WHY should you believe lattice calculations?

lattice gauge theory gives a definition of quantum field theory; that only tells you that, in principle, some day, some one will solve QCD

In the meantime, the real question is why you should believe anyone's error bars — and how to tell by looking at a paper what's going on.

As we noted last time, lattice QCD, on a computer is a multi-scale problem

$$L^{-1} \lesssim m_q \ll \Lambda_{QCD} \ll m_b \sim a^{-1}$$

and because we live in 4D spacetime,

$$L \sim 1\text{–}4 \text{ fm}$$

$$0.2\text{–}0.4 < \frac{m_q}{m_s} < 2$$

$m_{PS}^2(m_s) = 2m_K^2$

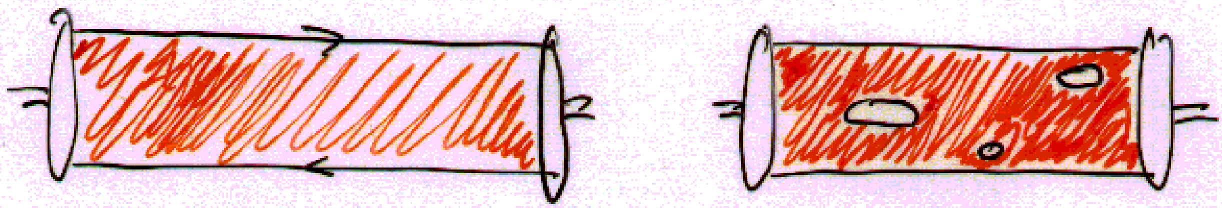
$$a^{-1} \sim 2\text{–}5 \text{ GeV}$$

$$m_b a \gtrsim 1 \quad \text{or} \quad m_Q \sim m_c$$

And then, until recently, there was the quenched approximation.

Except quenching, effective field theories are used to control uncertainties in going from computer to nature.

Quenched Approximation



valence quarks and gluons exact

sea quarks CPU intensive, so "quenching" means to omit them and compensate:

$$g_0^2 \longrightarrow g_0^2 \varepsilon$$

so it is a bit like a dielectric approx.

As with a dielectric, can only hope it's good over a limited range. For example

$$\beta_0 = 11 - \tfrac{2}{3} n_f$$

so g_0^2 runs differently

Only argued to be partly under control in

B_K, α_s, m_Q, $\mathcal{F}_{B\to D^*}(1)$, $\cancel{\xi = \frac{f_{B_s}^2 B_s}{f_{B_d}^2 B_d}}$

The only way out is too use CPU to compute the sea quark contribution.

ASK + PBM (1993) use it to learn how to estimate the other uncertainties.

(If you are pragmatic, you should note that quark models leave out more physics than quenched approx. So, if you are using with quark model mx elements, don't sneer at quenched mx elements.)

For the other systematics, EFT:

a	Symanzek EFT
m_Q	HQET or NRQCD
m_q	chiral perturbation theory
L	general hadronic FT (Lüscher)

rule on systs.

This lecture will cover the EFT's.

Tomorrow's will tell you how to compute

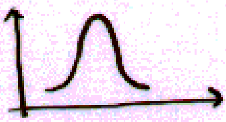
$$m_{PS}(m_q, m_{\bar{q}}) \quad \ldots$$

$$\langle 0 | A^\mu | \pi \rangle \qquad \langle \bar{K} | Q | K \rangle$$

$$\langle 0 | A^\mu | B \rangle \qquad \langle \bar{B} | Q | B \rangle$$

$$\langle D^{(*)} | J^\mu | B \rangle \qquad \langle \pi | V^\mu | B \rangle$$

via Monte Carlo.

Think of the output of the MC as a distribution 

and imagine that someone has "analysis codes" to put these distributions through today's theoretical formula.

In fact we have classes (in perl + C^{++}) to do exactly this.

WHAT is an effective field theory?

