TOWARDS A SYSTEMATIC ANALYSIS OF K-K MODES

work in progress
with G.G. Ross

3+1 DIMENSIONS : $SO(3,1) \simeq SU(2) \times SU(2)^*$

Weyl spinors $\nu_L \sim (\underset{\sim}{2},\underset{\sim}{1})$; $e_L \sim (\underset{\sim}{2},1)$; $\bar{e}_R \sim (2,1)$
OR $e_R \sim (1,2)$.

notation for Weyl spinors : ν , e , $\bar{e}$, ... ALL LEFT-HANDED.

4+1 DIMENSIONS : $SO(4,1)$ NO CHIRAL STRUCTURE
$SO(4,1) \sim Sp(4)^*$.

ONLY ONE TYPE OF FERMIONS

$\Psi \sim \underset{\sim}{4}$ of $SO(4,1)$ COMPLEX $\bar{\Psi} \equiv \Psi^\dagger \gamma_0$

conjugate spinor: $\Psi^c \equiv i\gamma_5\gamma_2 \Psi^*$

$$\underset{\sim}{4} \times \underset{\sim}{4} = \underset{\sim}{1} + \underset{\sim}{5} + \underset{\sim}{10}$$

Lorentz invariants $\underset{\sim}{1}$ $5 = 4_v + \underset{\sim}{1}$ $10 = 6 + 4_v$

4 LORENTZ - INVARIANT FERMION BILINEARS

5-DIM. KINETIC TERM

$$S_D \qquad \int d^4x \int dy \;\; \bar{\Psi}\, \gamma^A \partial_A \Psi$$

dimension $\quad [4+1] \left[\left(\frac{3}{2}+\frac{1}{2}\right) + 1 + \left(\frac{3}{2}+\frac{1}{2}\right)\right]$

WRITE Ψ IN TERMS OF WEYL SPINORS

$$\Psi = \begin{pmatrix} \eta \\ \bar{\xi} \end{pmatrix}(x,y) \qquad \eta, \bar{\xi} \sim (\underset{\sim}{2},\underset{\sim}{1})$$

$$S_D = \int d^4x \int dy \; (\eta^\dagger \dot{\eta} + \xi^\dagger \dot{\xi} + \cdots \qquad)$$

INVARIANCES OF DIRAC KINETIC TERM :

- LEPTON NUMBERS $\qquad \eta \;\; L=1 \quad ; \quad \xi \;\; L'=-1$

  $L+L'$ commutes with $SO(4,1)$.

- DISCRETE P_S : $\quad y \to -y$

  $\eta \to \bar{\xi} \qquad \xi \to -\bar{\eta}$

- DISCRETE π_S : $\quad y \to -y$

  $\eta \to \pm\eta \qquad \xi \to \mp\xi$.

KK-2

LORENTZ-INVARIANT MASS TERMS

		L	$\mathcal{P}_5$	π_5
DIRAC MASS :	$\bar{\Psi}\Psi = \eta^\dagger \bar{\xi} + \xi^T \eta$	$\Delta L = 0$	even	odd
MAJORANA MASS :	$\overline{\Psi^c}\Psi = \eta^T \eta - \xi^\dagger \xi^*$	$\Delta L = 2$	even	even
DIRAC VECTOR :	$\bar{\Psi}\gamma_5\Psi = \eta^\dagger \eta + \xi^\dagger \xi$	$\Delta L = 0$	odd	even
MAJORANA VECTOR :	$\overline{\Psi^c}\gamma_5\Psi = i(\eta^T \eta + \xi^\dagger \xi^*)$	$\Delta L = 2$	odd	even

ALL INVARIANTS DISTINGUISHED BY SYMMETRIES

STARTING FROM DIRAC KINETIC TERM, CAN GENERATE

THREE DIFFERENT LORENTZ-INVARIANT MASS TERMS

DIRAC MASS M_D

MAJORANA MASS M

" VECTOR MASS " A_5

FERMIONS IN HIGHER DIMENSIONS : 6, 7, ... 11.

ADD MORE FERMIONS : Spinor ⊃ Spinor ⊃ Spinor, ...

e.g. 2 EXTRA DIMENSIONS $SO(5,1)$

$\Psi \sim 4, \bar{4}$ of $SO(5,1)$

IN 4+1 LANGUAGE IT IS LIKE ADDING MORE MULTIPLETS

KALUZA-KLEIN EXPANSIONS.

$$\eta(x,y) = \eta_0(x)c_0 + \sum_{n=1}^{\infty}\left(\eta_n^c(x)c_n(y) + \eta_n^s(x)s_n(y)\right)$$

$$\xi(x,y) = \xi_0(x)c_0 + \sum_{n=1}^{\infty}\left(\xi_n^c(x)c_n(y) + \xi_n^s(x)s_n(y)\right)$$

$$\left.\begin{aligned}\eta(x,y+2\pi R) &= \eta(x,y)\\ \xi(x,y+2\pi R) &= \xi(x,y)\end{aligned}\right\}\ \text{circle compactification}$$

OR: $\xi(x,y+2\pi R) = -\xi(x,y)$ ORBIFOLD COMPACTIFICATION

$$\xi(x,y) = \sum_{n=1}^{\infty}\xi_n^s(x)s_n(y) \qquad s_n(y+2\pi R) = -s_n(y)$$

KK4

CIRCLE COMPACTIFICATION

$$c_0 = \frac{1}{\sqrt{2\pi R}} \quad ; \quad c_n(y) = \frac{1}{\sqrt{2\pi R}} \cos\left(\frac{ny}{R}\right) \quad ; \quad s_n(y) = \frac{1}{\sqrt{2\pi R}} \sin\left(\frac{ny}{R}\right)$$

FOR GENERALITY, ASSUME $\boxed{\text{GAUGED } U(1)}$, $\langle A_5 \rangle \equiv A_5$ constant

$\boxed{\text{DIRAC MASS } M_D}$, $\boxed{\text{MAJORANA MASS } M}$. LET

$$\Xi^T \equiv (\, i\xi_0,\ \eta_0,\ i\xi_1^c, \eta_1^c, i\xi_1^s, \eta_1^s, \ldots, i\xi_n^c, \eta_n^c, i\xi_n^s, \eta_n^s, \ldots\ldots \,)$$

GET MAJORANA MASS :

$$\Xi^T \mathcal{M} \Xi$$

$$\mathcal{M} = \left[\begin{array}{cc|cccccc}
M & D & 0 & 0 & \cdots & \cdots & & 0 \\
D & M & 0 & 0 & \cdots & & & 0 \\
\hline
0 & 0 & M & D & 0 & K_n & \cdots & \\
\vdots & \vdots & D & M & -K_n & 0 & \cdots & \\
\vdots & \vdots & 0 & -K_n & M & D & \cdots & \\
0 & 0 & K_n & 0 & D & M & \cdots & \\
 & & \vdots & \vdots & \vdots & \vdots & &
\end{array}\right]$$

$$K_n = -i\frac{n}{R}$$

$$D = -iM_D - gA_5$$

DIAGONALIZATION :

$$M_0^{\pm} = \sqrt{(M \pm g A_5)^2 + M_D^2}$$ two "small" eigenvalues

$$\chi_0^{\pm} = \frac{1}{\sqrt{2}}(\eta_0 \pm i\xi_0)$$ eigenfunctions

TWO COPIES WITH

$$M_n^{\pm} = \sqrt{\left[\left(M^2 + \frac{n^2}{R^2}\right)^{1/2} \pm g A_5\right)^2 + M_D^2}$$

$$\chi_n^{\pm} = \frac{1}{2}\left[(\eta_n^c + i\xi_n^s) \mp e^{-i\varphi_n}(\eta_n^s + i\xi_n^c)\right]$$

$$\tan\varphi_n = \frac{n}{MR}$$

$$\lambda_n^{\pm} = \frac{1}{2}\left[(i\xi_n^c - \eta_n^s) \mp e^{-i\varphi_n}(\eta_n^c - i\xi_n^s)\right]$$

$$(M_0^+, M_0^-, M_1^+ \; M_1^- \; M_1^+ \; M_1^-, \ldots, M_n^+, M_n^-, M_n^+, M_n^-, \ldots)$$

THIS INFINITE TOWER OF MASSES WILL BE CHANGED BY BRANE-BULK COUPLINGS.

BRANE - BULK COUPLINGS

$$S_{B\text{-}B} = \frac{1}{(M_s)^A} \int d^4x \; [\text{STANDARD-MODEL INVARIANT}] \cdot [\text{BULK FIELDS}(x, y=0)]$$

"STRING SCALE" - TO MAKE DIMENSIONS COME OUT RIGHT.

[BULK FIELDS] IS A FERMION → [SM-INVARIANT] IS FERMION

e.g. $L_i H$ $\quad L = \begin{pmatrix} \nu \\ e \end{pmatrix}_i$ $\quad H =$ Higgs $\quad \Delta L = 1$

$\bar{u}\,\bar{d}\,\bar{d}$ $\quad \Delta B = -1$

etc...

EXAMPLE OF INTEREST IS $L_i H$ $\quad \Delta L = 1 \quad \Delta B = 0$

dimensionless

$$S_{B\text{-}B} = \frac{h_i}{\sqrt{M_s}} \int d^4x \; L_i H(x) \, \Psi(x, y=0) + \frac{g_i}{\sqrt{M_s}} \int d^4x \; L_i H \, \Psi^c(x, y=0)$$

(two terms break lepton number)

→

$$\nu_n^T \left[\kappa_1^i \left(\eta_0 + \sum_{n=1} \eta_n^c \right) + \kappa_2^i \left(\xi_0 + \sum_{n=1} \xi_n^c \right) \right]$$

$$\kappa_1^i = \frac{h_i \langle v \rangle}{\sqrt{M_s 2\pi R}} \qquad \kappa_2^i = \frac{g_i \langle v \rangle}{\sqrt{M_s 2\pi R}}$$

SMALL FOR $M_s R \gg 1$

$$v^T \equiv (\alpha_0, \beta_0, \alpha_1, \beta_1, \gamma_1, \delta_1, \dots \alpha_n, \beta_n, \gamma_n, \delta_n, \dots)$$

$$\alpha_0 = \tfrac{1}{2}(\mu_1 + i\mu_2) \qquad \alpha_n = \tfrac{1}{2}(\mu_1 + i e^{i\varphi_n}\mu_2) \qquad \beta_n = \tfrac{1}{2}(\mu_1 - i e^{i\varphi_n}\mu_2)$$

$$\beta_0 = \tfrac{1}{2}(\mu_1 - i\mu_2) \qquad \gamma_n = \tfrac{1}{2}(\mu_1 e^{i\varphi_n} + i\mu_2) \qquad \delta_n = \tfrac{1}{2}(\mu_1 e^{i\varphi_n} - i\mu_2)$$

$$\mathcal{M} = \begin{pmatrix} 0 & v^T \\ v & D \end{pmatrix} \qquad \begin{pmatrix} v \\ v_{KK} \end{pmatrix}$$

$$D = \mathrm{diag}(M_0^+, M_0^-, \dots, M_n^+, M_n^-, M_n^+, M_n^-, \dots)$$

Need to diagonalize $\mathcal{M}\mathcal{M}^\dagger$

find characteristic equation

$$\det(\mathcal{M}\mathcal{M}^\dagger - x) = (v^\dagger v - x)\left(1 + v^\dagger \frac{1}{D^2 - x} v\right) - v^\dagger \frac{D^2}{D - x} v$$

