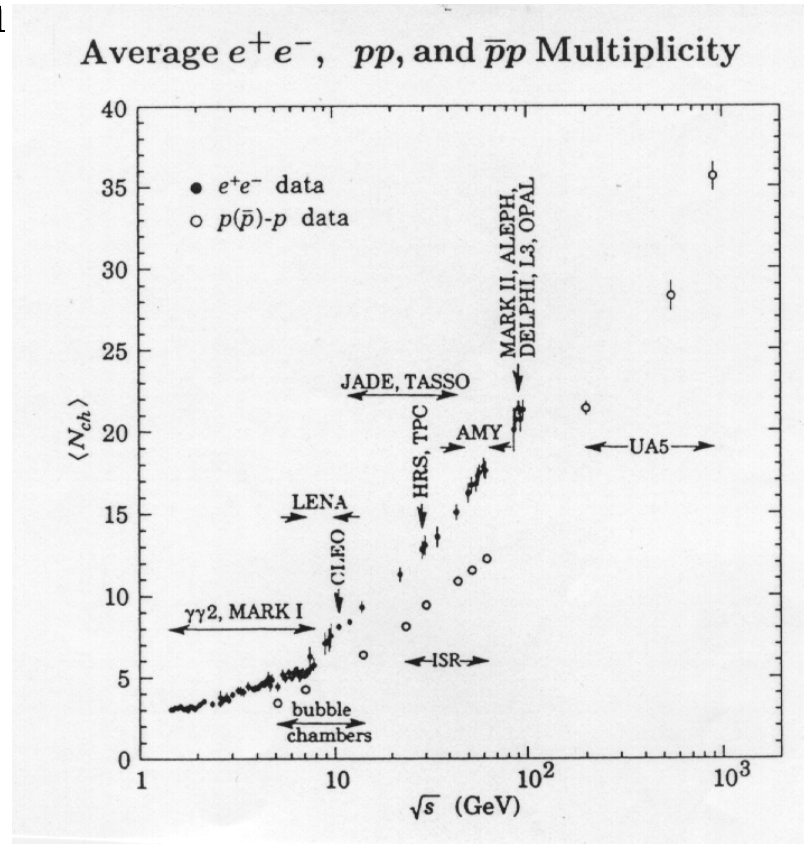
$$X_o \,/\, l_I << 1$$

$$f_o = 1/3$$

Basis of EM/HAD separation

Neutral fraction

Multiplicity of secondary pions
Depends only on log of energy

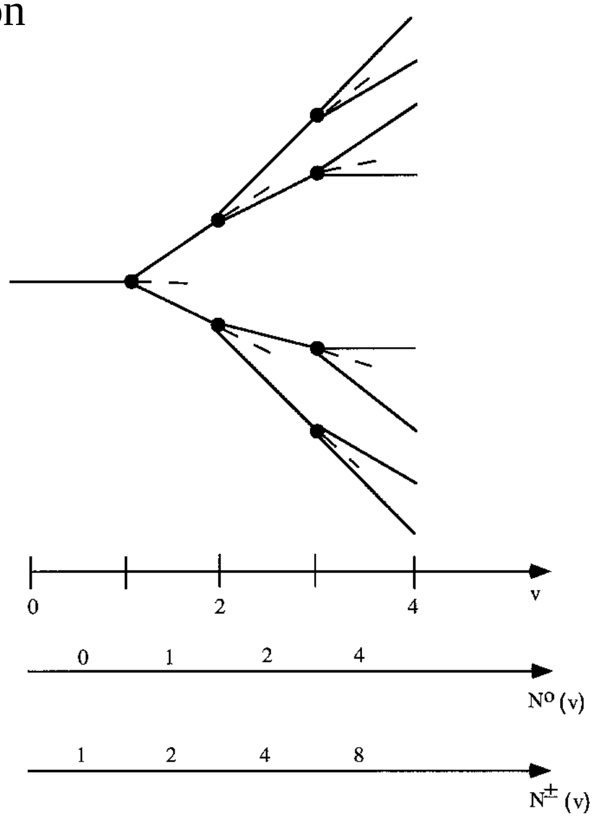


Hadronic Cascade

$$e(v) = E / N^{v},\ N = \langle N \rangle$$
$$N^{o}_{(v)} \sim N f_{o} \left[N(1 - f_{o}) \right]^{v}$$
$$N^{\pm}_{(v)} \sim \left[N(1 - f_{o}) \right]^{v+1}$$

Ignore fluctuations in multiplicity, Energy sharing, and interaction Points

Simple Model, <N> = 3



$$E_{o} \sim \sum_{v} e(v) N^{o}_{(v)}$$

$$E_{o} \sim E f_{o} \sum_{v} [1 - f_{o}]^{v},\ total\ neutral\ energy$$

$$"f_{o}" \sim f_{o} \sum_{o}^{v_{max}} (1 - f_{o})^{v} \quad \rightarrow 1$$

Neutrals are quickly absorbed.
Charged pions transport energy of shower

Hadronic Cascade - II

Simplified model for a hadronic cascade developed
by a 250 GeV incident pion.

Generation v	$\varepsilon(v)$ GeV	$N^{\pm}(v)$	$N^{o}(v)$	Eo(v) (GeV)
0	250	1	0	0
1	28	6	3	84
2	3.1	36	18	56
3	0.35	216	108	38
				178 GeV

$f_o = 1/3$, "f_o" = 0.71

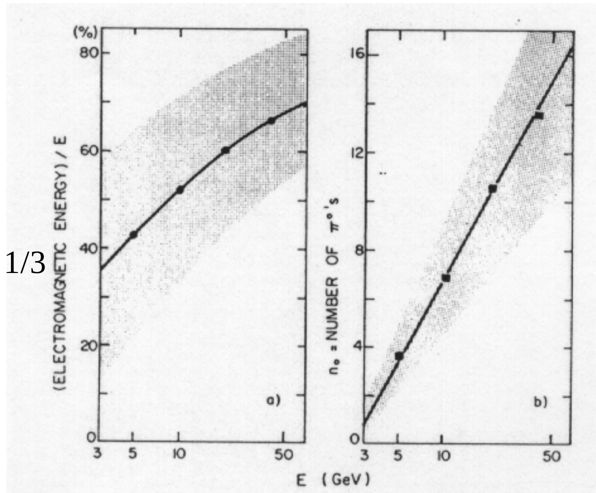
Ignore energy deposited by charged pions

Total path length ~ $[\sum N^{+-}]\lambda$

Energy lost in ionization =
Path length * dE/dx ~ 49.6 GeV
Ionization fraction ~ 0.2

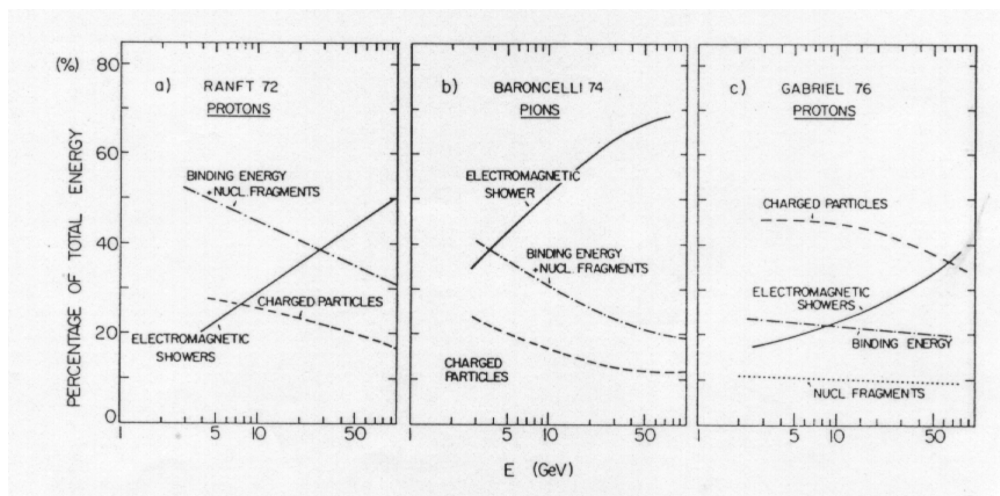
Ignore binding energy and nuclear fragments

Monte Carlo Models



fo = π^o / ($\pi^+\pi^o\pi^-$) ~ 1/3

In EM cascade, P_T ~ Me and
nuclei are inert
In hadron cascade, P_T ~ 400 MeV
And nuclei are disrupted
B ~ 8 MeV/nucleon



Hadron Showers, Longitudinal

as E →, ν_{max} →
ν_{max} [ln(<N>)] = ln(E/E_{TH})]

ex. E = 250 GeV, <N> = 8
ν_{max} ~ 3.3
Fewer generations than EM
And lower cascade multiplicity

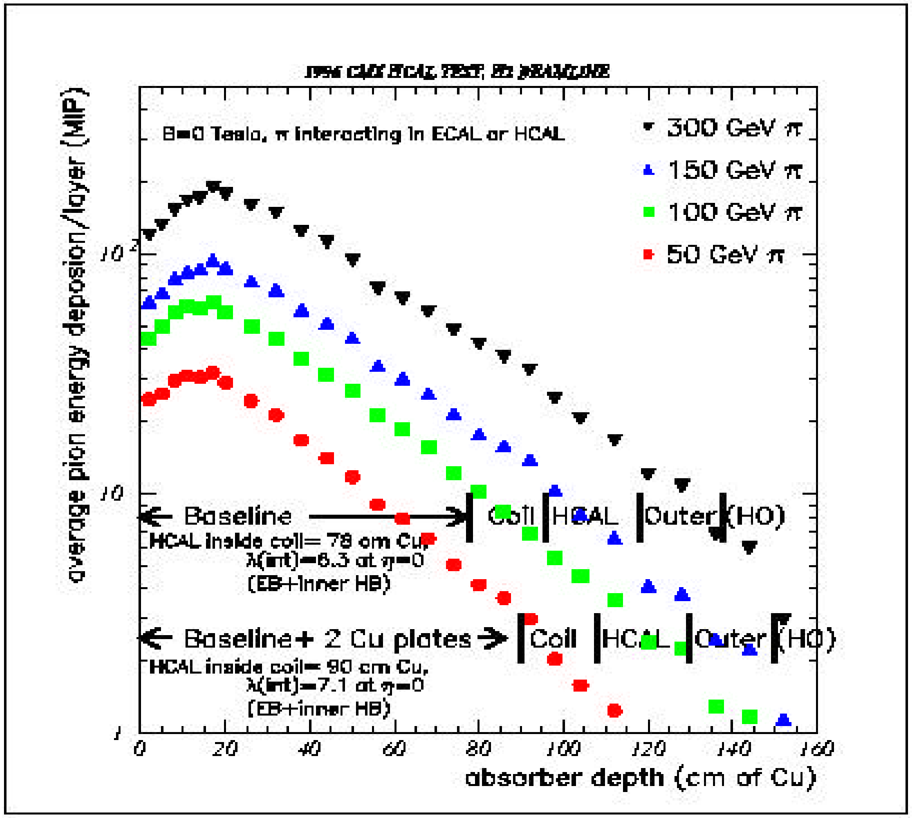


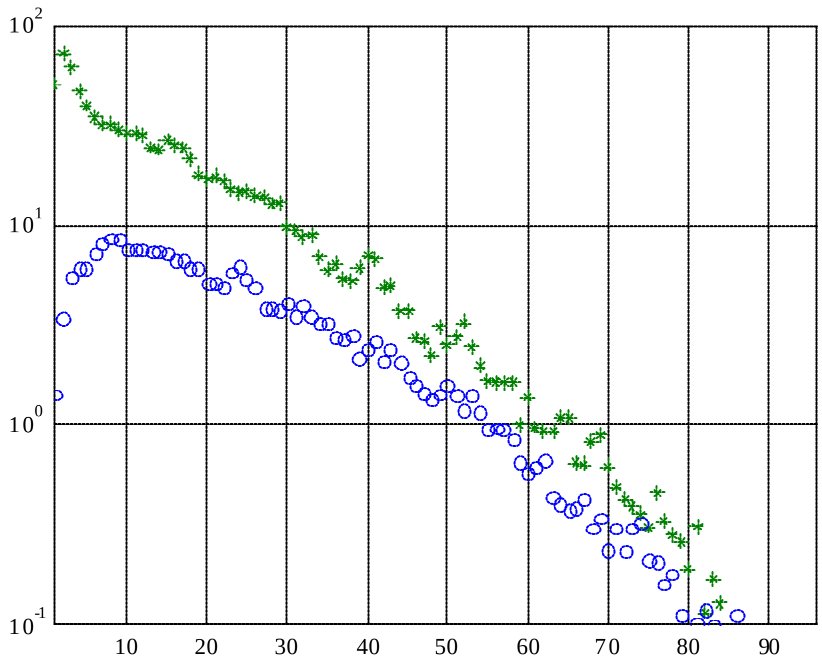
Fig. 26. Average 50, 100, 150 and 300 GeV pion shower profiles as a function of calorimeter absorber depth.