Wilson: integrate out short-distance physics and lump those effects into couplings (or short-distance coefficients).

Weinberg: identify relevant degrees of freedom.

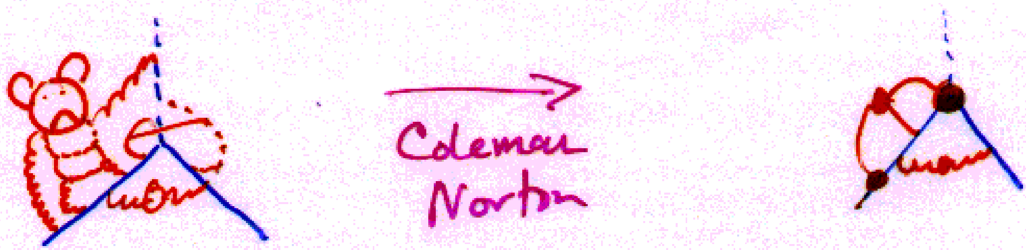
Then most general parametrization of S matrix is a (non-renormalizable) quantum field theory of those fields

$\doteq$ "has the same physics as"

$$\mathcal{L}_{\text{underlying}} \doteq \mathcal{L}_{\text{effective}}$$

$$\mathcal{L}_{\text{effective}} = \sum_n K_n(\text{short}) \underset{\mu}{\wr} \mathcal{O}_n(\text{long})$$

What about singularities?



Everything we do at Fermilab assumes this formula, with $\mathcal{L}_{\text{underlying}} = \mathcal{L}_{\text{Nature}}$ and experiments analyzed with $\mathcal{L}_{\text{eff du jour}}$.

In the case of lattice calculations

$\mathcal{L}_{\text{underlying}} = \mathcal{L}_{\text{computer}}$

we know what it is, and the problem is to get from it to QCD

Chapter along these lines for Shifman's Handbook of QCD, coming to arXiv soon.

Lattice spacing effects – Symanzik

Although the $a \to 0$ limit defines QCD, what we really need is a description of lattice spacing effects for a^{-1} of few GeV.

$$\mathcal{L}_{lat} \doteq \mathcal{L}_{sym}$$

$$\mathcal{L}_{sym} = \mathcal{L}_{QCD} + \sum_n a^{\dim \mathcal{O}_n - 4} K_n \, \mathcal{O}_n$$

– a continuum field theory

$$K_n = K_n(g^2, ma, \underbrace{\{c_0\}}_{\text{get to these in a minute}}; \mu a)$$

For example, at dimension 5 there is one operator

$$\mathcal{O}_5 = \bar{q}\, i\sigma_{\mu\nu} F^{\mu\nu} q$$

That means

$$m_N(a) = m_N(0) + a \langle p | O_5 | p \rangle K_5 + O(a^2)$$

$$\langle 0 | A^\mu_{lat} | \pi_{lat} \rangle = Z_A^{-1} \langle 0 | \mathcal{A}^\mu | \pi \rangle + a K_A \partial^\mu \langle 0 | P | \pi \rangle + a Z_A^{-1} K_5 \int d^4x \langle 0 | T\, O_5 \mathcal{A}^\mu | \pi \rangle$$

These formula show that lattice effects are reduced by

- making a smaller
- combining data sets and extrapolating
- "improving" the lattice $\mathcal{L}_{lat}$

$$\mathcal{L}_{lat} = \mathcal{L}_{Wilson} + \sum_n a^{\dim O_n - 4} c_{0n} O^n_{lat}$$

$$0 = K_n(c_{0m}) \Rightarrow c_{0m} = c_{0n}(g^2, ma)$$

EFT shows once $K_n = 0$ for one observable, that effect is removed everywhere

Heavy Quarks and HQET

For the b quark $m_b a \gtrsim 1$ on accessible lattice spacings. Two main strategies

1) ad hoc set $m_Q \sim m_c$ and extrapolate up, guided by expansion in $1/m_Q$

This is fine if $ma \ll 1$ and $\Lambda/m \ll 1$

In practice $0.4 < ma < 0.8$

$$1 < \frac{m}{\Lambda} < 2$$

Also, how does $(ma)^n$ extrapolate in $1/m$?