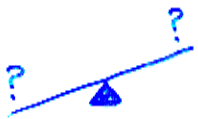
$$- v^\dagger \frac{1}{D^2 - x} v \, v^\dagger \frac{D^2}{D^2 - x} v + \left| v^T \frac{D}{D^2 - x} v \right|^2$$

STERILE NEUTRINO

Kaluza Klein Tower

SMALL MASS

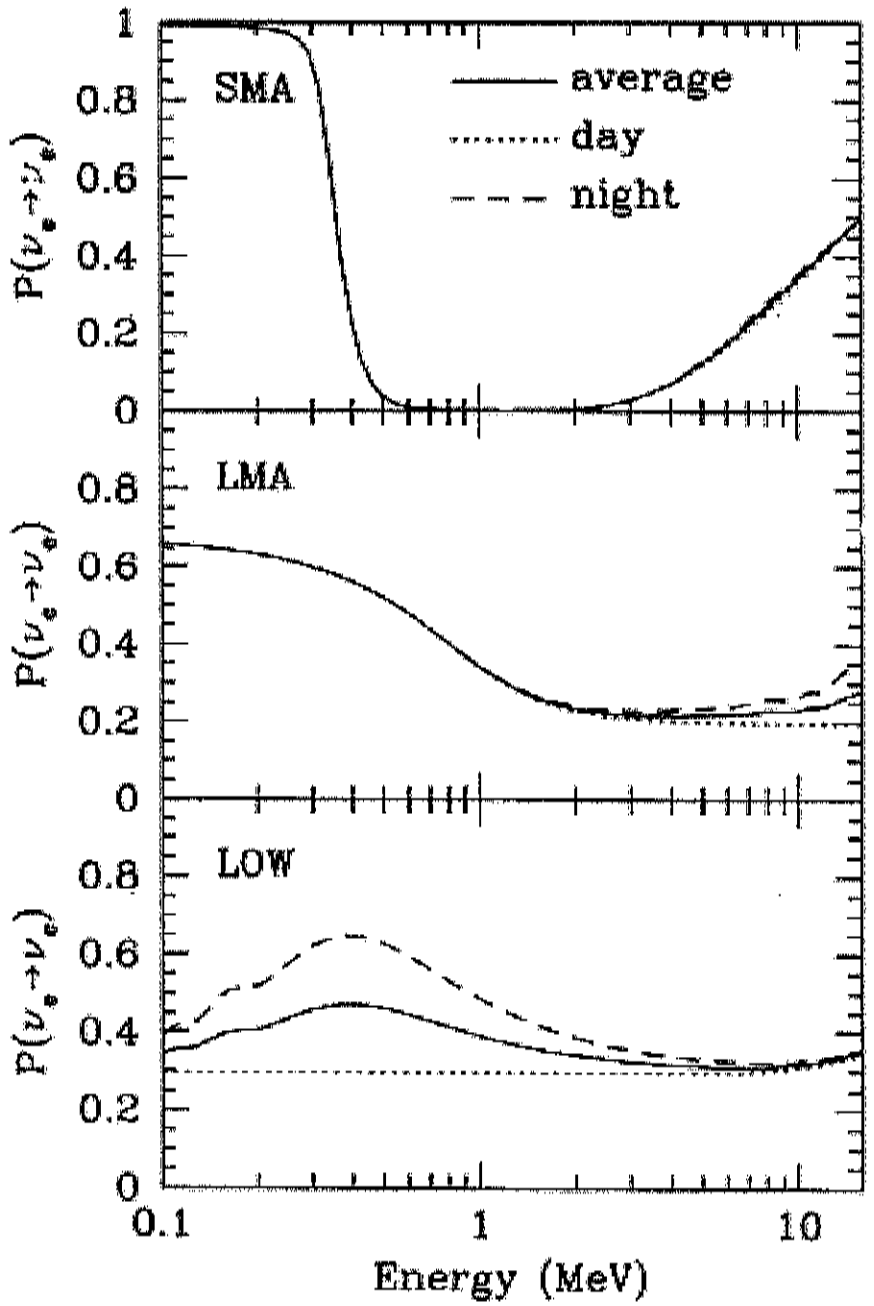
MIXING WITH ν_e, ν_μ, ν_τ.



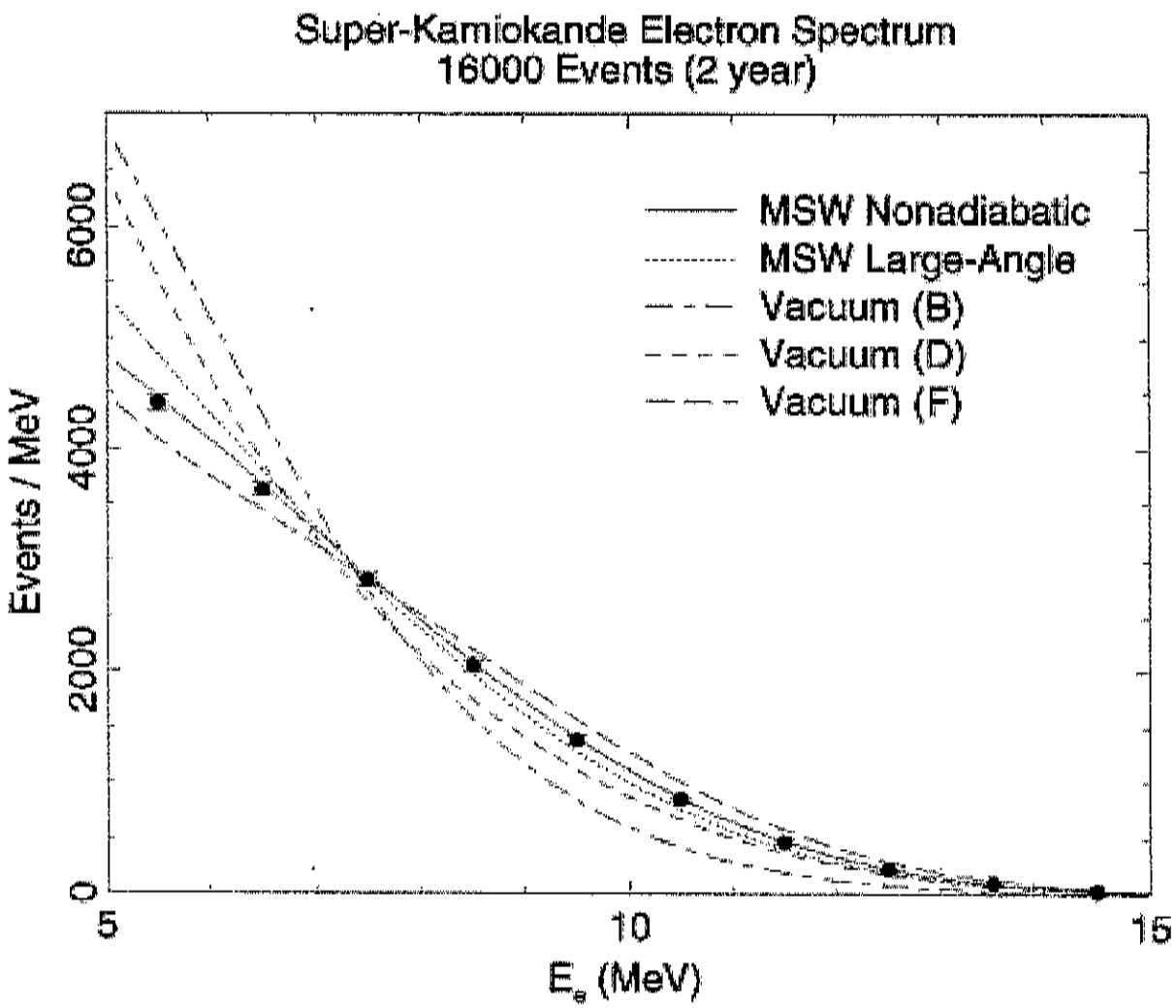
EACH EASY

BOTH HARD . $ $

Survival Probabilities



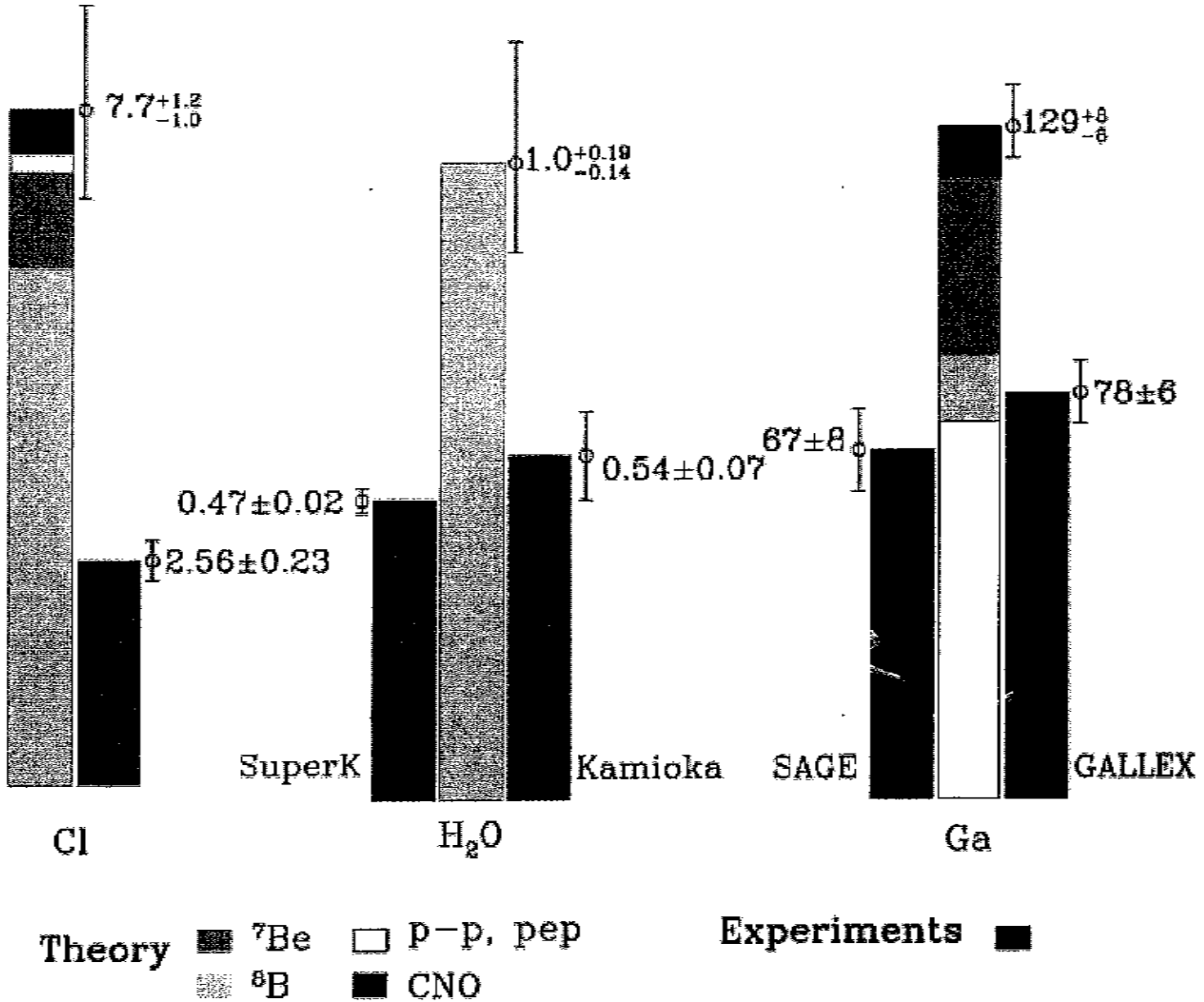
Bahcall


Super-Kamiokande Electron Spectrum
16000 Events (2 year)
MSW Nonadiabatic
MSW Large-Angle
Vacuum (B)
Vacuum (D)
Vacuum (F)
Events / MeV
0
2000
4000
6000
5
10
15
E_e (MeV)