Falloff with length scale λ ~ 15 cm in Cu

Profiles and Cascades

$$\left(\frac{dE}{E}\right)=\left[\frac{u^a e^{-u}}{\Gamma(a+1)}\ f_o\ du+\frac{w^c e^{-w}}{\Gamma(c+1)}(1-f_o)dw\right]$$

$$w=dv, d\sim 1 \qquad (Eq.\ 11.7)$$

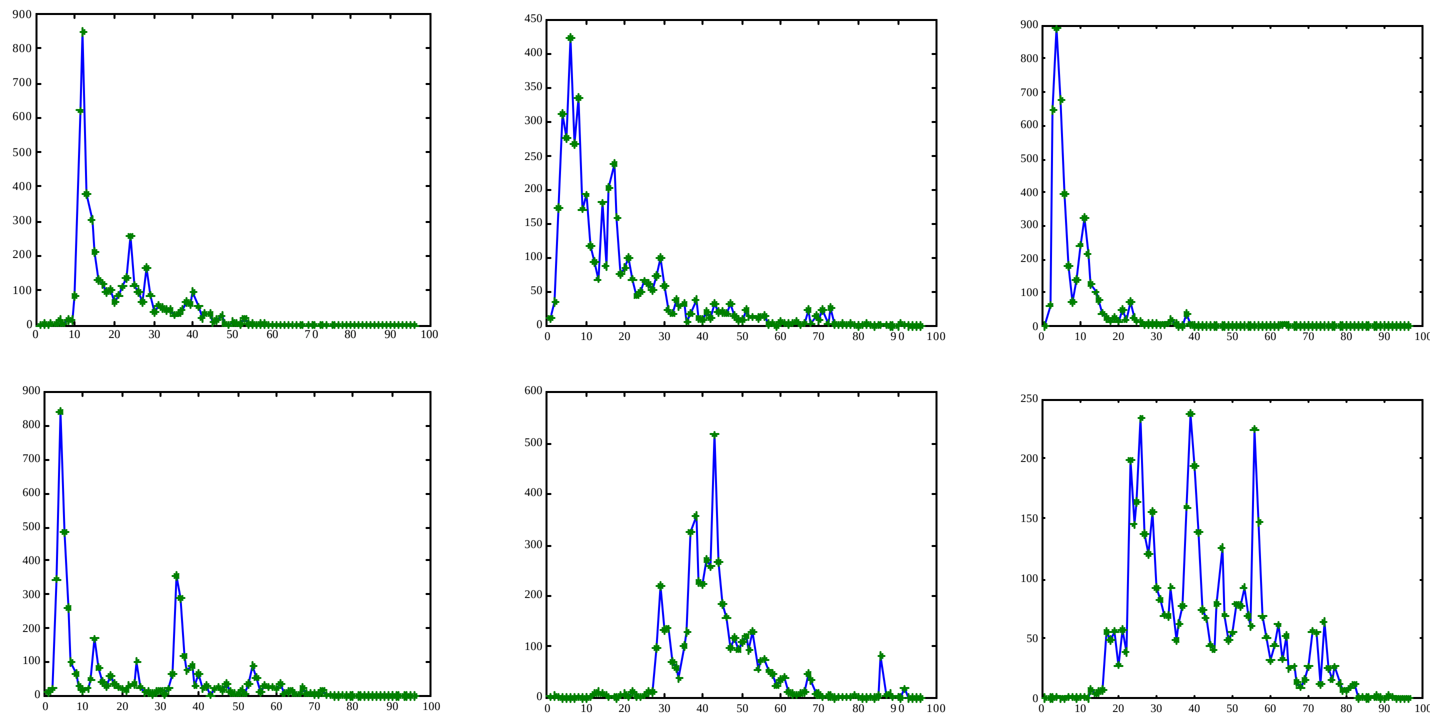


Profile with an EM (Xo)
and a hadronic (λ) conponent

Profile in calorimeter and
Profile with interaction point
Subtracted. → see EM in first
Interaction of cascade, later
Generations washed out by
Fluctuations

Individual Cascades, Clusters

Pb calorimeter with Xo/λ ~ 30
EM shower contained in ~ 6 samples



96 layers of sampling
each read out by a phototube

large fluctuations are seen
w.r.t. the smooth profile

hadron shower must be understood
event by event because the fluctuations
are large in a hadron shower

Energy Resolution

$$dE/E = a/\sqrt{E} \oplus b \equiv \sqrt{a^2/E + b^2}$$

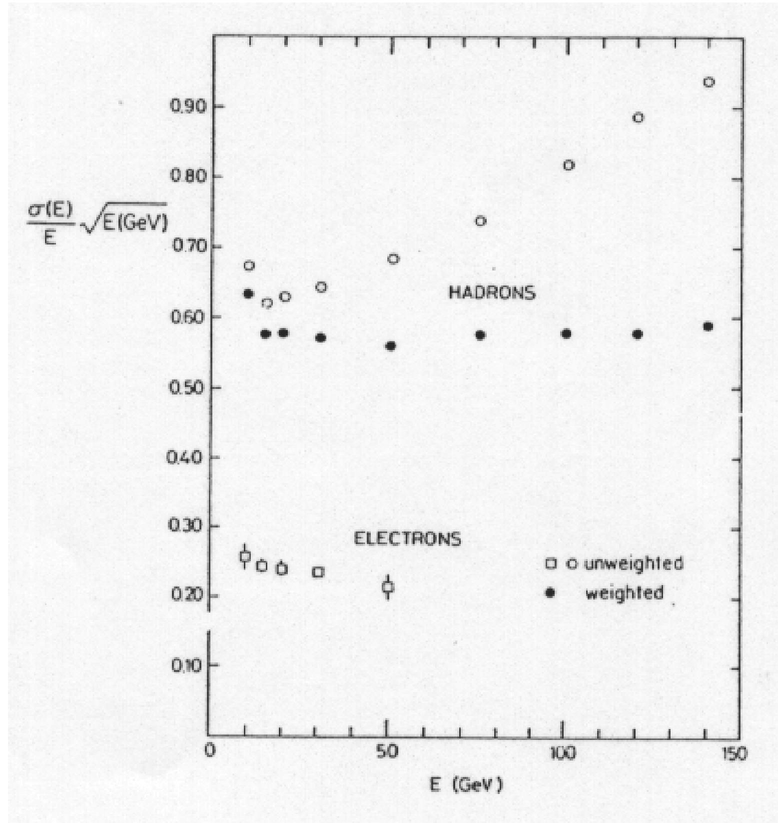
$$a_h/a_e \sim \sqrt{E_{TH}/E_C} \quad (Section 11)$$

$$\sim 6$$

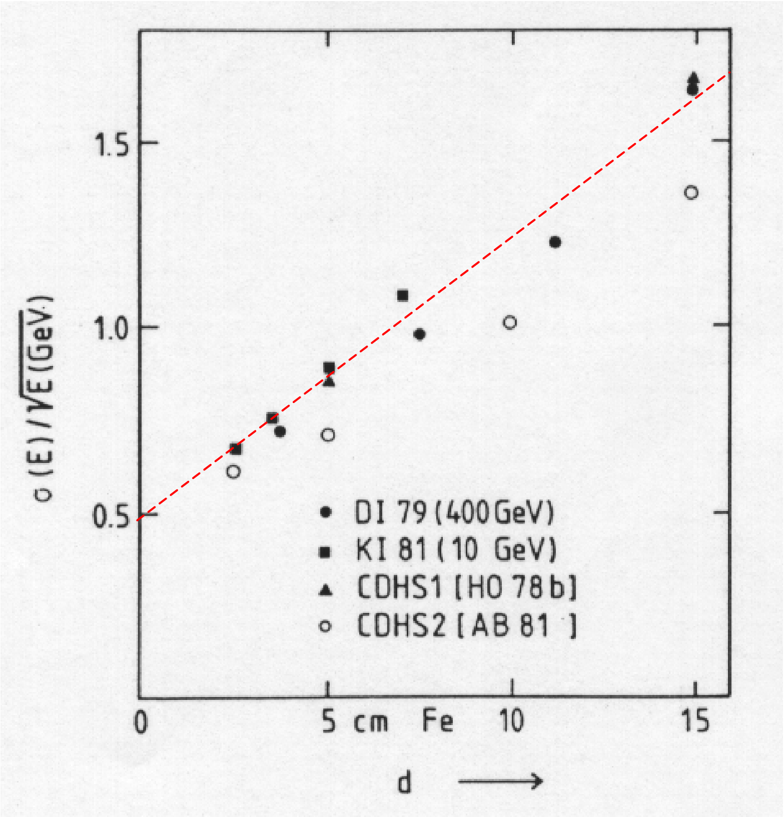
$E_{TH} \sim 280$ MeV
Ec ~ 7 MeV

→ expect ~ 50 % stochastic coefficient

in addition, there are fluctuations between ionization/binding/photons and interaction multiplicity/neutral fraction



Sampling Fluctuations



The stochastic coefficient scales roughly as sqrt of sample thickness as expected, but with a non-zero intercept. Sampling fluctuations are not the full story.

dE/E and HCAL

80% stochastic coefficient +
4% constant term

Is the device inhomogeneous?

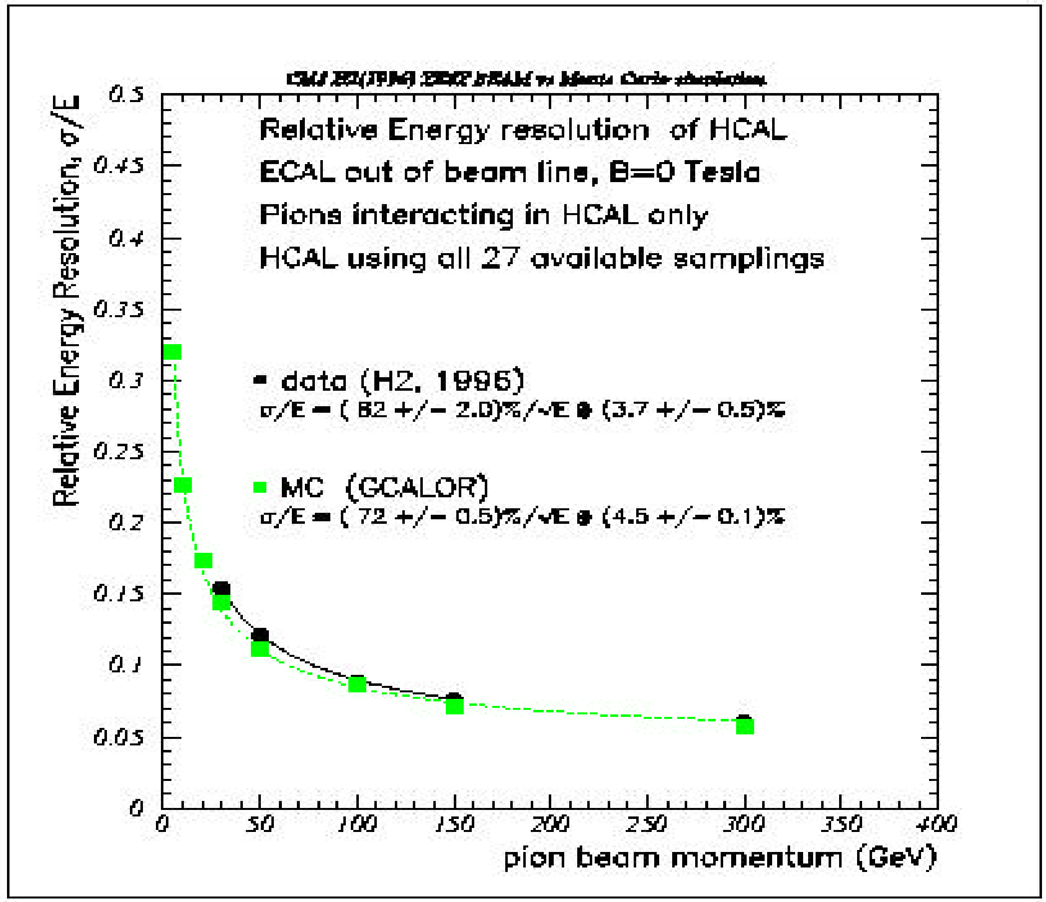


Fig. 32. Relative resolution of the calorimeter for pions and comparison with MC simulation.

Sampling Uniformity and dE/E

Relation of constant term to the r.m.s. of the sampling medium (scintillator tiles)

→ constant term < 3 % since uniformity < 7 % was achieved

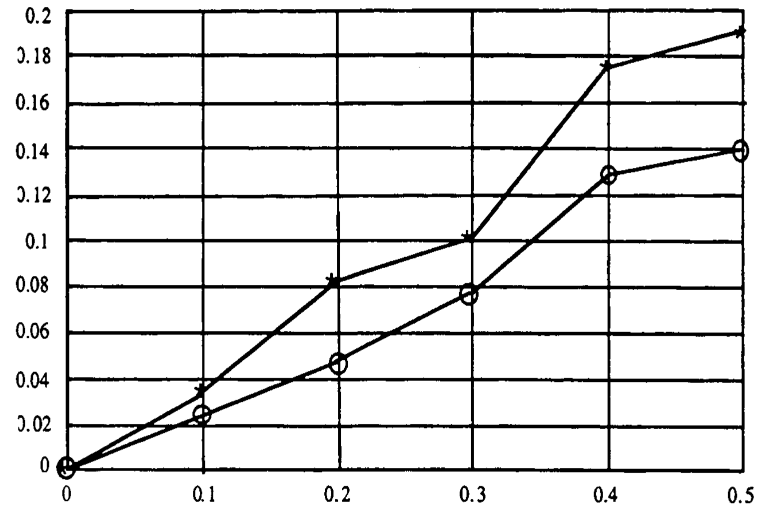


Fig. 6. 6: Induced constant term in the fractional energy error in the HCAL (y axis) as a function of tile manufacturing quality (fractional rms of light yield, x axis). The star symbols correspond to global calibration case and the open circles correspond to the local calibration case.

Non-Compensation

$$E \sim \left[e'' f_o'' + h(1 - '' f_o'') \right] E_{IN}$$

$$\left(\frac{dE}{E} \right)_{df_o} \sim \left| e/h - 1 \right| '' df_o ''$$

What if the medium responds differently to EM shower (e) and the hadron cascade (h) ? Recall ionization, binding and nuclear fragments

Typically a medium responds to EM More efficiently than HAD because of Time delay in fragments and loss of Neutron energy to the sampling.

$$d'' f_o '' \sim d\, f_o \sim \sqrt{\langle N^o \rangle} / \langle N \rangle \sim 0.17$$

$$\left(\frac{dE}{E} \right)_{d\, f_o} \sim \left| e/h - 1 \right| \left[\sqrt{\langle N^o \rangle} / \langle N \rangle \right]$$

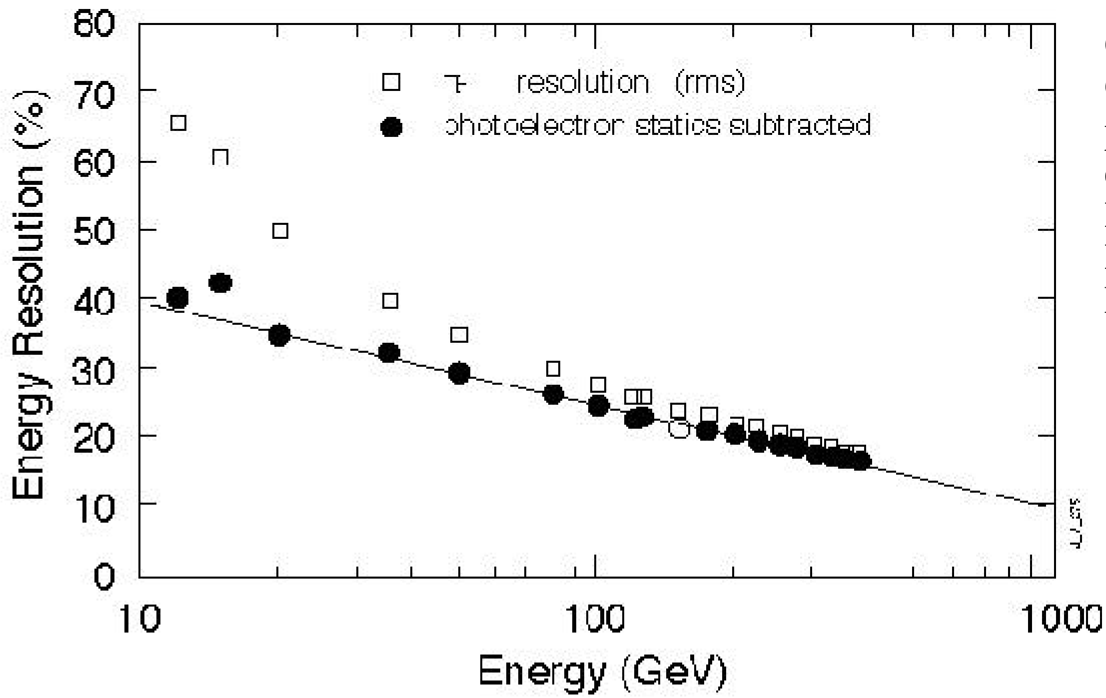
Fluctuations in the neutral fraction Induce a “constant term” in the Energy resolution dE/E