2) build in HQET/NRQCD from outset

If $ma \not\ll 1$ it is not lattice gauge theory that breaks down, it is Symanzik's LE$\mathcal{L}$.

many "small" terms $\sim (ma)^n$ not small!

So replace Symanzik w/ HQET

For $m_Q \gg \Lambda_{QCD}$

$$\mathcal{L}_{QCD} \doteq \mathcal{L}_{HQET} = \sum_n C_n^{QCD}(m_Q; \mu/m_Q)\, \mathcal{O}_n^{(HQET)}$$

$$\mathcal{L}_{lat} \doteq \mathcal{L}_{HQET} = \sum_n C_n^{lat}(m_Q, m_Q a; \mu/m_Q)\, \mathcal{O}_n^{(HQET)}$$

The operators are the same, but the short-distance coefficients are different

$$\begin{tikzcd}
\text{lat} \arrow[d, "C_n^{lat} \overset{!}{\doteq} C_n^{QCD}"'] & & \text{HQET} \arrow[ll, bend right, "C_n^{lat}"'] \arrow[dll, bend left, "C_n^{QCD}"] \\
\text{QCD} & &
\end{tikzcd}$$

In practice, matching is done in perturbation theory, which is accurate at short distances.

Light Quark Effects

For heavy quarks we exploited HQ symmetry

For light quarks we exploit chiral symmetry

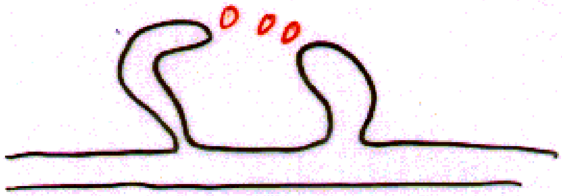
$$\mathcal{L}_{QCD} \doteq \mathcal{L}_{\chi PT}$$

for $p^2, m_{PS}^2 \ll \Lambda_{QCD}^2$.

The biggest concern here are chiral logarithms

In unquenched case, χPT fixes their coefficients to other matrix elements.

In quenched approximation, the chiral logs are mistreated



would be η'

Finite Volume – Lüscher

Here one takes Weinberg's point of view and posits a general hadronic field theory

Sensible if $L\Lambda \gg 1$ $L m_{ps} \gg 1$

Two interesting effects – polarization and "scattering".

Polarization – virtual exchanges "around the wor

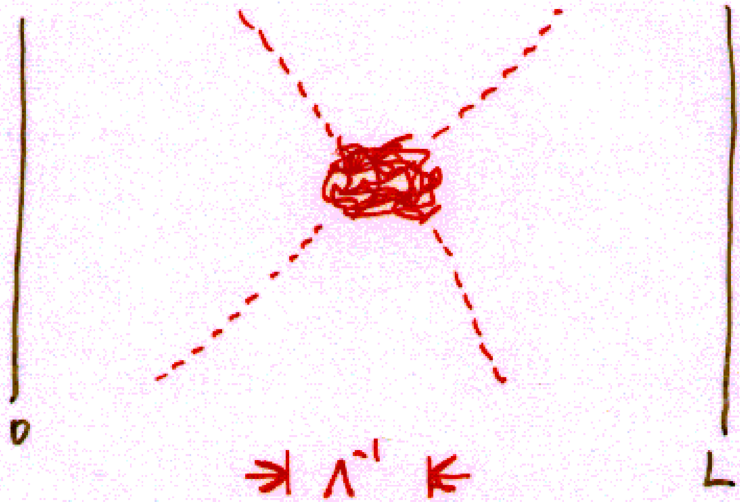
these are classically forbidden, and consequently exponentially suppressed

$$m_N(L) = m_N(\infty) + 3\,\frac{g_{\pi N}^2\, m_N}{16\pi^2}\, e^{-\mu L}$$

$$\mu^2 = m_\pi^2 - \frac{m_\pi^4}{m_N^2}$$

easy to obtain at one loop

Scattering is more complicated - have to think about a two-body wave function in a finite volume



Infinite volume - any energy allowed

phase shifts come in from matching w.f. at range R of scattering.