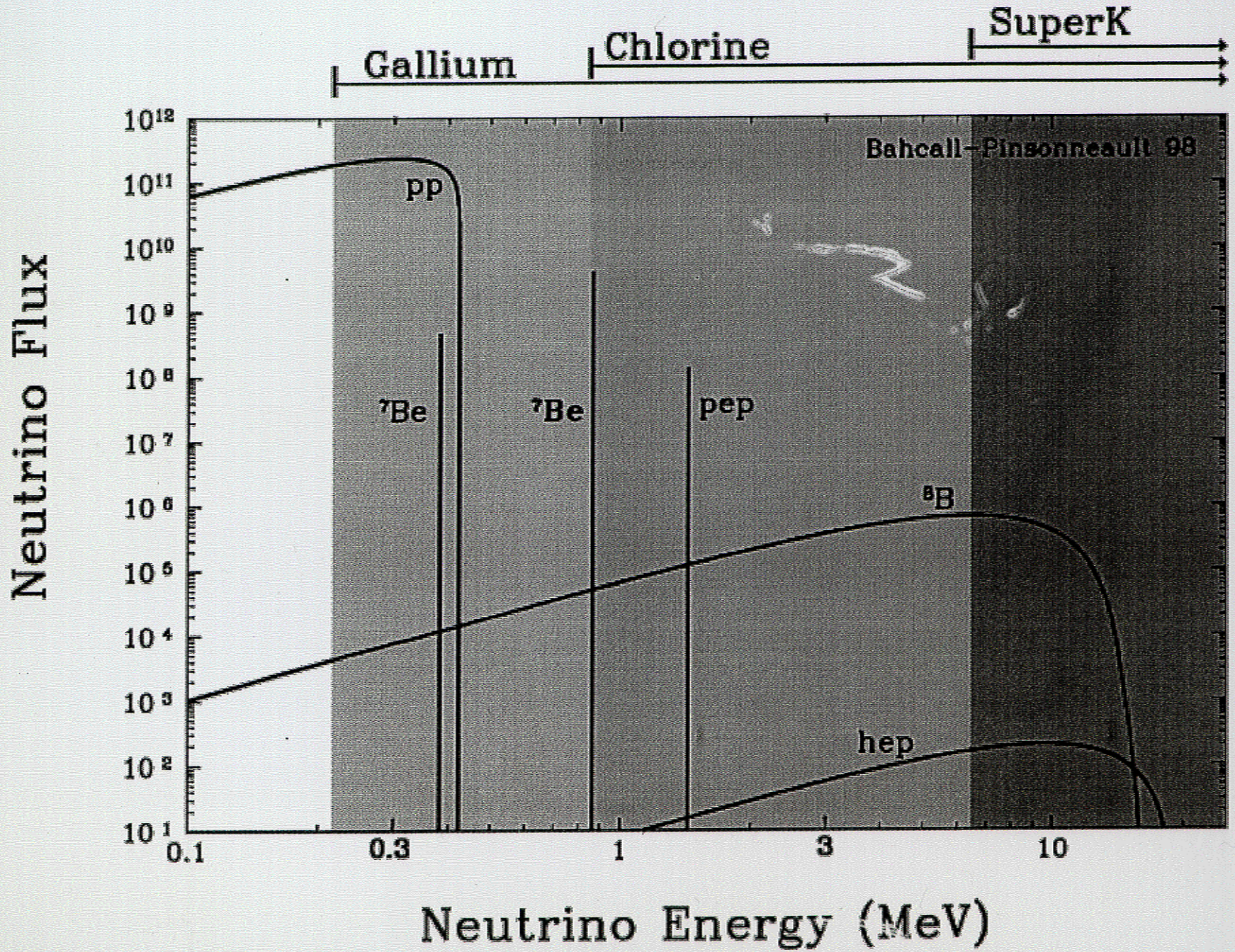
(Langacker)

Total Rates: Standard Model vs. Experiment

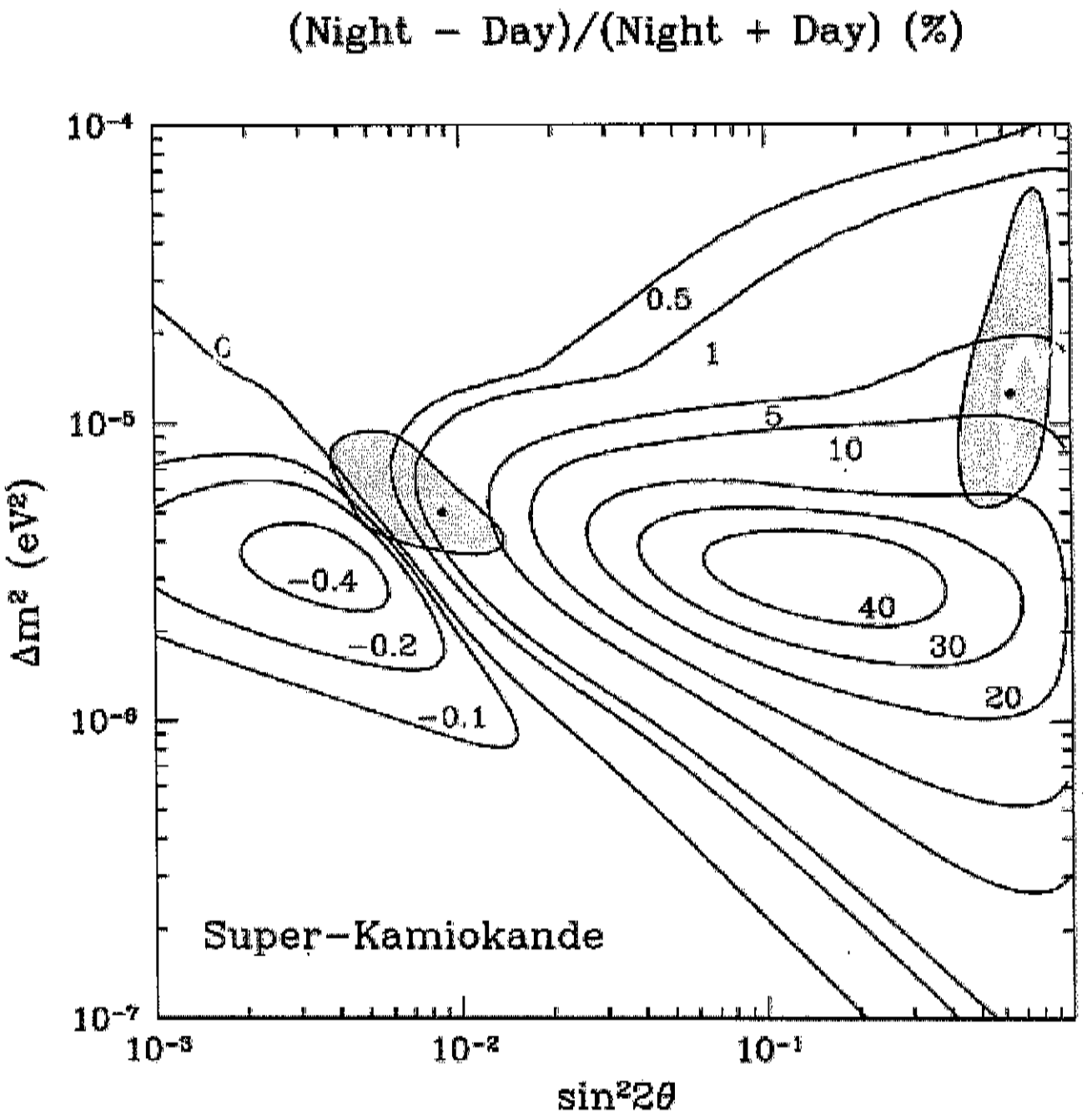
Bahcall–Pinsonneault 98



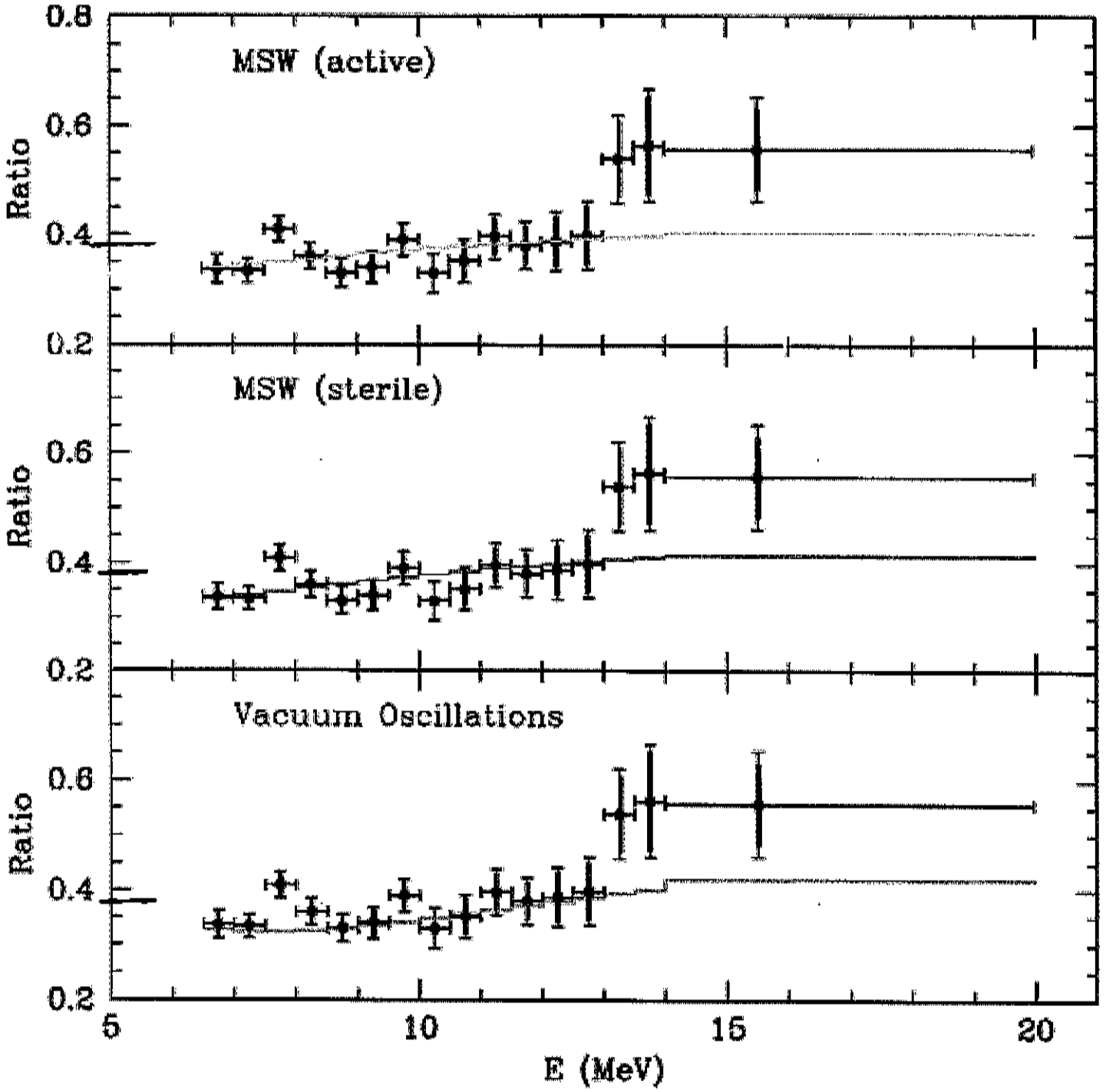
Bahcall



Solar neutrino energy spectrum


(Night − Day)/(Night + Day) (%)
0
0.5
1
5
10
20
30
40
−0.4
−0.2
−0.1
Super−Kamiokande
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-3}
10^{-2}
10^{-1}
Δm^2 (eV^2)
$\sin^2 2\theta$