If e/h = 1.2, then a 6.8 % constant Term is induced.

$$\left(\frac{dE}{E} \right)_{d\, f_o} \sim 1/\sqrt{\ln(E)} \rightarrow 0 \ as\ E \rightarrow \infty$$

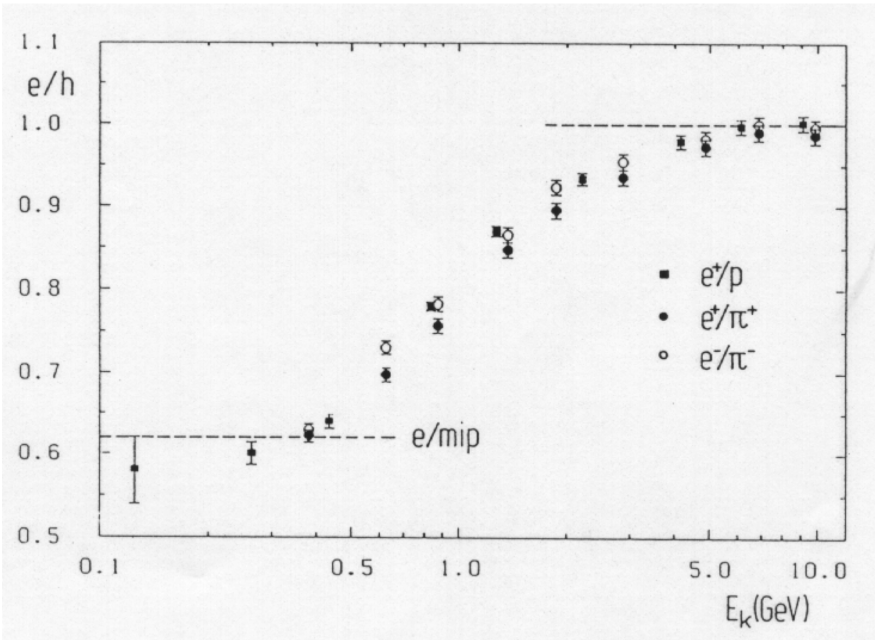
n.b. not “constant”
at high energies, as “fo” → 1,
“dfo” → 0 and dE/E → 0

Quartz Samples and dE/E



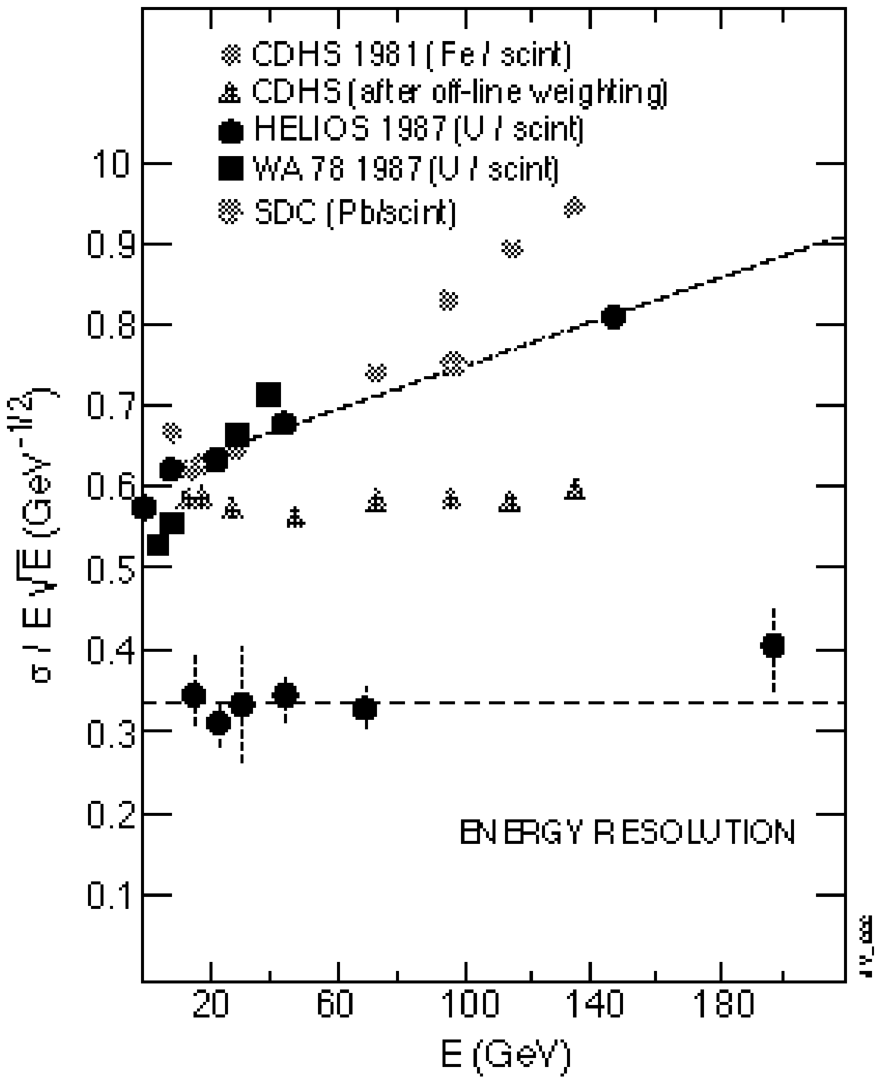
Quartz calorimeter responds to Cerenkov light of fast e in the EM part of the shower. h ~ 0.
dE/E ~ “dfo”/”fo”
Expect resolution determined by Neutral fluctuations. Note the ln(E) behavior.

Non-Compensation - II



At low energy, e/h decreases. As E → E_{TH} ~ 280 MeV from above, e/h falls even for devices designed to have e/h ~ 1 for E > 5 GeV. Intrinsic physics

dE/E and e/h



U/scint can be made compensating

Fe/scint is typically not compensating

Fe HCAL and e/h

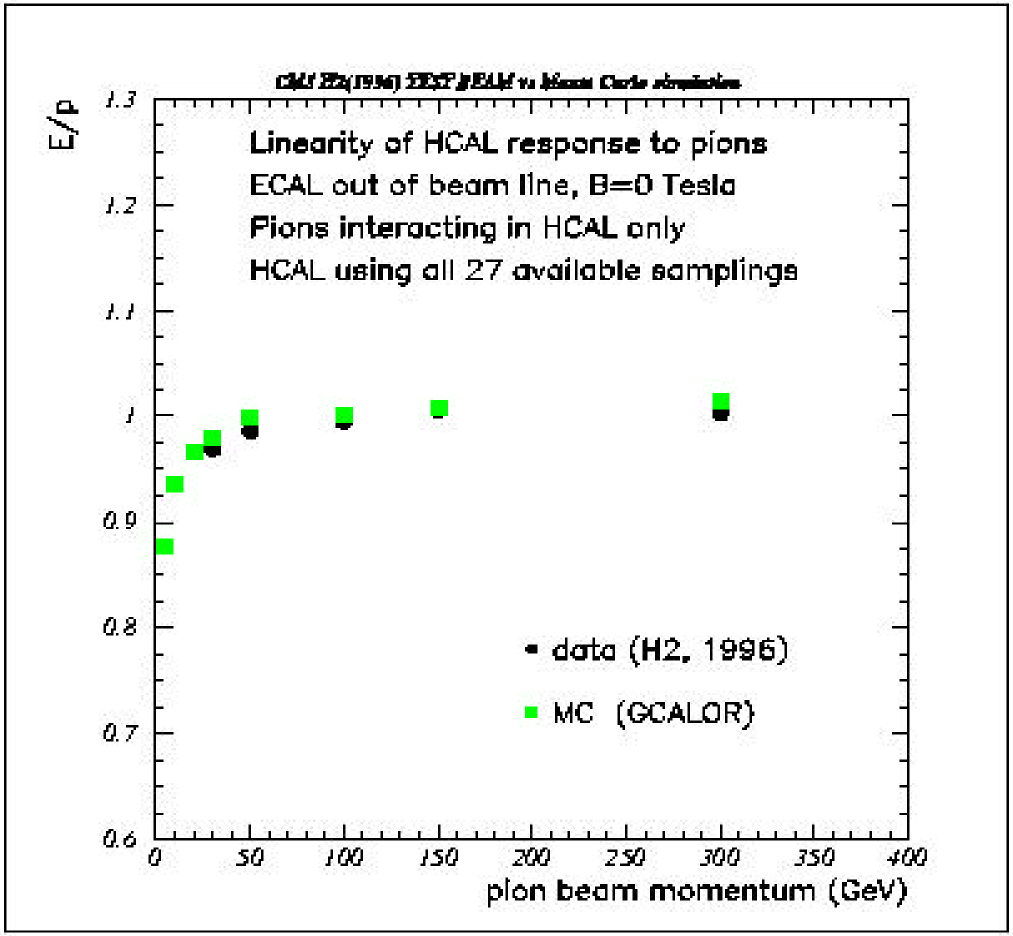


Fig. 31. Linearity of energy response of pions and comparison with MC simulation.

Non - linearity due to non-compensation

p **= e"fo" + h(1-"fo")**

e = e

p**/e = 1 + (1-"fo")(h/e-1)**

if e/h=1 then p**/e =1**

if "fo"-->1 then p**/e --> 1**

if e/h>1 then p**/e < 1**

if "fo" = fo = 1/3 and e/h = 1.4, then p**/e = 0.81**

Fe HCAL and e/h

Fe calorimeter
Fit to non-linearity
using a parametrized
“fo(E)”.
Find that e/h ~ 1.4 for Fe

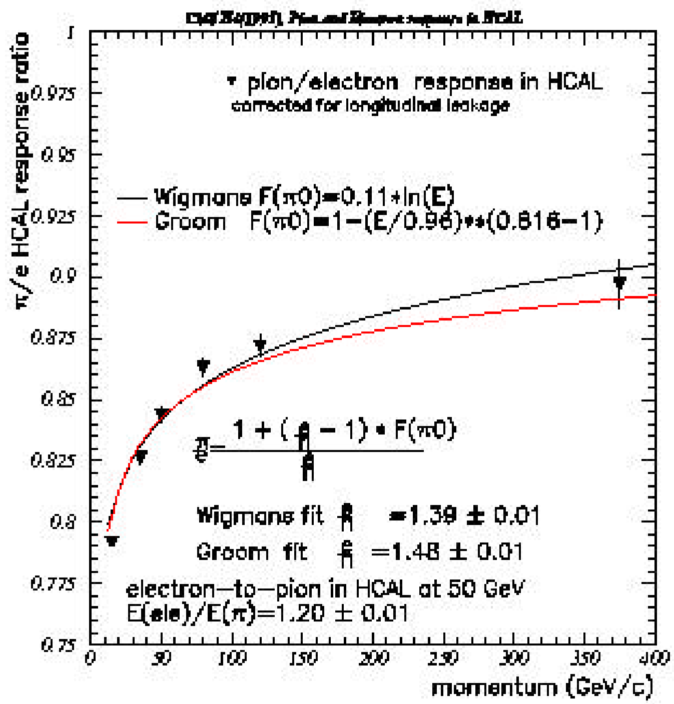


Fig. 25. H4(1995) data: the pion/electron ratio of response of the copper sampling prototype HCAL as a function of beam momentum. The calorimeter consists of the Inner HCAL (ten 3 cm Cu samplings followed by nine 6 cm Cu samplings), the magnetic coil mimic (8 cm Cu+29 cm Al) and the HCAL Outer (two 8 cm Cu and two 10 cm Cu samplings). The scintillator is 4 mm thick SCSN-81. The pion response of HCAL has been corrected for longitudinal leakage. We have assumed a linear electron response of HCAL, E(ele)/E(π at 50 GeV)=1.20+0.01. The extracted values of e/h correspond to the two different parameterisations of the average fraction of π^0's produced in pion induced showers.