Finite volume - discrete energies, and two places to match: R and L.

Same phase shifts at R

If there is only one partial wave

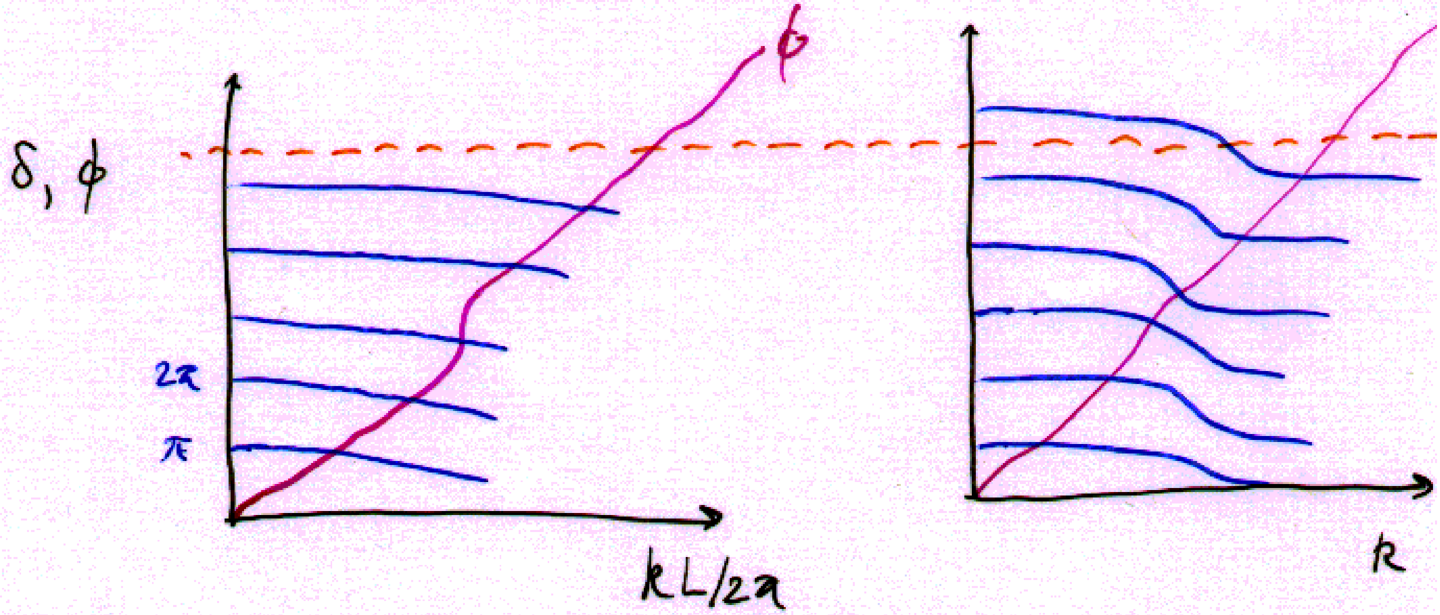
$$\delta(k) = n\pi - \phi(kL/2\pi)$$

dynamical from $R \sim \Lambda_{QCD}^{-1}$

kinematical from L

Derivation makes usual scattering theory look easy

$$E = 2\sqrt{m_\pi^2 + k^2}$$



w/o resonance   w/ resonance

Can, in principle, be used to calculate phases of $K \to \pi\pi$. But need large volumes

$$2\text{ fm} < L < 6\text{ fm}$$