(Bahcall)


MSW (active)
MSW (sterile)
Vacuum Oscillations
Ratio
0.8
0.6
0.4
0.2
5
10
15
20
E (MeV)

B. Acall

EIGENVALUES AND EIGENVECTORS

$\frac{v}{D} \ll 1$

$$\zeta = \frac{1}{\sqrt{1+v^\dagger D^2 v}} \left(1 , \frac{1}{D}\vec{v}^* \right) \qquad \mu_0 \simeq |v^T D^{-1} v| \qquad \mathcal{O}(v)\ \frac{v}{D}$$

$$\psi_0 = \left(\frac{|\alpha_0|}{M_0^+} , - \frac{\vec{\alpha}_0^*}{|\alpha_0|} \right) \qquad m_0 = \sqrt{M_0^{+2} + 2|\alpha_0|^2}$$

$$\chi_0 = \left(\frac{|\beta_0|}{M_0^-} , - \frac{\vec{\beta}_0^*}{|\beta_0|} \right) \qquad m_0' = \sqrt{M_0^{-2} + 2|\beta_0|^2}$$

- - - - - -

$$\psi_n^\pm = \left(\frac{\left(|\alpha_n|^2 + |\gamma_n|^2 \mp |\alpha_n^2 + \gamma_n^2|\right)^{1/2}}{\sqrt{2}\, M_n^+} , \frac{(\vec{\alpha}_n^* + \vec{\gamma}_n^*)|\alpha_n^2+\gamma_n^2| - (\vec{\alpha}_n + \vec{\gamma}_n)(\alpha_n^2+\gamma_n^2)}{\sqrt{2}\left(|\alpha_n|^2+|\gamma_n|^2 \mp |\alpha_n^2+\gamma_n^2|\right)^{1/2} |\alpha_n^2+\gamma_n^2|} \right)$$

$$m_n^\pm = \sqrt{M_n^{+2} + |\alpha_n|^2 + |\gamma_n|^2 \mp |\alpha_n^2 + \gamma_n^2|}$$

χ_n same as above, except $\alpha_n \to \beta_n$ $\gamma_n \to \delta_n$. $m \to m'$.

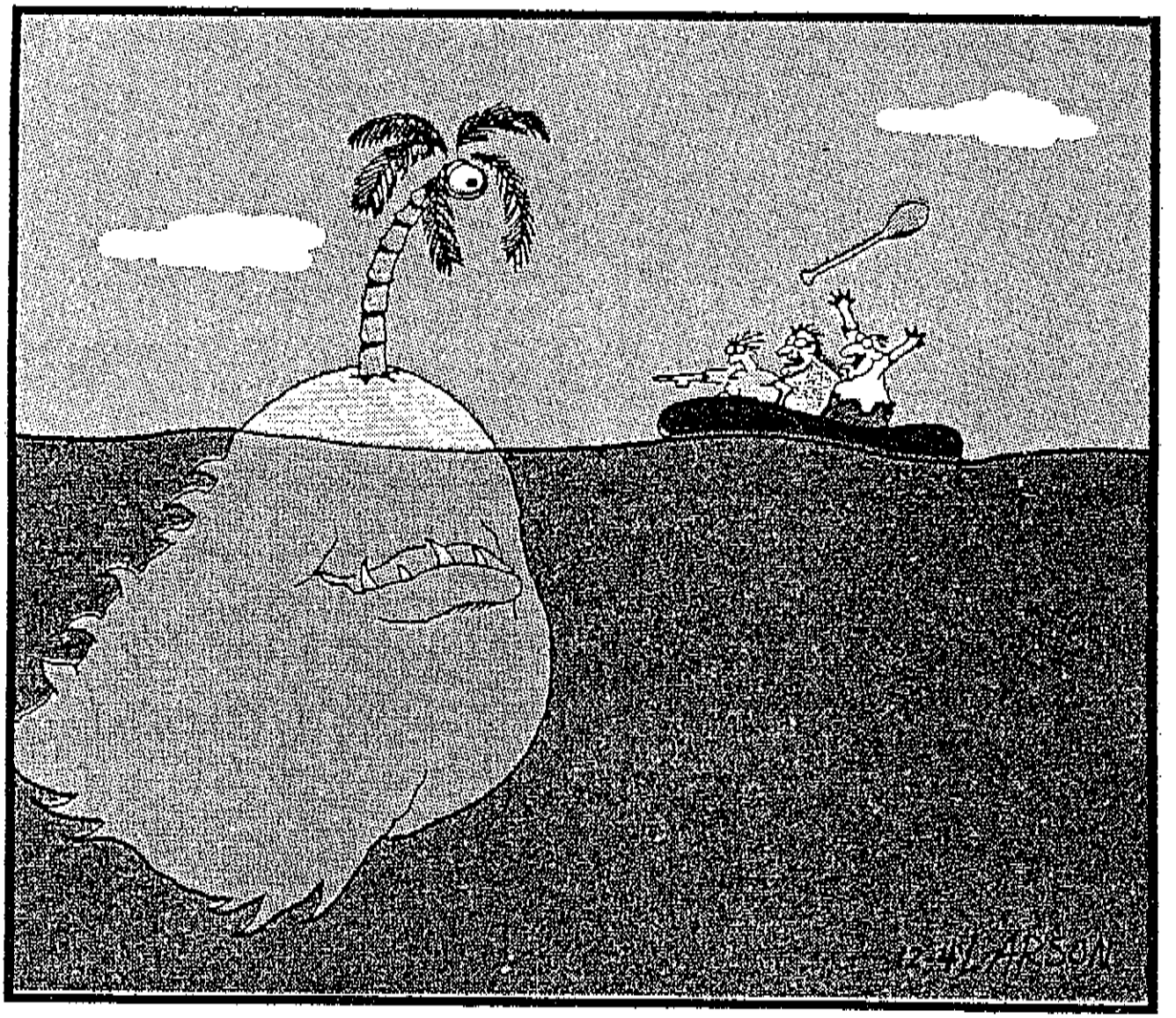
Generically

1 - light μ_0

2 $\left\{ \begin{array}{l} \text{heavy} \\ \text{heavy} \end{array} \right.$ m_0, m_0'

4 $\left\{ \text{heavier} \right.$ $m_n^\pm$, $m_n^{\pm\prime}$

By GARY LARSON



Where are the wild neutrinos?
Everywhere!

You may find it disconcerting...

Cosmic Gall

Neutrinos, they are very small,
They have no charge and have no mass
And do not interact at all
The earth is just a silly ball
To them, through which they simply pass,
Like dustmaids down a drafty hall
Or photons through a sheet of glass.
They snub the most exquisite gas,
Ignore the most substantial wall,
Cold shoulder steel and sounding brass,
Insult the stallion in his stall,
And scorning barriers of class,
Infiltrate you and me! Like tall
And painless guillotines, they fall
Down through our heads into the grass.
At night, they enter at Nepal
And pierce the lover and his lass
From underneath the bed - you call
It wonderful; I call it crass.

John Updike

THREE NEUTRINO LECTURES

P. Ramond

Fermilab & University of Florida

May-June 2000

- A BRIEF NEUTRINO HISTORY

- STANDARD MODEL NEUTRINOS

  mass & magnetic moment.

- HOW NEUTRINOS CAN GET MASSES

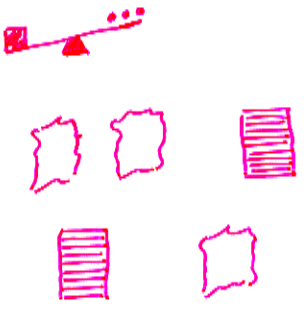
  - LEPTONIC HIGGS MODELS

  - STERILE NEUTRINO MODELS

    massive :

    massless :

    light :



- NEUTRINO OSCILLATIONS

  description and limits

  MSW effect

- EXPERIMENTAL OUTLOOK

DISCLAIMER

" I SHALL AVOID AS MUCH AS I CAN, THOSE QUESTIONS WHICH, THOUGH THE HAVE ELICITED THE SKILL OF MATHEMATICIANS, HAVE NOT ENLARGED OUR KNOWLEDGE OF SCIENCE "

J. Clerk Maxwell 1873.

References.

Books :

B. Kayser et al. "Physics of Massive Neutrinos" 1989

J. Bahcall Neutrino Astrophysics, Cambridge 1990

F. Boehm, P. Vogel "Physics of Massive Neutrinos", Cambridge 1987,

P. Ramond "Journeys Beyond the Standard Model", Perseus 1999.

Gribier, Spiro, Vignaud "La Lumière des neutrinos"

Articles :

L. M. Brown Physics Today 31, 23 (1978)

P. Radvanyi, "Neutrinos, Dark Matter and the Universe, J. Tran Thanh Van, Éditions Frontières.

NEUTRINO HISTORY

- neutrinos have been around since the earliest moments of our universe, but their existence was noticed by humans only in the XXth century.

- POSTULATED BY W. PAULI, DEC 1930

"Dear Radioactive Ladies and Gentlemen..."

"desperate remedy..." to explain

- exclusion principle for the nuclear wavefunction (N14, Li6)
- continuous β-spectrum

"I don't feel secure enough to publish anything about this idea..."

"Only those who wager can win..."

"However, I cannot appear in Tübingen since I am indispensable here on account of a ball..."

- written 5 days after divorcing his first wife, actress Kate Deppner

- a year or so later, Pauli started seeing Carl Jung

- THERE IS A SPIN 1/2 NEUTRAL PARTICLE THAT LIVES INSIDE THE NUCLEUS

  THIS TAKES CARE OF THE SYMMETRY OF THE NUCLEAR WAVEFUNCTION

- IT IS EMITTED IN β-RADIOACTIVITY

  THIS TAKES CARE OF THE CONTINUOUS ELECTRON SPECTRUM

CALL IT NEUTRON (lives inside nucleus by magnetic interactions ?)

MASS $\lesssim$ .01 PROTON MASS

1932

- CHADWICK DISCOVERS HEAVY NEUTRAL NUCLEON : NEUTRON!

1933

- FERMI : PAULI'S PARTICLE IS NOT BOUND INSIDE NUCLEON BUT IS EMITTED IN β-PROCESS

  IT IS LIGHT : NEUTRINO is born!

- DETECTION OF $\bar{\nu}_e$ Cowan, Reines 1956

$$\bar{\nu}_e + p \rightarrow n + e^+$$

e^+ → annihilates back to back γ's .5 MeV detected by pmt's

n → slowed by Cadmium $\gamma\gamma\gamma$ produced later

look for coincidences.