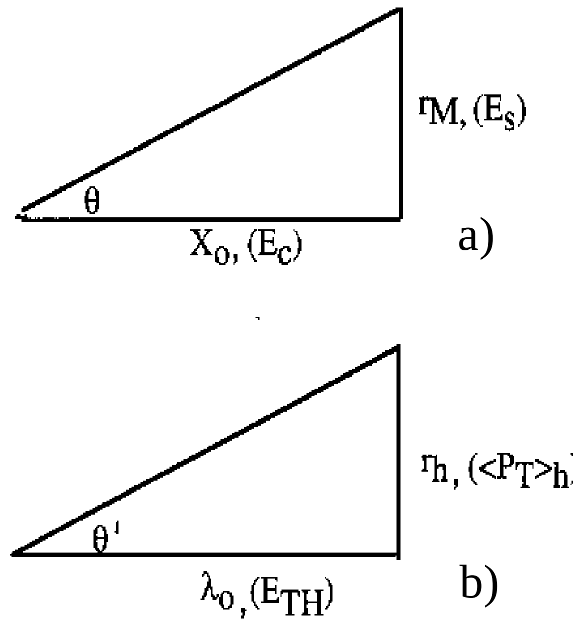
Transverse Size

$$\langle q \rangle \sim \langle p_T \rangle_h / e(v)$$

$$\langle q' \rangle_{SM} \sim \langle p_T \rangle_h / E_{TH}$$

$$r_h \sim l_I \langle p_T \rangle_h / E_{TH}$$

$$\sim l_I \left[\frac{\langle p_T \rangle_h}{2m_p} \right] \sim l_I$$



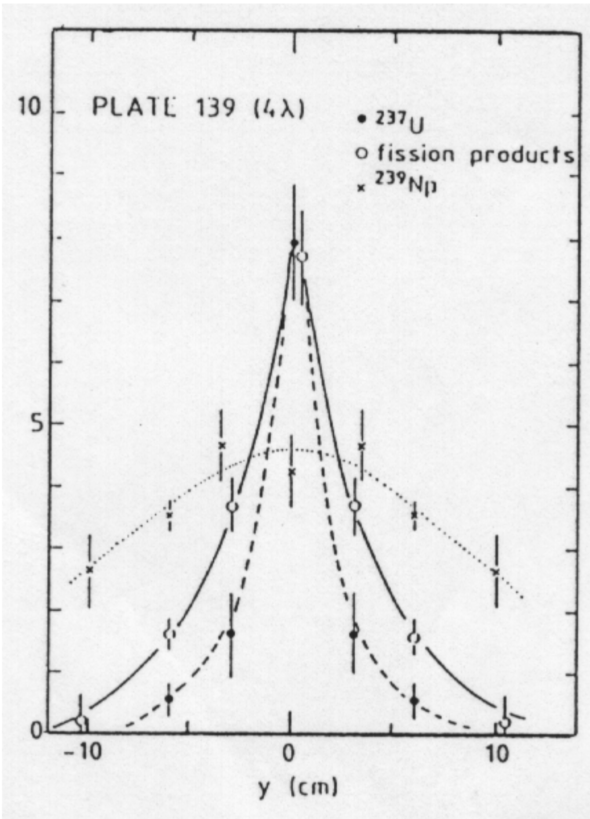
EM

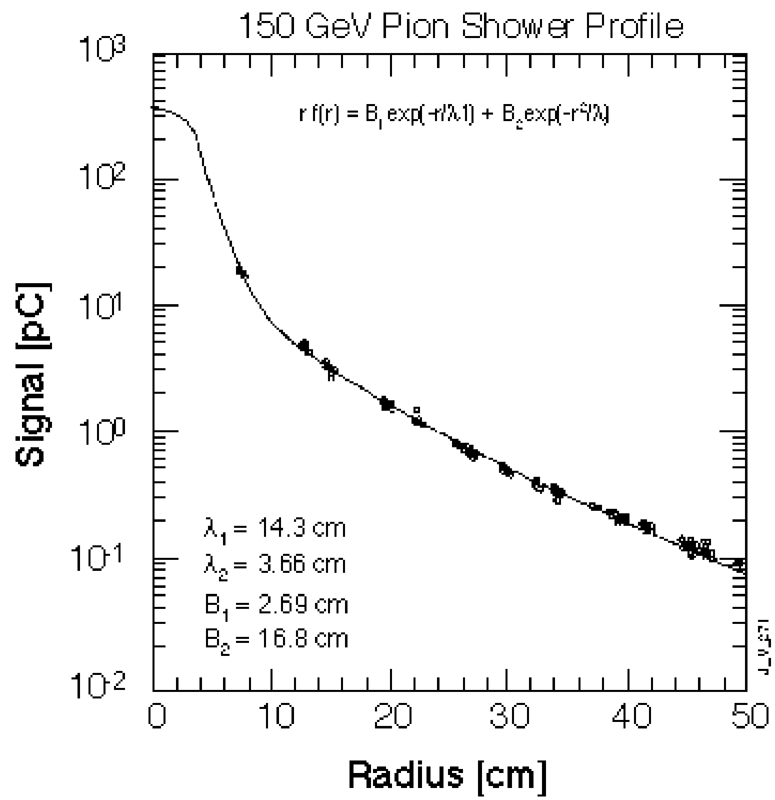
Hadronic

Expect that a hadron shower
Has a transverse size ~ λ

r_M ~ 2 cm for crystals
λ ~ 15 cm for Cu
EM showers are transversely
smaller than hadronic showers

Transverse Size - II

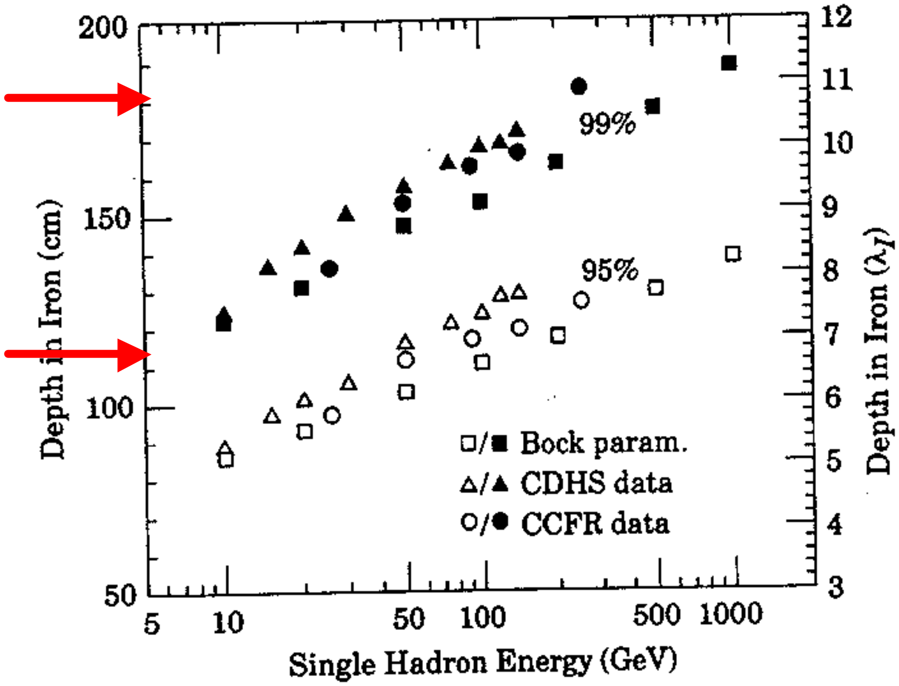




Transverse distribution has A EM "core" and a hadronic "tail" – 2 components

n.b. there are 2 distance scales, one ~ r_M and the other ~ λ. Data is integrated over all depths in the shower

Energy Leakage



For 95 % containment, the Fluctuation in the leakage is

- 5 %.

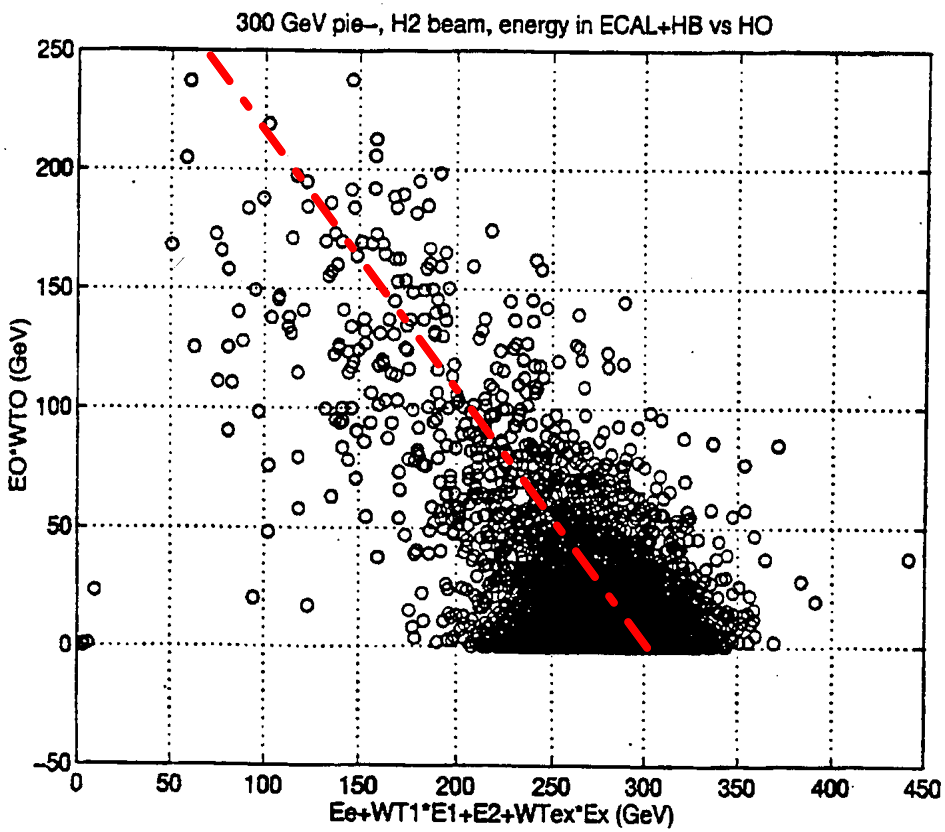
With 7 λ total depth, single pions are

- 95 % contained for energies < 100 GeV

As the LHC is a 7 TeV + 7 TeV machine, That is a bit thin.

CMS - Leakage and “Catcher”

4 λ



CMS outer calorimetry
300 Gev pions
“exit weighting” to
oversample late developing showers

Figure. 7: Scatter plot of energy inside the solenoid vs. the energy outside the solenoid in the HO layers for single 300 GeV pions.

7 λ

dE/E and Energy Leakage

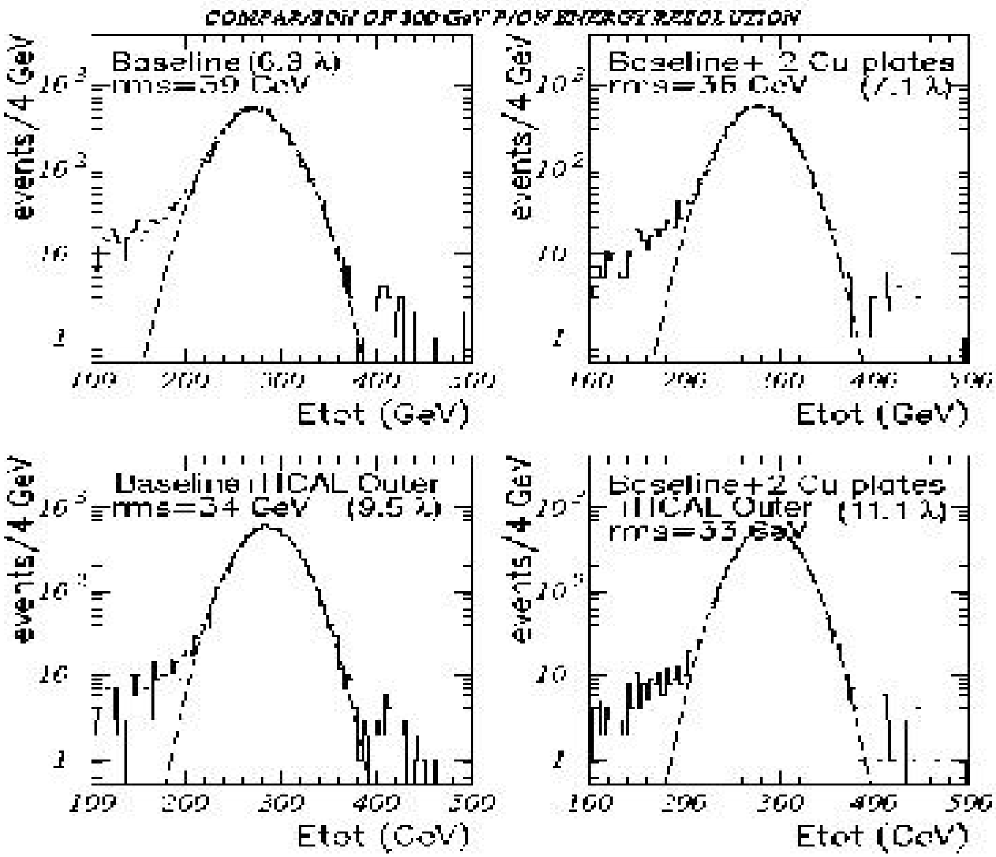
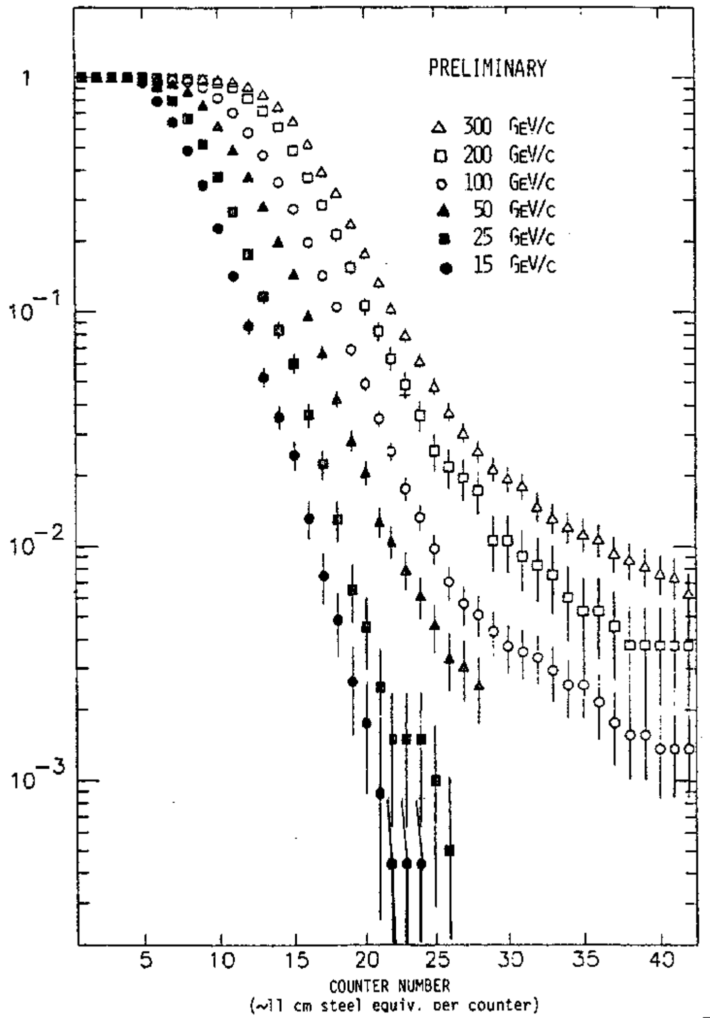


Fig. 27. Comparison of energy resolution (rms) for 300 GeV pions for different HCAL sampling configurations: Baseline Inner HCAL, Baseline Inner HCAL + 2 plates, Baseline Inner HCAL + HO, and Baseline Inner HCAL + 2 plates + HO.

Leakage "tails" are reduced
By adding in the outer calorimeter
With suitable relative weights.

Intrinsic Neutrino “Leakage”



“punch through” displays a length scale ~ λ except for very deep into high energy showers.

There is a component that does not fall off rapidly in depth and which has a fraction which rises rapidly with energy.