- 1962 DETECTION OF $\bar{\nu}_\mu$

- 1968 DETECTION OF SOLAR ν_e R. Davis

$$\nu_e + {}^{37}Cl \rightarrow {}^{37}Ar + e^-$$

inert, radioactive (Pontecorvo, 1945)

find half as many as predicted by Solar model.

- 1987 SUPERNOVA NEUTRINO BURST DETECTED IN PROTON DECAY MONITORING DEVICES ! IMB, KAMIOKA,

- 1990-95 SOLAR ν_e SPECTRUM EXPLORED SAGE, GALLEX

  DEFICIT CONFIRMED !

- WEAKLY INTERACTING NEUTRINOS $\lesssim 3$ SLAC, CERN.

- 1998 SUPERK EFFECT OF OSCILLATIONS OBSERVED IN NEUTRINOS PRODUCED BY COSMIC RAYS.

$$\nu_e + e^- \to \nu_e + e^-$$

$$\nu_{\mu,e} + N \to \mu, e + X$$

$\nu_\mu/\nu_e \sim .6$ OF THEORY

$\# \nu_\mu$ IS ZENITH-ANGLE DEPENDENT !

EVIDENCE FOR $\nu_\mu \to ?$ OSCILLATIONS !

NEUTRINO MASSES ARE A REALITY !

STANDARD MODEL NEUTRINOS

$$L_i = \begin{pmatrix} \nu_i \\ \ell_i \end{pmatrix} \qquad i = 1, 2, 3 \qquad \text{FAMILY INDEX}$$

$$\bar{\ell}_i$$

QUANTUM NUMBERS:

- LEFT-HANDED : WEYL FERMION $\sim (\underset{\sim}{2}, \underset{\sim}{1})$ $SU(2) \times SU(2)$ LORENTZ-GROUP
- WEAK DOUBLET $(I_W = \frac{1}{2}) \sim \underset{\sim}{2}$ ν_i $I_{3W} = +\frac{1}{2}$
- COLOR SINGLETS
- THREE LEPTON NUMBERS $L = L_e + L_\mu + L_\tau$ anomalous, B-L conserved.
  $L_e - L_\mu$ ✓
  $L_\mu - L_\tau$ ✓

NO RIGHT-HANDED NEUTRINOS

NEUTRINOS ARE LEFT-HANDED

ANTINEUTRINOS ARE RIGHT-HANDED

STUDY LEPTON BILINEARS FOR:

- POSSIBLE MASS TERMS
- POSSIBLE YUKAWA COUPLINGS

REVIEW LORENTZ GROUP :

$\vec{J}$: ROTATIONS

$\vec{K}$: BOOSTS

$\vec{J} + i\vec{K}$: $SU(2)$

$\vec{J} - i\vec{K}$: $SU(2)$

label particles (fields) under $SU(2) \times SU(2)$

$\not{P}$	$(\underset{\sim}{2}, \underset{\sim}{1})$	: WEYL FERMION	e.g. left-handed neutrino
$\not{P}$	$(\underset{\sim}{1}, \underset{\sim}{2})$	: $\overline{\text{WEYL}}$ FERMION	right-handed antineutrino
P	$(\underset{\sim}{2}, \underset{\sim}{2})$	: VECTOR	photon, gauge boson.
$\not{P}$	$(\underset{\sim}{3}, \underset{\sim}{1})$	: TENSOR	magnetic moment, field strengths, ...
$\not{P}$	$(\underset{\sim}{1}, \underset{\sim}{3})$	: TENSOR	

(description conserves CP in all cases)

FERMION BILINEAR :

$$(\underset{\sim}{2}, 1) \otimes (\underset{\sim}{2}, \underset{\sim}{1}) = (1, 1)_A \oplus (\underset{\sim}{3}, \underset{\sim}{1})_S$$

LORENTZ-INVARIANT MASS LIKE

LORENTZ TENSOR MAGNETIC MOMENT

$f_i \sim (2,1)$

$f_i f_j$ SYMMETRIC IN $i \leftrightarrow j$

$\boxed{\text{MAJORANA MASS}}$

$\Delta L = 2$ L = overall lepton number

example: TWO SPECIES f_1, f_2

MAJORANA MASS : $(f_1 \quad f_2)\begin{pmatrix} A & C \\ C & B \end{pmatrix}\begin{pmatrix} f_1 \\ f_2 \end{pmatrix}$

$A, B, C \neq 0$ VIOLATE L_1 AND L_2

$A, B \neq 0$ $C = 0$ VIOLATE L_1 AND L_2

- $A = B = 0$ $C \neq 0$ VIOLATE $L_1 + L_2$
  PRESERVES $L_1 - L_2$

$f_1 = e_L$ $f_2 = \bar{e}_L$ $L_1 - L_2$ = electric charge.

DIRAC MASS $(e_L \quad \bar{e}_L)\begin{pmatrix} 0 & m \\ m & 0 \end{pmatrix}\begin{pmatrix} e_L \\ \bar{e}_L \end{pmatrix}$

$$\begin{pmatrix} \text{MAJ} & \text{DIRAC} \\ \text{DIRAC} & \text{MAJ} \end{pmatrix}$$

C-conserving.

NEUTRINO MAJORANA MASS

$\nu_i \nu_j$: $I_{3W} = \frac{1}{2} + \frac{1}{2} = 1$: member weak isotriplet

SYMMETRIC IN FAMILY INDICES

VIOLATE TOTAL LEPTON NUMBER $\Delta L = 2$

NEUTRINO MAJORANA MASS REQUIRES

LORENTZ - SCALAR
WEAK ISOTRIPLET

SM CONTAINS ONLY HIGGS THAT ARE WEAK DOUBLETS

TREE - LEVEL MAJORANA MASS

IS NATURALLY ZERO IN STANDARD MODEL

RADIATIVE CORRECTIONS : MAKE ISOTRIPLET OUT OF TWO HIGGS DBLTS.

$L_i \vec{\tau} L_j$		$H \vec{\tau} H$		
-1	-1	$+1$	$+1$	HYPERCHARGE WORKS
1	1	0	0	LEPTON NUMBER DOES NOT WORK !

NO NEUTRINO MASSES IN GWS STANDARD MODEL !!

HENCE :

LEPTON NUMBER CONSERVATION → MASSLESS NEUTRINOS

TRUE STATEMENT IN STANDARD MODEL

TO ENDOW NEUTRINOS WITH MASS, IT IS NECESSARY TO ADD NEW DEGREES OF FREEDOM TO THE STANDARD MODEL !

BASIC CHOICES :

- ADD RIGHT-HANDED (STERILE : ELECTROWEAK SINGLET) NEUTRINOS FOR EACH SPECIES : THE FERMION WAY
- ADD HIGGS WITH LEPTON NUMBER : THE SCALAR WAY

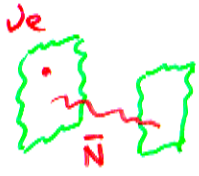
  LEPTONIC HIGGS

NEED TO EXPLAIN WHY NEUTRINOS ARE SO LIGHT!

THEORETICAL ROAD MAP

for

SMALL NEUTRINO MASSES



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CHANGES IN SPACE-TIME GEOMETRY

Branes
New dimensions
Gravity at low energy

WINDOW INTO PHYSICS AT VERY SHORT DISTANCES

GUT, SO(10)

Proton decay

⋮

LEPTONIC HIGGS

LEPTONIC HIGGS MODELS

STRATEGY : • ADD EXTRA HIGGS

- ENDOW HIGGS WITH LEPTON NUMBERS BY YUKAWA COUPLINGS
- BREAK LEPTON NUMBER WITH HIGGS INTERACTIONS.

POSSIBLE YUKAWA COUPLINGS :

$L_{(i}\vec{\tau} L_{j)} \cdot \vec{T}$ $\qquad$ $\vec{T}$: isotriplet HIGGS with $Y=2$

lepton number -2

family symmetric

$L_{[i} 1 L_{j]}$ $\quad S^{+}$ $\qquad$ S^{+} : isosinglet HIGGS with $Y=2$ $(Q=+1)$

lepton number -2

family antisymmetric

$\vec{\ell}_{(i} \vec{\ell}_{j)}$ $\quad S^{--}$ $\qquad$ S^{--} : isosinglet HIGGS with $Y=-4$ $(Q=-2)$

lepton number $+2$

family symmetric

Cubic interactions : all involve mass parameter.

- $H\vec{\tau}H \cdot \vec{T}$
- $S^{--} S^{+} S^{+}$ $\qquad$ ALL BREAK L : $\Delta L = 2$
- $\vec{T}\,\vec{T}\,S^{--}$

Quartic Interactions:

$$H^\dagger \vec{\tau} H \cdot \vec{T}^* \times \vec{T} \quad , \quad H^\dagger \vec{\tau} H \cdot \vec{T}^* S^+ \quad , \quad H \vec{\tau} H \cdot \vec{T}^* S^{--}$$

$$+ \cdots$$

SOME MODELS: ALL NEED CUBIC COUPLING TO BREAK L

ALL HAVE TUNABLE MASS PARAMETER

- S^+ ONLY: neutrino Majorana mass: antisymmetric in family generated at one loop

  (Zee model)

- T ONLY WITH CUBIC TERM: explicit breaking of L, tunable parameter

  symmetric matrix

  mass generated at loop order

  WITHOUT CUBIC TERM: could have spontaneous breaking of lepton number: Majoron model

  (this triplet Majoron model ruled out by Z-width)

etc...

MODELS WITH RIGHT-HANDED NEUTRINOS

ADD FOR EACH FAMILY (or more) $\bar{N}_a$ $a = 1, 2, 3, \ldots ?$

$\bar{N}_a$ are Weyl fermions that are STANDARD MODEL SINGLETS

(sterile neutrinos)

COUPLING TO THE STANDARD MODEL

$$L_i H_u \bar{N}_a$$

- endows $\bar{N}_a$ with lepton numbers

simplest: $L_e H \bar{N}_e + L_\mu H \bar{N}_\mu + L_\tau H \bar{N}_\tau$

WHAT ARE THEIR MASSES ?

HOW MANY ARE THEY ?

$$\underbrace{\bar{N}_1 \quad \bar{N}_2 \quad \bar{N}_3}_{\text{Dirac partners of the normal s.m. neutrinos}} \qquad \underbrace{\bar{N}_4, \ldots \bar{N}_8}_{\text{other unaccompanied sterile neutrinos.}}$$

or they could couple as Dirac pairs, or not

TWO NATURAL CHOICES :

- $\bar{N}$'s ARE MASSLESS

  natural in the sense that it preserves lepton number

- $\bar{N}$'s ARE MASSIVE

  - if $\bar{N}$'s have no quantum numbers, how can they be light ?

    electrons are light because their mass depends on the scale of electroweak breaking, ...

  - if they have quantum numbers, what are they ?

- e.g. 1 $\bar{N}$ / family is hallmark of GUT SO(10)

  SO(10) symmetry broken $\sim 10^{16}$ GeV $\rightarrow$ expect mass $\sim 10^{16}$ GeV.

- can $\bar{N}$'s have quantum numbers associated with space-time geometry ?