1 % at 300 GeV

due to $\pi \rightarrow \mu\, \nu$ decays of cascade pions

n.b. earth’s atmosphere -
We live at the bottom of a very diffuse ~ 10 λ calorimeter – the atmosphere.
Cosmic ray muons supply the background dose of ~ 0.2 rad/yr natural background

Muons and Single Samples

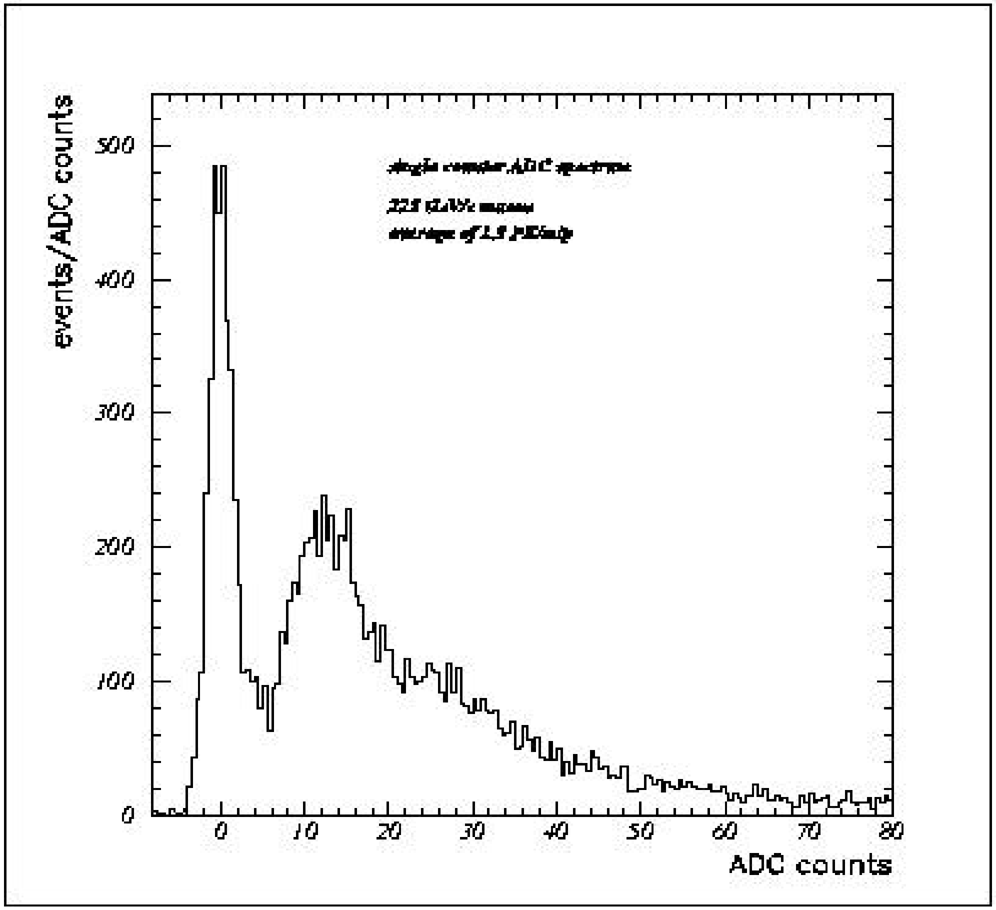


Fig. 3. H2(1995) Test Beam results: ADC spectrum of 225 GeV/c muons in a single counter. Based on the observed inefficiency of the counter, we estimate the average number of photo-electrons per minimum ionizing particle to be 1.3 PE/mip.

Single scint tile test
1.3 p.e./MIP

$\exp[-\langle N\rangle] = 1 - \varepsilon$

$\langle N\rangle = 3$ is 95 % efficient

Muons in Sampling Calorimeters

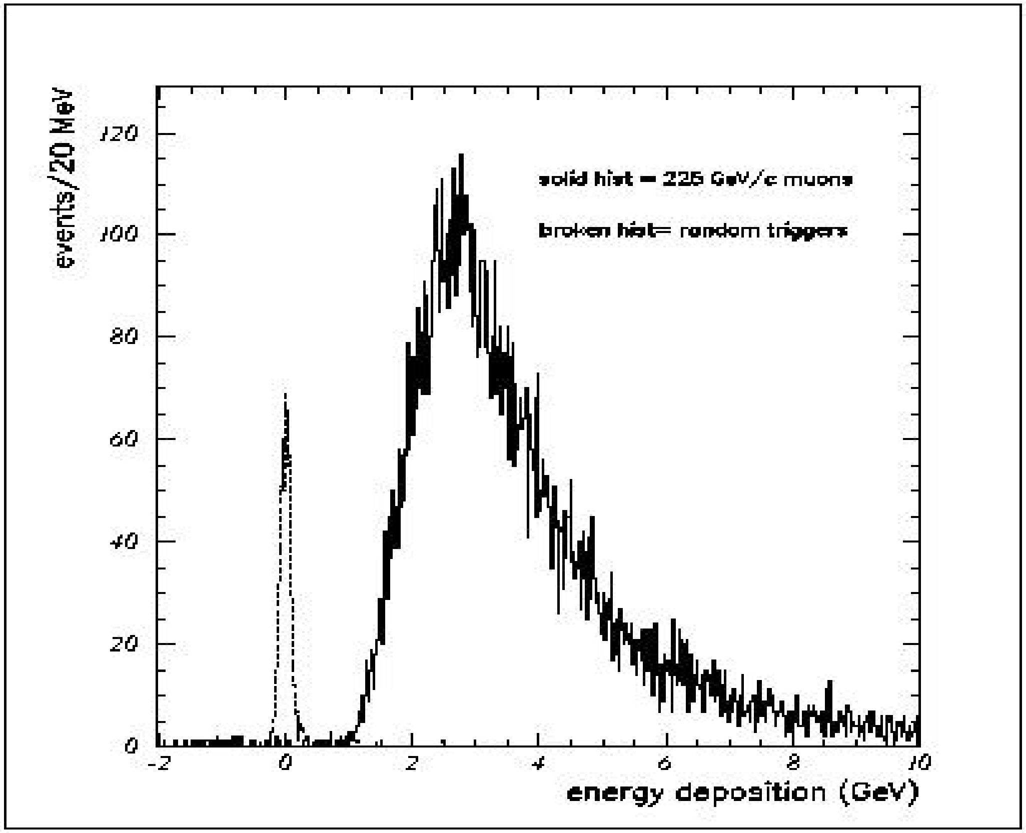
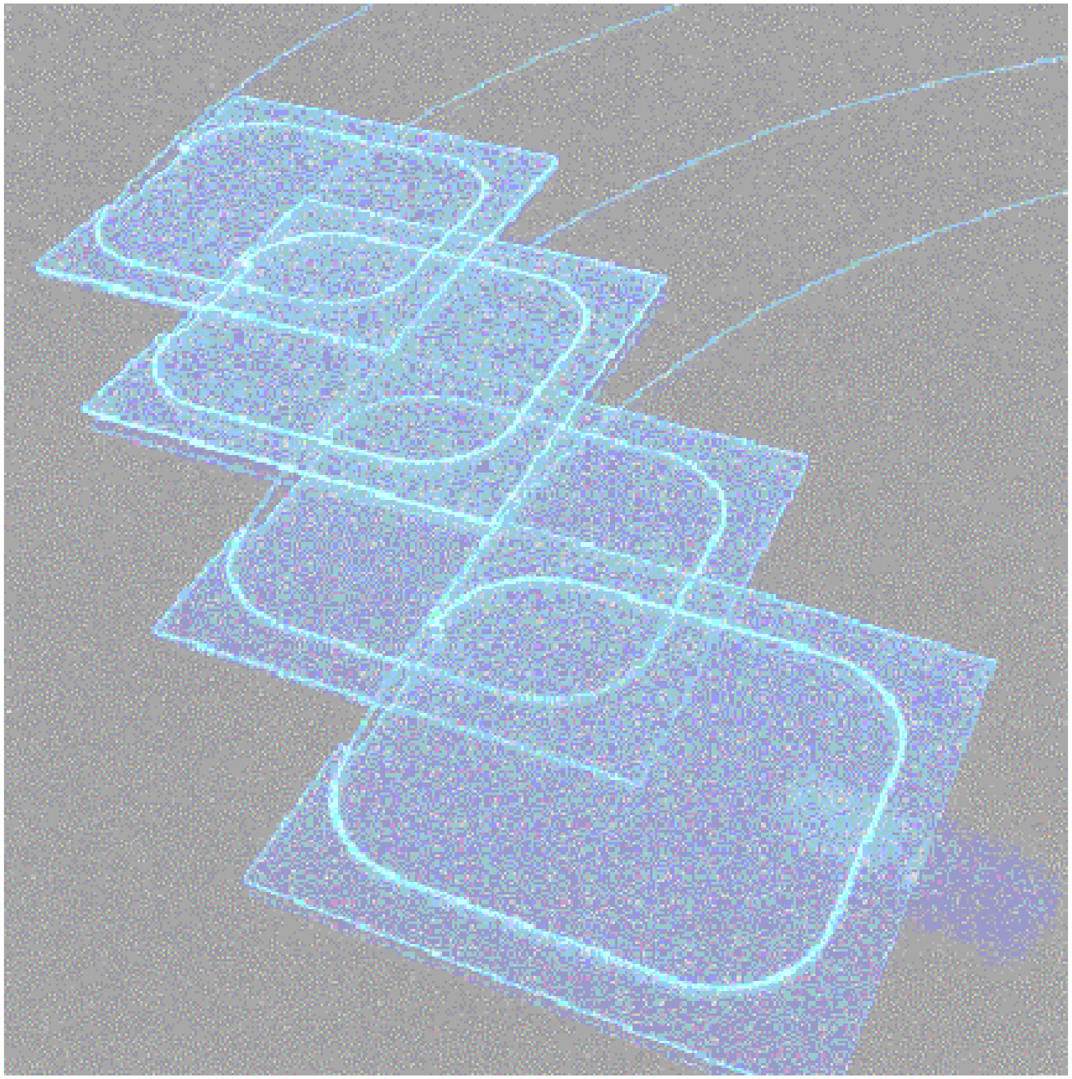


Fig. 8. H2(1995) Test Beam results: Energy deposited by 225 GeV/c muons in HCAL. Broken line shows the energy reconstructed in the HCAL for random triggers (pedestal events). The pedestal peak has RMS width of 80 MeV equivalent energy.

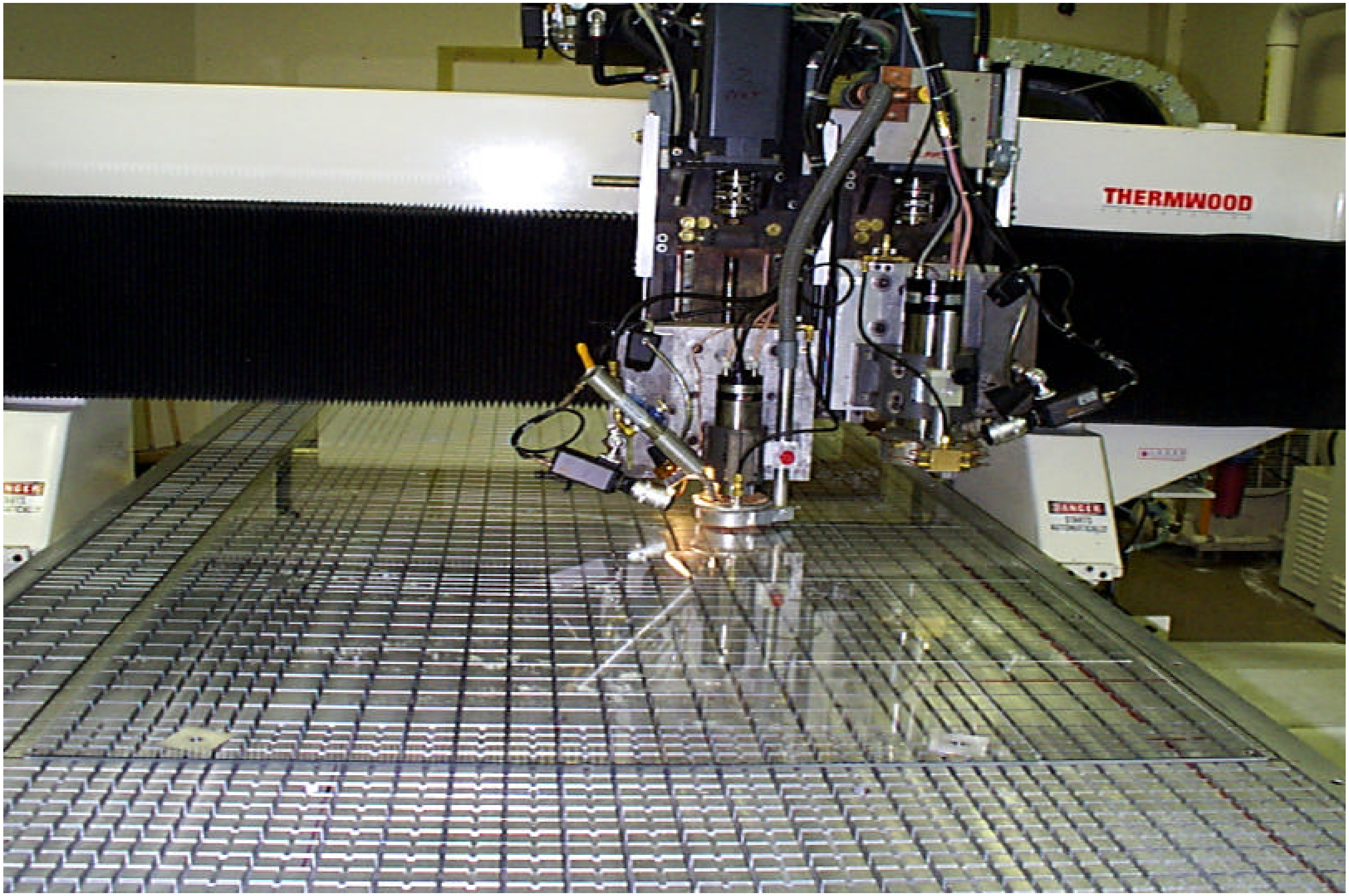
Muon calibration for the full HCAL in depth

$<E_{\mu}> \sim 3$ GeV

Tile/WLS Fiber Calorimetry

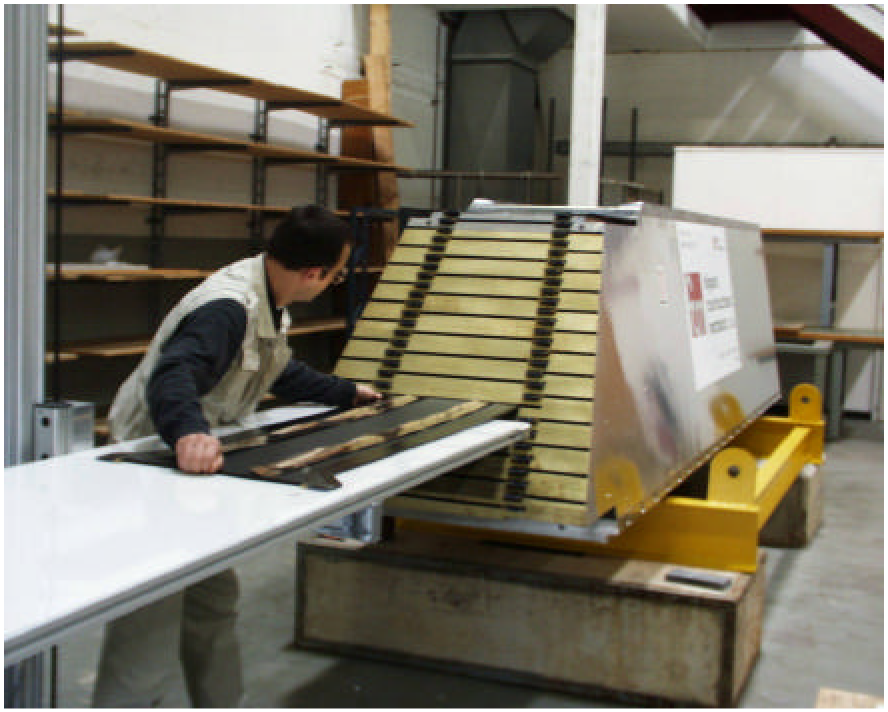


HCAL - Tiles in Lab5



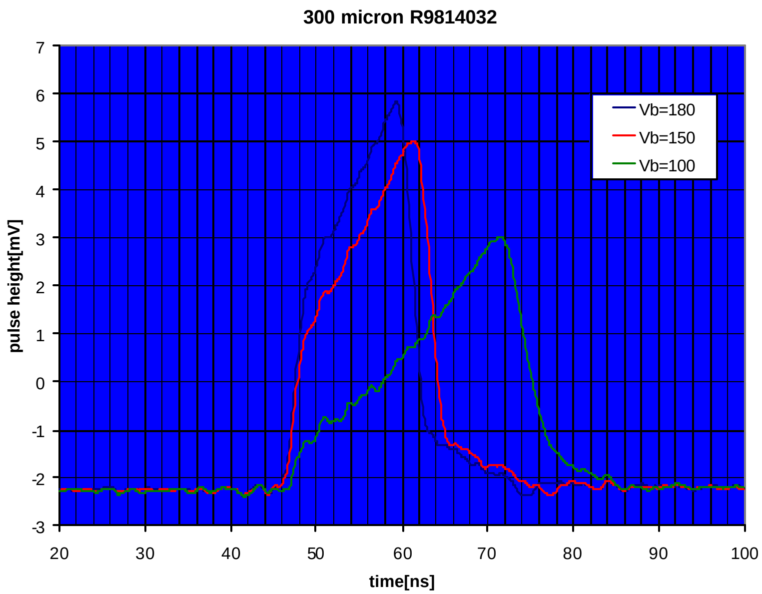
PPP1 and PPP2 at CERN

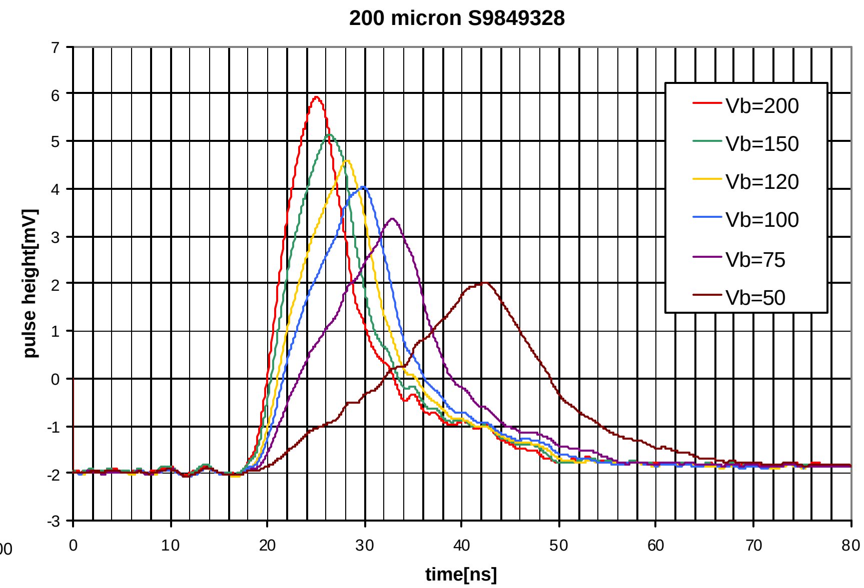
HCAL - PPP1, Scintillator

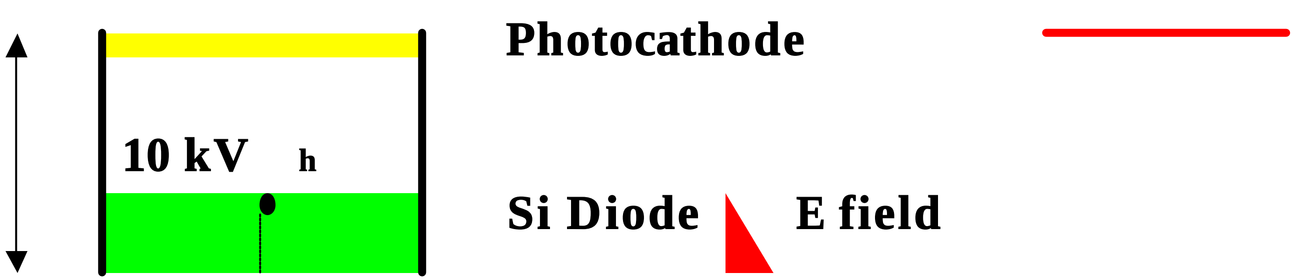


- absorber done in industry to FNAL design
- scintillator done in Lab5
- wedge assembled at CERN

Pulse Formation - Bias



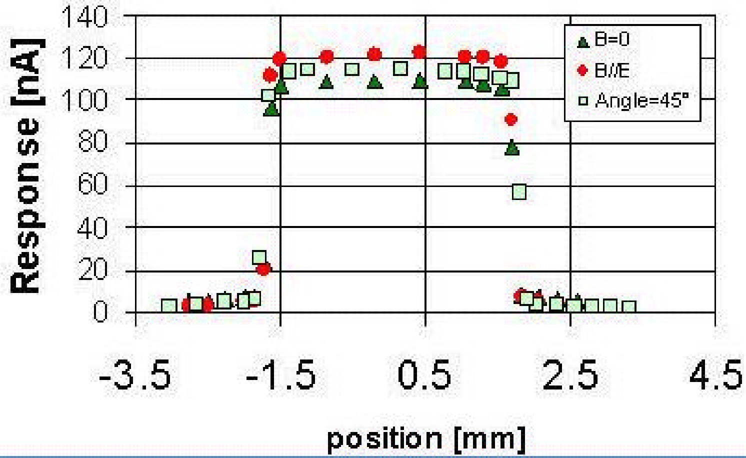


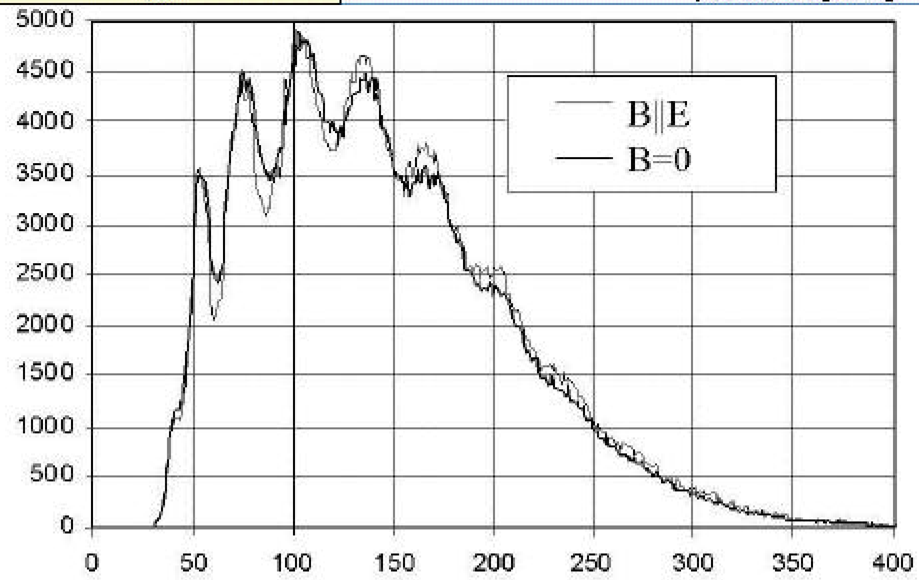


Backscatter in B Field

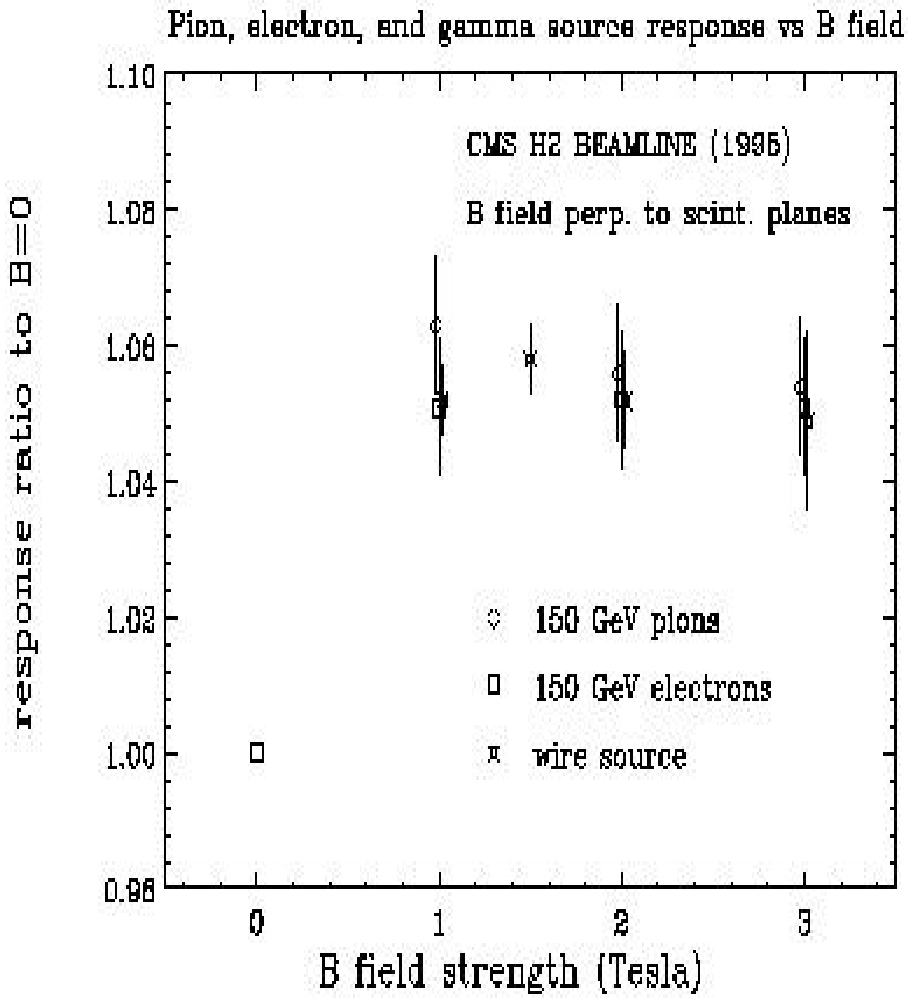
Backscatter focus yields 10% more signal with magnetic field on

Off-angle gain is reduced by longer path length through surface layer





Scintillator and B Fields



Scintillators get brighter in B fields

For B perpendicular to sampling plates, π, e, μ, γ all show ~ 6 % increase

The effect saturates for B > 2 T

HCAL and e/p in B Fields

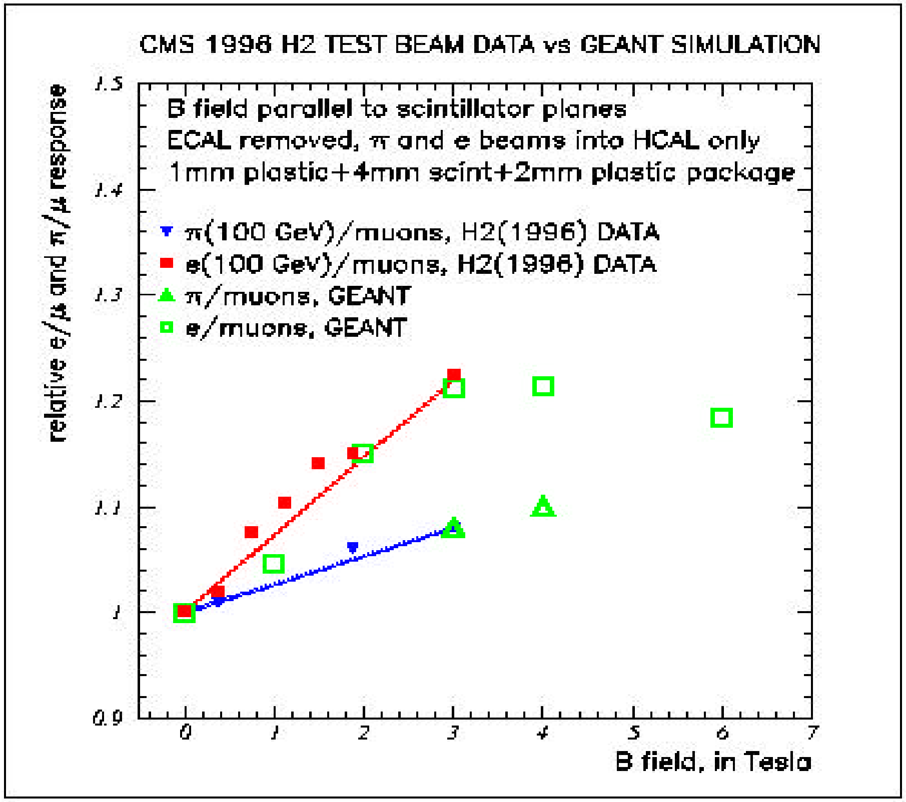


Fig. 7. Effect of B field on the average energy response of the tile/fiber calorimeter to pions, electrons (H2 data) (divided by the muon response) and comparison with GEANT predictions. B field lines were parallel to the scintillator plates (Barrel configuration). The overall scintillator brightening B field effect cancels when the ratio of electrons to muons is taken (upper curve), thus illustrating the increase from curling low energy electrons in the shower. The ratio of hadrons to muons (lower curve) shows a smaller increase thus indicating that the effect is a function of the electromagnetic fraction in the shower.