(I) MINIMAL EXTENSION : THREE DIRAC PARTNERS

$\bar{N}_1, \bar{N}_2, \bar{N}_3$

NO MAJORANA MASSES FOR $\bar{N}_{1,2,3}$

ADD TO THE STANDARD MODEL THE YUKAWA INTERACTIONS :

$$\sum_{i=1}^{3} \left(y_{ei} L_e H_u \bar{N}_i + y_{\mu i} L_\mu H_u \bar{N}_i + y_{\tau i} L_\tau H_u \bar{N}_i \right)$$

IN ELECTROWEAK VACUUM : $\langle H_u \rangle = v_u$

$$\longrightarrow \quad (\nu_e \quad \nu_\mu \quad \nu_\tau) \underbrace{\begin{pmatrix} m_{e1} & m_{e2} & m_{e3} \\ m_{\mu 1} & m_{\mu 2} & m_{\mu 3} \\ m_{\tau 1} & m_{\tau 2} & m_{\tau 3} \end{pmatrix}}_{M} \begin{pmatrix} \bar{N}_1 \\ \bar{N}_2 \\ \bar{N}_3 \end{pmatrix}$$

$m_{ei} = y_{ei} v$ etc... $M \sim \Delta I_W = \frac{1}{2}$

$$(\nu_e \quad \nu_\mu \quad \nu_\tau) \; U_\nu^\dagger \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} V_{\bar{N}} \begin{pmatrix} \bar{N}_1 \\ \bar{N}_2 \\ \bar{N}_3 \end{pmatrix}$$

unitary matrices

THREE DIRAC MASSES : m_1, m_2, m_3

$$: \quad m_1 \nu_1 \bar{\nu}_1 + m_2 \nu_2 \bar{\nu}_2 + m_3 \nu_3 \bar{\nu}_3$$

MIXING ANGLES :

CHARGED LEPTON MASS MATRIX

$$\begin{pmatrix} e & \mu & \tau \end{pmatrix} \begin{pmatrix} m_{ee} & m_{e\mu} & m_{e\tau} \\ m_{\mu e} & m_{\mu\mu} & m_{\mu\tau} \\ m_{\tau e} & m_{\tau\mu} & m_{\tau\tau} \end{pmatrix} \begin{pmatrix} \bar{e} \\ \bar{\mu} \\ \bar{\tau} \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} e & \mu & \tau \end{pmatrix} U_e^{\dagger}}_{\text{mass eigenstates}} \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} \underbrace{V_{\bar{e}} \begin{pmatrix} \bar{e} \\ \bar{\mu} \\ \tau \end{pmatrix}}_{\text{mass eigenstates.}}$$

unitary matrix

$$\longrightarrow \quad m_e e\bar{e} + m_\mu \mu\bar{\mu} + m_\tau \tau\bar{\tau}$$

WEAK CHARGED CURRENT

$$W_\mu \cdot \left(j_\mu^{\text{quark}} + j_\mu^{\text{leptons}} \right)$$

$$j_\mu^{\text{leptons}} = \begin{pmatrix} \nu_e^{\dagger} & \nu_\mu^{\dagger} & \nu_\tau^{\dagger} \end{pmatrix} \sigma_\mu \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} :$$

weak eigenstates

$$j_\mu^{\text{leptons}} = (\nu_1^\dagger \quad \nu_2^\dagger \quad \nu_3^\dagger)\,\gamma_\mu\, \underbrace{U_\nu U_e^\dagger}\begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}$$

LEPTON MIXING MATRIX

like CKM for lepton sector: MNS matrix

(Maki, Nakagawa, Sakata 1962)

$$U_{MNS} \equiv U_\nu U_e^\dagger$$

THIS IS WHAT YOU MEASURE !

3 MIXING ANGLES

1 $\not{CP}$ PHASE

just like CKM

COMPARE WITH "DATA"

DIRECT LIMITS ON MASS :

	" ν_e "	MASS: $m_1 \lesssim$ eV	TREMENDOUSLY
kinematics :	" ν_μ "	MASS: $m_2 \lesssim$ 160 KeV	SMALL
	" ν_τ "	MASS : $m_2 \lesssim$ 18 MeV	$y_{ei} <<<<1$

MIXING ANGLES :

$\nu_\mu - \nu_\tau$	MAXIMALLY MIXED	SuperK
$\nu_e - \nu_\mu$	UNKOWN	SOLAR
$\nu_e - \nu_\tau$	UNKOWN	SOLAR
$\not{CP}$	$(\text{UNKOWN})^2$	

$$\left(\pi^+ \to \mu^+ + \nu_\mu \;\Longrightarrow\; \pi^+ \to \mu^+ + \frac{1}{\sqrt{2}}(\nu_2 + \nu_3) \right)$$

NOTHING WRONG WITH THIS SCENARIO, EXCEPT THAT m_1, m_2, m_3

AND THEREFORE THE YUKAWA COUPLINGS MUST BE VERY SMALL

WHY ?

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II SECOND TYPE OF MODEL :

THREE "DIRAC" PARTNERS $\bar{N}_1, \bar{N}_2, \bar{N}_3$

WITH ARBITRARY MAJORANA MASSES :

$$(\bar{N}_1 \;\; \bar{N}_2 \;\; \bar{N}_3)\; M_0 \begin{pmatrix} \bar{N}_1 \\ \bar{N}_2 \\ \bar{N}_3 \end{pmatrix}$$

M_0 HAS NO ELECTROWEAK QUANTUM NUMBER : $M_0 \sim \Delta I_W = 0$

WHAT IS ITS ORDER OF MAGNITUDE ?

ITS STRUCTURE ?

M_0 SCALE IS THAT OF THE CUT-OFF OF THE STANDARD MODEL

whatever that is

- cut-off is 10 TeV : $M_0 \sim$ 10 TeV — Technicolor, Extra dimensions
- 10^7 GeV — 10^7 GeV — Extra dimensions
- 10^{16} GeV — $M_0 \sim 10^{16}$ GeV — SO(10), extra dim, string
- $M_{Planck} \simeq 10^{18}$ GeV — $M_0 \sim 10^{18}$ GeV — Gravity, String, etc...

WHICH IS IT ? TRY $M_0 \gg$ EW SCALE .

DIAGONALIZE NEUTRINO MASS MATRIX

$$\left(\nu_e \quad \nu_\mu \quad \nu_\tau \quad \bar{N}_1 \quad \bar{N}_2 \quad \bar{N}_3 \right) \left(\begin{array}{c|c} 0 & M \\ \hline M & M_0 \end{array}\right) \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \bar{N}_1 \\ \bar{N}_2 \\ \bar{N}_3 \end{pmatrix}$$

EW STRUCTURE :

$$\left(\begin{array}{c|c} \Delta I_W = 1 & \Delta I_W = \frac{1}{2} \\ \hline \Delta I_W = \frac{1}{2} & \Delta I_W = 0 \end{array}\right)$$

$$M \ll M_0 \qquad \frac{\Delta I_W = 1/2}{\Delta I_W = 0} \equiv \varepsilon \ll 1$$

SOLVE IN PERTUBATION THEORY :

3 "LIGHT" EIGENVALUES $\sim$ $M^T \dfrac{1}{M_0} M \equiv U_\nu^T \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix} U_\nu$

3 "HEAVIES" $\sim$ $M_0 = V_0^T \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{pmatrix} V_0$

LIGHT MASSES $\sim$ (NORMAL MASS) $\left(\dfrac{\Delta I_W = \frac{1}{2}}{\Delta I_W = 0} \right)$

AS BEFORE

$$\mathcal{U}_{MNS} = \mathcal{U}_\nu \mathcal{U}_e^\dagger$$

from charged lepton matrix

from ~~[illegible]~~ matrix

IMPORTANT DIFFERENCE : • ν MASS MATRIX IS MAJORANA

CANNOT ABSORB PHASES IN $\nu\,\nu$

• $\mathcal{U}_\nu$ IS UNITARY TO ORDER ε^2.

$$\mathcal{U}_{MNS} = (\text{PHASE MATRIX})\begin{pmatrix}\text{3 angles +}\\ \text{1 phase}\end{pmatrix}(\text{PHASE MATRIX})$$

CANNOT BE ABSORBED IN ν'S BECAUSE OF MAJORANA MASS

absorbed in right-handed charged leptons

$\downarrow$

TWO EXTRA $\not{CP}$ PHASES

CAN ONLY APPEAR IN

$\Delta L = 2$ PROCESSES

GET : 3 ROTATION ANGLES

ONE FLAVOR-CHANGING $\not{CP}$ PHASE

TWO PARTICLE-ANTIPARTICLE $\not{CP}$ PHASES.

NEUTRINO MAGNETIC MOMENTS

IF NEUTRINOS HAVE MASS, THEY SHOULD HAVE MAGNETIC MOMENTS

electron example:

electron is charged so the Dirac equation is

$$\gamma^\mu (\partial_\mu - ie A_\mu) \psi_e = m_e \psi_e$$

gives rise to magnetic moment $\mu = \frac{\hbar e}{2 m_e c}$

Landé g-factor = 2 + radiation corrections.

neutrino case:

neutrino is electrically neutral, so its Dirac equation is

$$\gamma^\mu \partial_\mu \psi_\nu = m_\nu \psi_\nu$$

no "classical" magnetic moment.

its Landé g-factor is 0 + radiation corrections.

How to estimate radiation corrections?

- WITH ONE MAJORANA NEUTRINO, IT IS NOT POSSIBLE TO MAKE A MAGNETIC MOMENT:

Weyl neutrino ~ (2,1) $\qquad (2,1)\cdot(2,1) = (1,1)_A + \cancel{(3,1)_S}$

forbidden by spin-statistics.

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GRAND UNIFIED APPROACH TO ν- MASSES

naturally appears with $SO(10)$, E_6, ...

STANDARD MODEL : $SU(2) \times SU(3)^c \times U(1)$

3 families

$$\begin{pmatrix} \nu \\ e \end{pmatrix} \quad \bar{d} \qquad \bar{u} \quad \begin{pmatrix} u \\ d \end{pmatrix} \quad \bar{e} \qquad \bar{N}$$

$$\underbrace{(2,1^c)_{-1} \quad (1,\bar{3}^c)_{2/3}}_{\bar{5}} \qquad \underbrace{(1,\bar{3}^c)_{-4/3} \quad (2,3^c)_{1/3} \quad (1,1^c)_2}_{10} \qquad \underbrace{(1,1^c)_0}_{1}$$

SU(5): $\bar{5}$ $\quad$ 10 $\quad$ 1

SO(10) $\quad$ 16

DEFINITE IMPLICATIONS FOR MASSES AND MIXING ANGLES.

RELATES QUARK TO LEPTON MIXINGS

$\bar{5} \times 10$ contains charge $-1/3$ and -1 masses

10×10 contains charge $2/3$ masses

SU(5) MASS RELATIONS : (Georgi-Jarlskog)

$$(\bar{5}_1 \;\; \bar{5}_2 \;\; \bar{5}_3)\begin{pmatrix} 0 & 5_H & 0 \\ 5_H & 45_H & 0 \\ 0 & 0 & 5_H \end{pmatrix}\begin{pmatrix} 10_1 \\ 10_2 \\ 10_3 \end{pmatrix}$$

Higgs particle.