For B parallel to the sampling plates,
π, e, μ, γ all show different effects.
μ, γ show only the 6 % brightening
e show ~ 20% increased response,
while 100 GeV π show ~ 10% increase –
after correcting for brightening

π are < e since "fo" < 1.
Recall shower energy deposited by
soft e in the EM clusters. Foe Ec ~ 7 MeV,
the radius of curvature is ~ 7.7. mm

HCAL - Path Length/Loopers

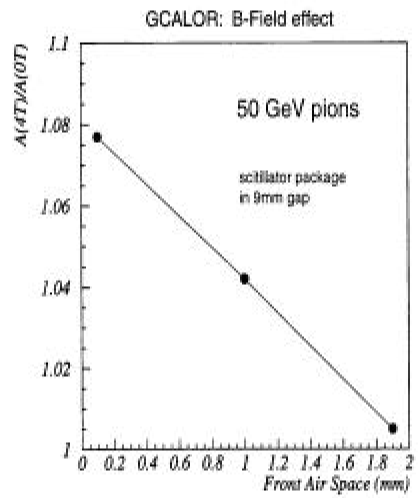
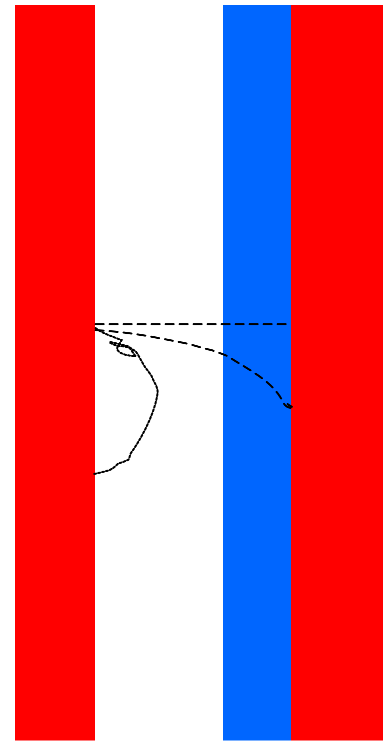


Fig. 1.35: Monte Carlo study on response of HB to 50 GeV pions in 4 Tesla field relative to response in 0 Tesla field with different air space between upstream absorber and scintillator package placed in 9mm gap between absorbers. The scintillator package consists of a 2mm plastic front cover plate, a 4mm scintillator and a 1mm plastic back cover plate.



Can tune the effect out by playing "loopers" off against path length – but there are open spaces in the calorimeter which is inefficient

n.b. mm scale

Increased path length gives increased light. This competes with "loopers" if there is a gap between the plate and the scint

LHC - Radiation Fields

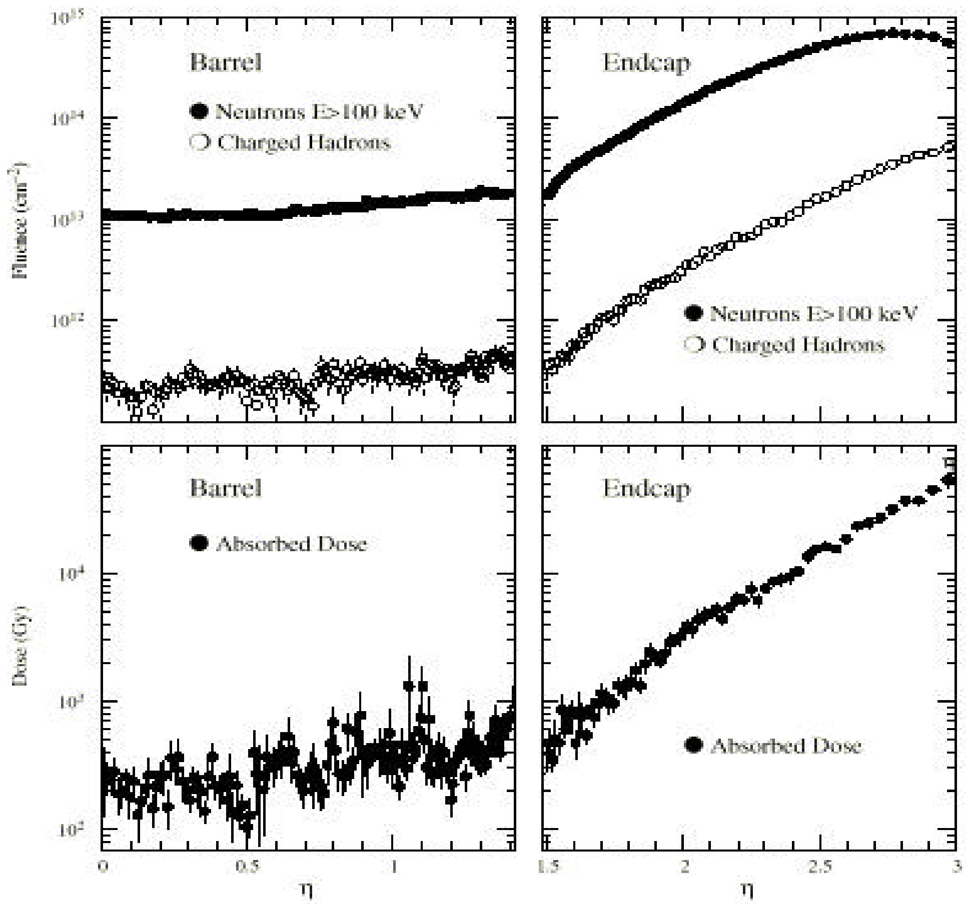


Fig. A.7: Neutron (E > 100 keV) and charged hadron fluence and absorbed dose immediately behind the crystals as a function of pseudorapidity. The values are obtained in an aluminium–air mixture. Values correspond to an integrated luminosity of 5×10^5 pb^{-1}.

dose < 10 Mrad over the lifetime of the LHC

note the large n background inherent in pp machines

dose ~ exp(3η)

LHC - HCAL Dose

HCAL dose is monotonic, falling
From max in ECAL with length
Scale ~ λ. The dose is due to soft
pions, $P_T \sim 0.4$ GeV, $\eta \sim 3$, $P \sim 4$ GeV

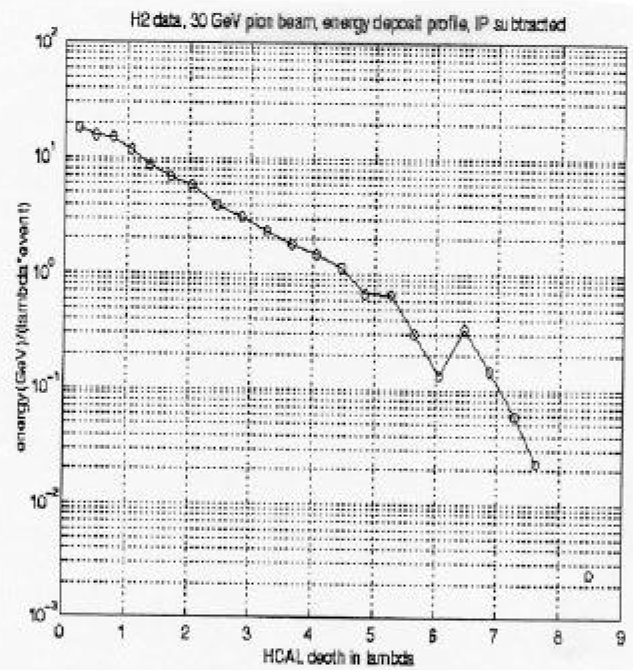
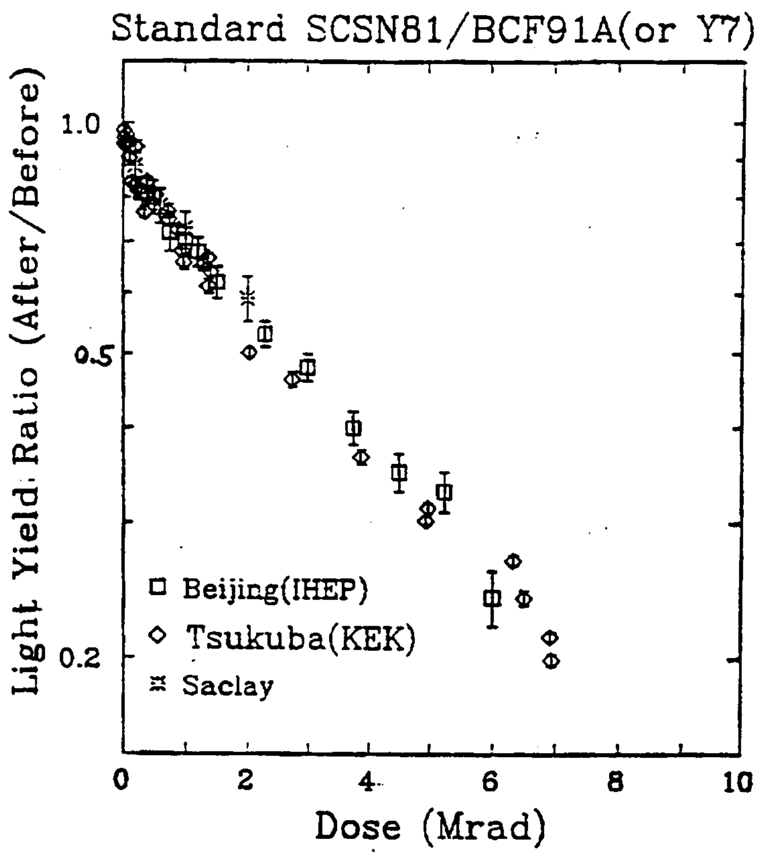


Fig. 1.9: Energy deposit as a function of depth for 30 GeV pions from the H2 test beam.

Scintillator - Dose/Damage



Scintillator under irradiation forms Color centers which reduce the Collected light output (transmission loss).

LY ~ exp[-D/Do], Do ~ 4 Mrad

HCAL - Raddam and z Sampling

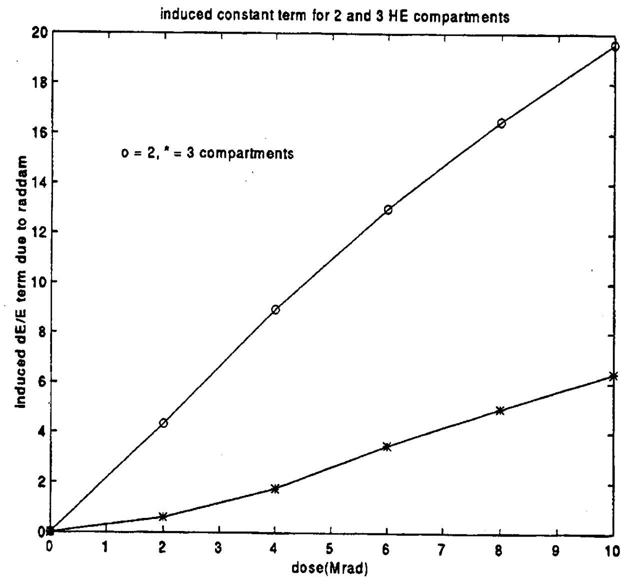
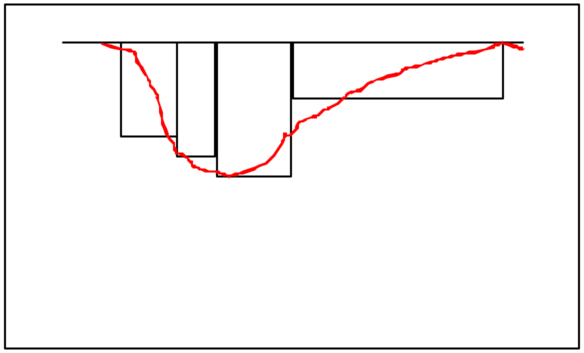


Fig. 1.10: Induced constant term in the energy resolution as a function of dose for 2 and 3 longitudinal compartments.

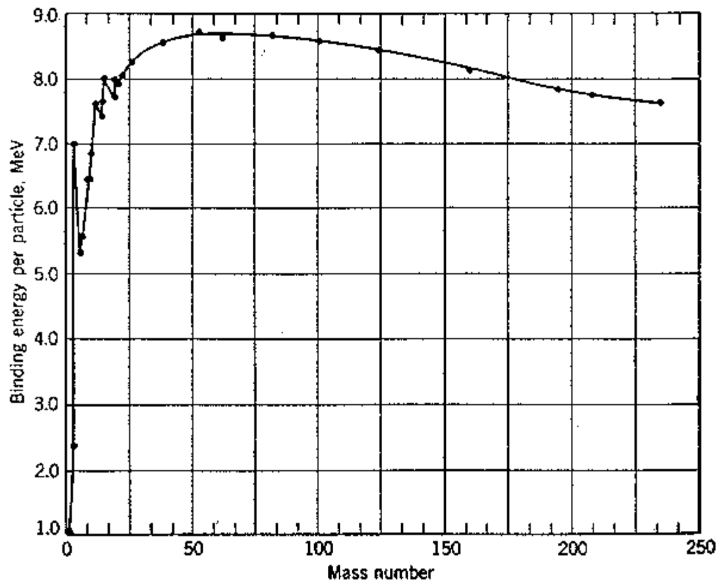
Solve Radiation Damage with Longitudinal Segmentation

Independent calibration and readout of
Each distinct segment.



For HCAL, at 5 Mrad maximum, dE/E goes from
10% → 2 % with 2 → 3 compartments

Neutrons



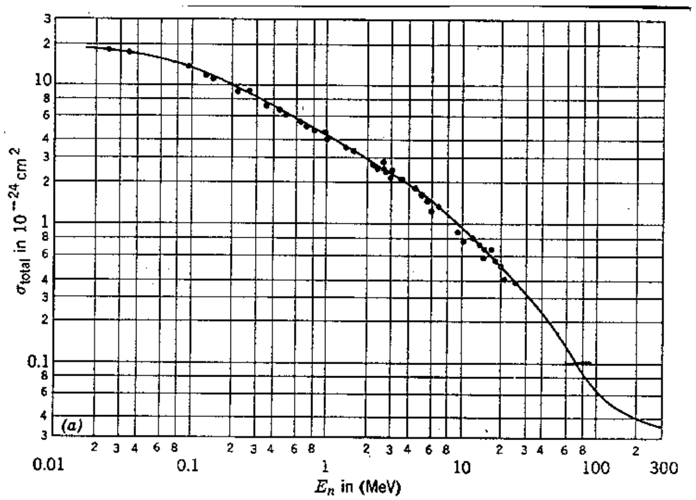
Recall hadrons disrupt the medium

For 100 GeV pion, there are ~ 500 n near hadron shower maximum with Tn ~ 1 MeV [B ~ 8 MeV/nucleon] , slow down and escape → sea of soft n at the LHC

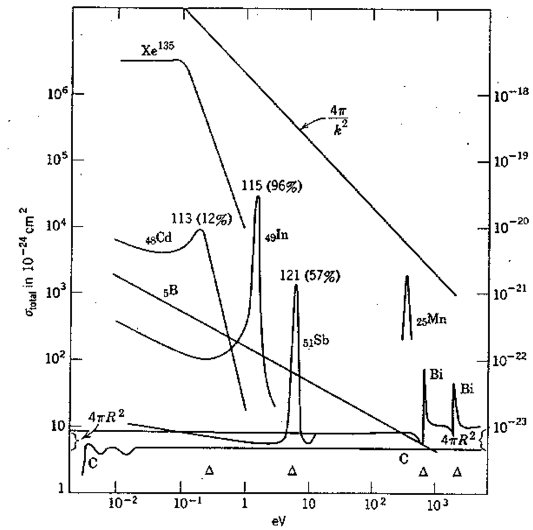
$$N_n \sim [5\, E(GeV)]$$

$$\langle T_n \rangle \sim 1\ MeV$$

Neutron Detection



$$\left(\frac{A-1}{A+1}\right)^2 \le \frac{T}{T_n} \le 1$$



S wave unitarity

Geometric cross section

Recall billiards: off cushion - T/Tn ~ 1
Off cue – T/Tn → 0

n.b. scintillator sampling “eats” the n

exothermic reactions to detect thermal n

$n + p \rightarrow D + g$ (2.2 MeV) and $n + {}^6Li \rightarrow {}^3H + a$ (4.76 MeV).

Standard Model

The Basic Constituents of the “Standard Model”

	Generations			**CHARGE Q ***	
MATTER (SPIN 1/2)+	$\begin{pmatrix} e \\ \nu_e \end{pmatrix}$	$\begin{pmatrix} m \\ \nu_m \end{pmatrix}$	$\begin{pmatrix} t \\ \nu_t \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	LEPTONS
	$\begin{pmatrix} u \\ d \end{pmatrix}$	$\begin{pmatrix} c \\ s \end{pmatrix}$	$\begin{pmatrix} t \\ b \end{pmatrix}$	$\begin{pmatrix} 2/3 \\ -1/3 \end{pmatrix}$	QUARKS
	QUANTA	**FORCE**	**COUPLING**	**# QUANTA**	**SYMBOL**
INTERACTIONS (SPIN 1)	Gluons	Strong	$\alpha_s = g_s^2$	8	g
	Photons	EM	$\alpha = e^2$	1	γ
	Weak Bosons	Weak	$\alpha_w = g_w^2$	3	$W^- \; Z^o \; W^+$

* Units are electron charge $|e|$.

\+ Units are $\hbar$.

SM - Detection, ID

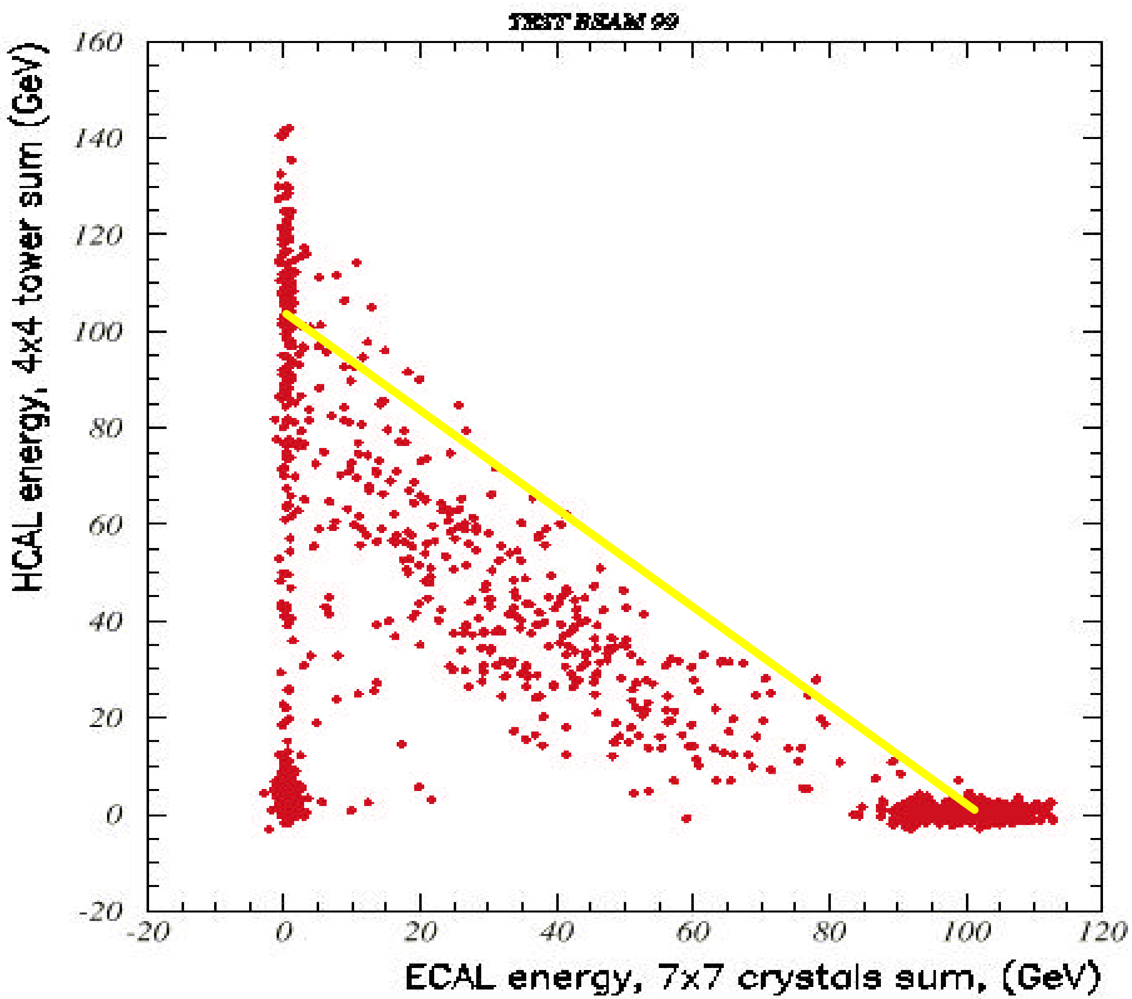
Detection/Identification Methods

SIGNATURE	DETECTOR	PARTICLE
Jet of Hadrons l_o	Calorimeter	u, c, $t \rightarrow Wb$ d, s, b g
“Missing” Energy	Calorimeter	v_c , v_m , v_t
Electromagnetic Shower, X_o	Calorimeter	e, g , $W \rightarrow ev$
Only Ionization Interactions, dE/dx	Muon Absorber	$m, t \rightarrow mv\bar{v}$ $Z \rightarrow mm$
Decay with $ct \geq 100\, mm$	Si Tracking	c, b, t

Detector Subsystems

Particle type	Tracking	ECAL	HCAL	Muon
g				
e				
m				
Jet				
Et miss				

HCAL + ECAL and Particle ID



e, m**,** p

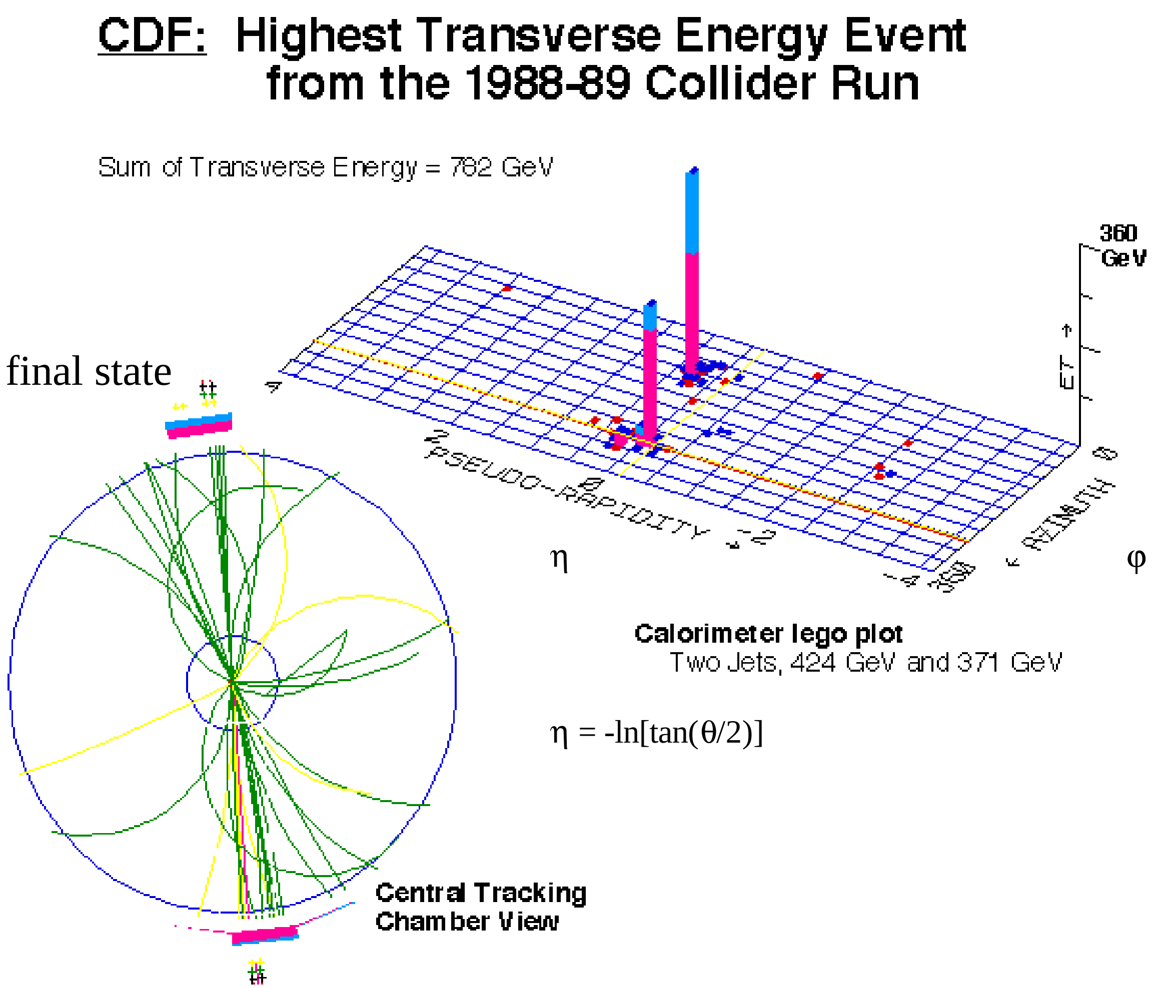
in

“ECAL” + HCAL

Jets and Calorimetry

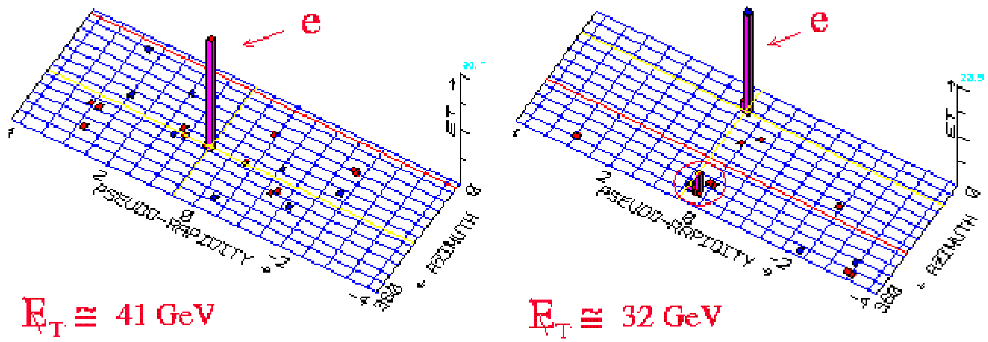
Jets

If quarks have Pt ~ 0, then final state
Is (2 body) “back to back”



$\eta = -\ln[\tan(\theta/2)]$

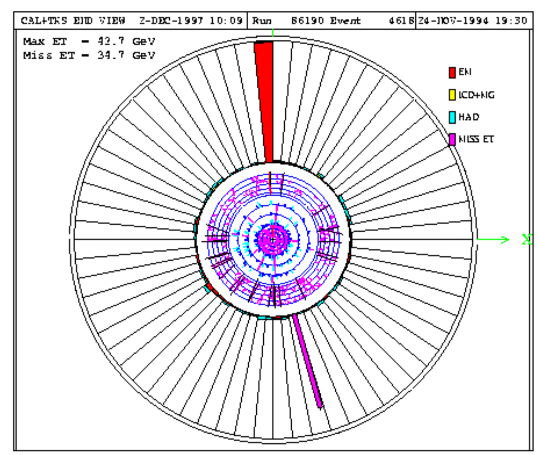
W --> e + n and Calorimetry



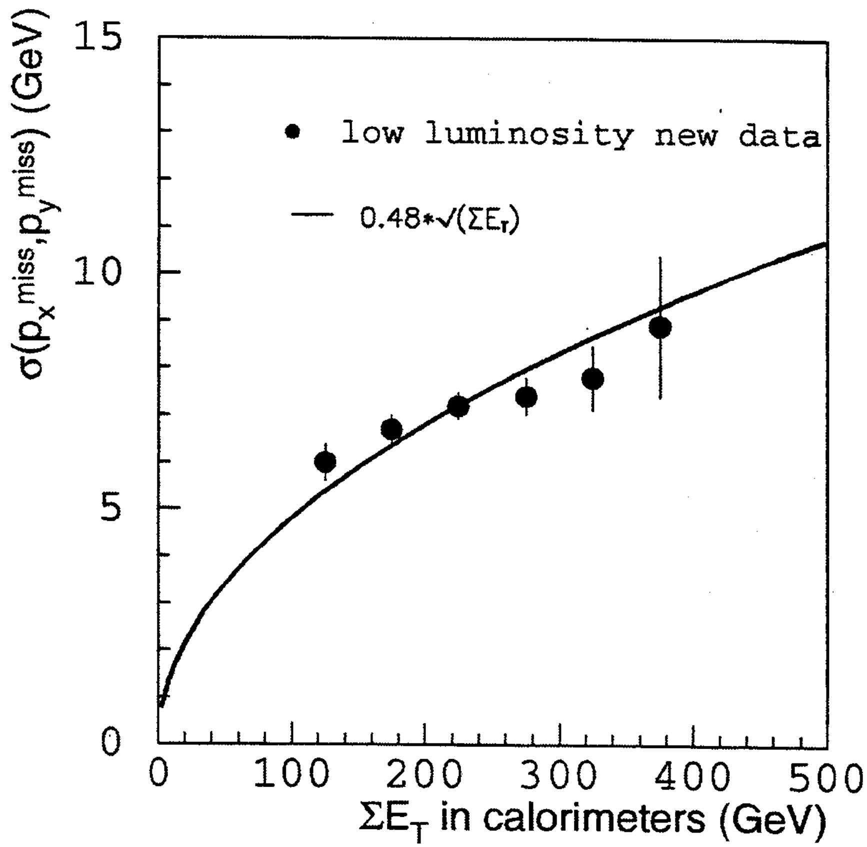
W $\rightarrow$ e ν

$M_W \sim 2$ Pt ~ 80 GeV

EM Calor (e) + HCAL (missing Et)



Calorimeters and Neutrinos



Missing Et is a global variable

dE/E ~ a/√Σ(Et)
summed over all Et in the event