C.G. coefficient.

$$\begin{pmatrix} 0 & R & 0 \\ R & S & 0 \\ 0 & 0 & T \end{pmatrix} \qquad \begin{pmatrix} 0 & R & 0 \\ R & -3S & 0 \\ 0 & 0 & T \end{pmatrix}$$

CHARGE $-1/3$ CHARGE -1

$$m_b = m_\tau$$

$$\left.\begin{aligned} m_d m_s &= m_e m_\mu \\ m_d - m_s &= 3(m_e - m_\mu) \end{aligned}\right\} \quad \frac{m_e}{m_\mu} = \frac{1}{9}\frac{m_d}{m_s}$$

also implies that $U_e \sim \begin{pmatrix} c & s & 0 \\ -s & c & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $\tan\theta_{e\mu} \sim \sqrt{\frac{m_e}{m_\mu}}$

SO(10) GENERALIZATIONS

L MASSES RESIDE IN

$$16 \times 16 = \underbrace{10}_{5+\bar{5}} + 120 + \underbrace{126}_{1+\bar{5}+10+\overline{15}+45+\overline{50}}$$

SO(5)

MAJORANA $\Delta I_W = 0$ MATRIX M_0

SO(10) CAN RELATE

- CHARGE $2/3$ TO CHARGE 0 DIRAC MASS MATRICES
- CHARGE $-1/3$ TO CHARGE -1 DIRAC MASS MATRICES
- MAJORANA $\Delta I_W = 0$ MASS MATRIX TO THE OTHERS

NEED 126 HIGGS REPRENTATION !!

confronting the main problem of GUT theories : UGLY HIGGS !

- early approach (1982 ; Harvey, Reiss, P.R.)

" DAMN THE HIGGS TORPEDOES "

three results (with $M_{top} \sim 20$ GeV!)

- long-lived b-quark $V_{cb} = \sqrt{\frac{m_c}{m_t}}$ (not quite right) $\sim$
- maximal $\nu_\mu - \nu_\tau$ mixing (decreases with m_t >) $\sim$
- $\nu_e - \nu_\mu$ mixing $\sim \sqrt{\frac{m_e}{m_\mu}}$ (too large for SMA) $\sim$

SO(10) WANTS TO WORK

EXAMPLE OF A MORE RECENT APPROACH (Babu, Barr)

$16_i \cdot 16_j \; \overline{126}_H$ comes from $(16_i \, \overline{16})_{45} (16_j \, \overline{16})_{45}$

EXPLAIN THE NEUTRINO SPECTRUM BY C.G. COEFFICIENTS:

$$\begin{pmatrix} 0 & C & 0 \\ C & B & 0 \\ 0 & 0 & A \end{pmatrix} \qquad \begin{pmatrix} 0 & \frac{3}{8}C & 0 \\ \frac{3}{8}C & -3B & 0 \\ 0 & 0 & \frac{3}{8}A \end{pmatrix} \qquad \begin{pmatrix} 0 & C & 0 \\ C & 0 & 0 \\ 0 & 0 & A \end{pmatrix} M$$

charge 2/3 matrix

charge 0 DIRAC

$\Delta I_W = 1/2$

M

$\Delta I_W = 0$ MAJORANA

MASS

M_0

$$M^T \frac{1}{M_0} M$$

find

$$\frac{m_{\nu_\mu}}{m_{\nu_\tau}} = 16 \frac{m_c}{m_t} \qquad \frac{m_{\nu_e}}{m_{\nu_\mu}} = \frac{1}{256} \frac{m_u}{m_c}$$

$\nu_e - \nu_\mu$ mixing <u>small</u> consistent with SMA

no mixing between ν_μ and ν_τ

need to rely on charge -1 matrix!

FROGGATT-NIELSEN APPROACH TO SMALL YUKAWA COUPLINGS :

EXTRA U(1) PHASE SYMMETRIES IN STANDARD MODEL

ELEMENTARY PARTICLES HAVE FLAVOR CHARGES :

example :

large top quark mass $\rightarrow$ $Q_3 \bar{u}_3 H_u \Rightarrow t\bar{t}\, m_{top}$

has zero FN charge(s)

"small" bottom quark mass $\rightarrow$ $m_b b\bar{b}$ comes from $Q_3 \bar{d}_3 H_d$

has non-zero FN charge.

Suppose the charge of $Q_3 \bar{d}_3 H_d$ is N

then invent an electroweak singlet field θ with charge -1

$$Q_3 \bar{d}_3 H_d \left(\frac{\theta}{M}\right)^N \quad \text{has zero charge}$$

mass to get right dimensions

if $\frac{\theta}{M} \sim \lambda$ (Cabibbo angle, say), then

$$\frac{m_b}{m_t} \sim \lambda^N$$

EXPONENTS TRACK CHARGES OF OPERATORS !

FERMION MASS HIERARCHIES

u	d	e
c	s	μ
t	b	τ

INTER FAMILY

INTRA FAMILY

INTRA-FAMILY

$$\frac{b}{t} \sim \lambda^3 \qquad \frac{b}{\tau} \sim 1 \equiv \lambda^0$$

AT 10^{16} GeV

WITH SUSY.

λ = Cabibbo angle $\sim .22$

(pretty bad expansion parameters)

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INTER-FAMILY

ORDERS OF MAGNITUDE

at $M_{GUT} \sim 10^{16}$ GeV

$$\frac{u}{t} \sim \lambda^{8} \qquad \frac{c}{t} \sim \lambda^{4}$$

$$\frac{d}{b} \sim \lambda^{4} \qquad \frac{s}{b} \sim \lambda^{2}$$

$$\frac{e}{\tau} \sim \lambda^{6} \qquad \frac{\mu}{\tau} \sim \lambda^{2}$$

C.K.M matrix

$$\mathcal{U}_{CKM} \quad \begin{pmatrix} 1 & \lambda & \lambda^{3} \\ \lambda & 1 & \lambda^{2} \\ \lambda^{3} & \lambda^{2} & 1 \end{pmatrix}$$

off-diagonal coefficients of order 1.

MODEST AIM:

LOOK FOR THEORY OF EXPONENTS ONLY

full theory would include prefactors, etc...

Q_i	$\begin{pmatrix} u \\ d \end{pmatrix}$	$\begin{pmatrix} c \\ s \end{pmatrix}$	$\begin{pmatrix} t \\ b \end{pmatrix}$	left-handed
$\bar{u}_i$	$\bar{u}$	$\bar{c}$	$\bar{t}$	right handed
$\bar{d}_i$	$\bar{d}$	$\bar{s}$	$\bar{b}$	
L_i	$\begin{pmatrix} \nu_e \\ e \end{pmatrix}$	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$	$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$	left-handed
$\bar{e}_i$	$\bar{e}$	$\bar{\mu}$	$\bar{\tau}$	right-handed.
[$\bar{N}_i$	$\bar{N}_e$	$\bar{N}_\mu$	$\bar{N}_\tau$]	right-handed sterile neutrinos.

STANDARD MODEL INVARIANTS

$Q_i \, \bar{u}_j \, H_u \longrightarrow$ CHARGE $2/3$ MASS MATRIX

$Q_i \, \bar{d}_j \, H_d \longrightarrow$ CHARGE $-1/3$ MASS MATRIX

$L_i \, \bar{e}_j \, H_d \longrightarrow$ CHARGE -1 MASS MATRIX

$L_i \, \bar{N}_j \, H_u \longrightarrow$ CHARGE 0 "DIRAC" MASS MATRIX

$\bar{N}_i \, \bar{N}_j \longrightarrow$ "MAJORANA" MASS MATRIX

RECONSTRUCTING YUKAWA COUPLINGS

CKM

QUARK MIXING MATRIX

$$\begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

CHARGE $-1/3$

$$\frac{d}{b} \sim \lambda^4 \qquad \frac{s}{b} \sim \lambda^2$$

$$\begin{array}{cc} & \begin{array}{ccc} & & \end{array} \\ Q \updownarrow & \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda^1 & \lambda^0 & 1 \end{pmatrix} \\ & \overleftrightarrow{\bar{d}} \end{array}$$

CHARGE $2/3$

$$\frac{u}{t} \sim \lambda^8 \qquad \frac{c}{t} \sim \lambda^4$$

$$\begin{array}{c} \uparrow \\ Q \\ \downarrow \end{array} \begin{pmatrix} \lambda^8 & \lambda^5 & \lambda^3 \\ \lambda^7 & \lambda^4 & \lambda^2 \\ \lambda^5 & \lambda^2 & 1 \end{pmatrix}$$

$$\leftarrow \bar{u} \rightarrow$$

FAMILY CHARGES

$$Q = (3, 2, 0) = \left(\tfrac{4}{3}, \tfrac{1}{3}, -\tfrac{5}{3}\right) - \tfrac{5}{3}(1, 1, 1)$$

$$\bar{u} = (5, 2, 0) = \left(\tfrac{8}{3}, -\tfrac{1}{3}, -\tfrac{7}{3}\right) - \tfrac{7}{3}(1, 1, 1)$$

$$\bar{d} = (1, 0, 0) = \left(\tfrac{2}{3}, -\tfrac{1}{3}, -\tfrac{1}{3}\right) - \tfrac{1}{3}(1, 1, 1)$$

TRACE

TRACE DOES NOT CONTRIBUTE TO INTERFAMILY HIERARCHY

UNIFORM DESCRIPTION OF TRACELESS PART

$\bar{d}$: $-B\,(2, -1, -1)$

Q : $-B\,(2, -1, -1) + 2\,(1, 0, -1)$

$\bar{u}$: $-B\,(2, -1, -1) + 2\,(1, 0, -1)$

B = BARYON NUMBER

$$X_{Q,\bar{u},\bar{d}} = -B(2,-1,-1) + \eta(1,0,-1)$$

$$\eta = \begin{cases} 0 & \text{for } \bar{d} \\ 2 & \text{for } Q, \bar{u} \end{cases}$$

η SAME FOR $Q, \bar{u}$

$X_{\bar{d}}$ SAME FOR 2nd AND 3rd FAMILY

GENERALIZE CHARGES TO LEPTONS

- STANDARD MODEL HAS SU(5) - CHARGES

CONSEQUENCE OF ANOMALY CANCELLATION

$\bar{5}$: $\bar{d}$ L ← lepton doublet

10 : Q $\bar{u}$ $\bar{e}$ ← lepton isosinglet

- BARYON NUMBER → B - L

POSTULATE CHARGES

$$X_{Q,\bar{u},\bar{d},L,\bar{e}} = (B-L)(2,-1,-1) + \eta(1,0,-1)$$

$$\eta = \begin{cases} 0 & \bar{d} \quad L \\ 2 & Q \quad \bar{u} \quad \bar{e} \end{cases}$$

L : $(-2, 1, 1)$ LEPTON DOUBLET

SAME CHARGE FOR 2nd & 3rd FAMILY!

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NEUTRINO MASSES AND MIXINGS

RIGHT-HANDED NEUTRINO MASS (MAJORANA):

$\bar{N}_i \bar{N}_j$: $\bar{N}$: $(A, B, 0)$ U(1) charge

$$\bar{N}\bar{N} \quad \begin{pmatrix} \lambda^{2A} & \lambda^{A+B} & \lambda^{A} \\ \lambda^{A+B} & \lambda^{2B} & \lambda^{B} \\ \lambda^{A} & \lambda^{B} & \lambda^{0} \end{pmatrix} =$$

$$= \begin{pmatrix} \lambda^{A} & 0 & 0 \\ 0 & \lambda^{B} & 0 \\ 0 & 0 & \lambda^{0} \end{pmatrix} \hat{M}^{(0)} \begin{pmatrix} \lambda^{A} & 0 & 0 \\ 0 & \lambda^{B} & 0 \\ 0 & 0 & \lambda^{0} \end{pmatrix}$$

not Cabibbo-suppressed

$\hat{M}^{(0)}$ = RIGHT-HANDED NEUTRINO MASS WITHOUT CABIBBO SUPPRESSIONS.

DIRAC MASS:

$$L\bar{N}H_u : \begin{pmatrix} \lambda^{2P} & 0 & 0 \\ 0 & \lambda^{-P} & 0 \\ 0 & 0 & \lambda^{-P} \end{pmatrix} \begin{pmatrix} \hat{\ } \end{pmatrix} \begin{pmatrix} \lambda^{A} & 0 & 0 \\ 0 & \lambda^{B} & 0 \\ 0 & 0 & \lambda^{0} \end{pmatrix}$$

not Cabibbo-suppressed

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LIGHT NEUTRINO MASS :

$$5\cancel{5}\,\frac{1}{\cancel{5}\cancel{5}}\,\cancel{5}5 = 55$$

$$(L\bar{N})\,\frac{1}{\bar{N}\bar{N}}\,(\bar{N}L) = M_\nu$$

CABIBBO SUPPRESSION

$$\begin{pmatrix}\lambda^{2p} & & \\ & \lambda^{-p} & \\ & & \lambda^{-p}\end{pmatrix}(\widehat{L\bar{N}})\begin{pmatrix}\lambda^{A} & & \\ & \lambda^{B} & \\ & & \lambda^{0}\end{pmatrix}\frac{1}{\begin{pmatrix}\lambda^{A} & & \\ & \lambda^{B} & \\ & & \lambda^{0}\end{pmatrix}(\widehat{\bar{N}\bar{N}})\begin{pmatrix}\lambda^{A} & & \\ & \lambda^{B} & \\ & & \lambda^{0}\end{pmatrix}}\begin{pmatrix}\lambda^{A} & & \\ & \lambda^{B} & \\ & & \lambda^{0}\end{pmatrix}(\widehat{\bar{N}L})\begin{pmatrix}\lambda^{2p} & & \\ & \lambda^{-p} & \\ & & \lambda^{-p}\end{pmatrix}$$

$$= \begin{pmatrix}\lambda^{2p} & & \\ & \lambda^{-p} & \\ & & \lambda^{-p}\end{pmatrix}\hat{M}_\nu\begin{pmatrix}\lambda^{2p} & & \\ & \lambda^{-p} & \\ & & \lambda^{-p}\end{pmatrix}$$

$$= \begin{pmatrix}\lambda^{4p} & \lambda^{p} & \lambda^{p} \\ \lambda^{p} & \lambda^{-2p} & \lambda^{-2p} \\ \lambda^{p} & \lambda^{-2p} & \lambda^{-2p}\end{pmatrix} \sim \begin{pmatrix}\lambda^{6p} & \lambda^{3p} & \lambda^{3p} \\ \lambda^{3p} & 1 & 1 \\ \lambda^{3p} & 1 & 1\end{pmatrix}$$

HENCE :

FAMILY STRUCTURE OF M_ν DETERMINED

BY LEPTON DOUBLET CHARGES !!

NO CABIBBO SUPPRESSION $\nu_\mu - \nu_\tau$

$$\left.\begin{matrix}\nu_e \to \nu_\mu \\ \nu_e \to \nu_\tau\end{matrix}\right\}\ \lambda^3 \quad \text{SUPPRESSION}$$

$\nu_e \to \nu_\mu \quad \mathcal{O}(\lambda^3)$

FNAL-35
T-34
B~~SER~~

$$\mathcal{U}_{MNS} = \mathcal{U}_\nu^\dagger \, \mathcal{U}_\ell$$

determined by Lepton doublet charges

determined also by lepton doublet charges

$$\mathcal{U}_{MNS} \sim \begin{pmatrix} 1 & \lambda^3 & \lambda^3 \\ \lambda^3 & 1 & 1 \\ \lambda^3 & 1 & 1 \end{pmatrix}$$

Alt-F : $\det\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \sim \mathcal{O}(\lambda^2)$

Neutrino masses : $\lambda^6 : 1 : 1$ $\qquad m \sim \frac{v^2}{M}$

- FLAVOR STRUCTURE OF NEUTRINO SECTOR

  DETERMINED BY FAMILY CHARGES OF LEPTON DOUBLETS!

- SCALE OF NEUTRINO MASSES SET

  BY CUT-OFF OF THEORY

NEUTRINO OSCILLATIONS

Some random references:

book: P. Ramond "Journeys beyond the Standard Model", Perseus, 1999.

articles: B. Kayser "On the Quantum Mechanics of Neutrino Oscillations" Phys. Rev. D24, 110 (1981)

P. Fisher et al "Neutrino Masses and Oscillations" hep-ph/9906244

A. Mann & Primakoff, Phys. Rev D15, 655 (1977).

VACUUM OSCILLATIONS

- formalism
- quantum-mechanical limitations

Neutrinos in Matter

- index of refraction
- oscillations in matter
- MSW effect

Other types of Oscillations.

EARLY OSCILLATIONS

1956 : Ray Davis Jr at Savannah reactor looks for ^{37}Ar in cleaning fluid caused by neutrino scattering (à la Pontecorvo)

$$\nu_e + {}^{37}Cl \rightarrow {}^{37}Ar + e^-$$

RUMOR : HE FOUND SOME Ar !

HOW CAN ν_e COME FROM A REACTOR ?

Pontecorvo : $\bar{\nu}_e$ oscillated into ν_e

(like $K^0 - \bar{K}^0$ system)

RUMOR GOES AWAY - IDEA OF OSCILLATION STAYS

1962 : Danby et al (Lederman, Schwartz) two neutrinos ν_e AND ν_μ

Nagoya (MNS) : look for ν_e in $\pi^+ \rightarrow \mu^+ + \nu_\mu$

$\nu_\mu \rightsquigarrow \nu_e$

new symbol $\rightsquigarrow$ "oscillates into"

flavor oscillations.

1968 : Ray Davis (again!) reports deficit of solar neutrinos

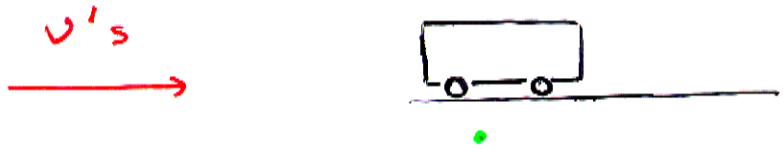
SNU $\equiv 10^{-36}$ captures per target atom / second

1969 : Gribov and Pontecorvo : IT IS OSCILLATIONS, FLAVOR OSCILLATIONS $\nu_e \rightsquigarrow \nu_\mu$?

VACUUM OSCILLATIONS

- state produced at $\vec{x}$ with momentum $\vec{p}$ is NOT a mass eigenstate.

As time evolves it flip-flops to another state and back to itself, and so at a later time, the state is a linear combination of the one originally produced and of other(s) into which it mixed. This decreases the probability that it is the original state.



	BIRTH PLACE	OBSERVATION
	$\vec{p}$	$\vec{p}$
	ν_μ	$\nu_\mu, \nu_e, \nu_\tau, \nu_s, \ldots$
process	$\pi^+ \to \mu^+ + \nu_\mu$	$\nu_e + p \to e^+ + n, \ldots$
	$t=0$	t
	$x=0$	$x=ct$

MASS EIGENSTATES OF MOMENTUM p_ν : TWO-LEVELS ONLY

$$i\frac{d}{dt}\begin{pmatrix}|\nu_1\rangle\\|\nu_2\rangle\end{pmatrix} = \begin{pmatrix}E_1 & 0\\0 & E_2\end{pmatrix}\begin{pmatrix}|\nu_1\rangle\\|\nu_2\rangle\end{pmatrix}$$

$$|\nu_1(t)\rangle = e^{-iE_1t}|\nu_1(0)\rangle \qquad |\nu_2(t)\rangle = e^{-iE_2t}|\nu_2(0)\rangle$$

M_1 $\qquad\qquad$ M_2

$$E_1 = \sqrt{M_1^2 + \vec{p}_\nu^{\,2}} \qquad E_2 = \sqrt{M_2^2 + \vec{p}_\nu^{\,2}}$$

$$E_1 \simeq p_\nu + \frac{M_1^2}{2p_\nu} \qquad \simeq p_\nu + \frac{M_2^2}{2p_\nu}$$

same momentum

$$\pi^+ \to \mu^+ + \nu_\mu$$

NOT A MASS EIGENSTATE BUT

$$|\nu_\mu\rangle = \cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle$$ ORIGINAL "BEAM"

orthogonal combination $\to$

$$|\nu_x\rangle = -\sin\theta\,|\nu_1\rangle + \cos\theta\,|\nu_2\rangle$$

at a later time this evolves to

$$|\nu_\mu(t)\rangle = \cos\theta\,|\nu_1(0)\rangle e^{-iE_1t} + \sin\theta\,|\nu_2(0)\rangle e^{-iE_2t}$$ "BEAM" at t

$$|\nu_x(t)\rangle = -\sin\theta\,|\nu_1(0)\rangle e^{-iE_1t} + \cos\theta\,|\nu_2(0)\rangle e^{-iE_2t}$$

Reverse the relationship

$$|\nu_1\rangle = \cos\theta\,|\nu_\mu\rangle - \sin\theta\,|\nu_x\rangle$$

$$|\nu_2\rangle = \sin\theta\,|\nu_\mu\rangle + \cos\theta\,|\nu_x\rangle$$

Then the "beam" is

$$|\nu_\mu(t)\rangle = \left(\cos^2\theta\, e^{-iE_1 t} + \sin^2\theta\, e^{-iE_2 t}\right)|\nu_\mu\rangle + \cos\theta\sin\theta\left(e^{-iE_2 t} - e^{-iE_1 t}\right)|\nu_x\rangle$$

TWO TYPES OF DETECTORS :

① "ν_μ" DETECTOR :

$$\text{Prob}(\nu_\mu \to \nu_\mu; t) = |\langle \nu_\mu | \nu_\mu(t)\rangle|^2$$

$$= 1 - \sin^2 2\theta \left(\frac{1 - \cos(E_1 - E_2)t}{2}\right)$$

② "ν_x" DETECTOR

$$\text{Prob}(\nu_\mu \to \nu_x; t) = |\langle \nu_\mu | \nu_x(t)\rangle|^2$$

$$\sin^2 2\theta \left(\frac{1 - \cos(E_1 - E_2)t}{2}\right)$$

DISTANCE TRAVELLED $x = t$ (light neutrinos)
(from birth to death)

OSCILLATE AS $\cos 2\pi \frac{x}{L}$

$$L = \frac{2\pi}{E_1 - E_2} \simeq \frac{4\pi P}{|M_1^2 - M_2^2|}$$ OSCILLATION LENGTH

$$i\frac{d}{dt}\begin{pmatrix} |\nu_\mu(t)\rangle \\ |\nu_x(t)\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} |\nu_\mu(t)\rangle \\ |\nu_x(t)\rangle \end{pmatrix}$$

$$\equiv \quad H_0 \begin{pmatrix} |\nu_\mu(t)\rangle \\ |\nu_x(t)\rangle \end{pmatrix}$$

$L = 2.47$ m for $E_\nu \sim 1$ MeV

$|M_1^2 - M_2^2| = 1\,(\mathrm{eV})^2$

QUANTUM MECHANICAL AND EXPERIMENTAL LIMITATIONS

① CAVEAT EXPERIMENTALISTS!

$$\pi^+ \to \mu^+ + \nu_\mu$$

is really

$$\pi^+ \to \mu^+ + \text{"}(\nu_1 + \nu_2)\text{"}$$

any decay will produce either ν_1 with prob $\cos^2\theta$

ν_2 $\sin^2\theta$

if you measure the kinematics of the decay too accurately, you will know which from their masses.

Then quantum mechanics says: no oscillations!

to resolve which ν is produced, need accuracy $\delta m^2 < |M_1^2 - M_2^2|$

but

$$\delta m^2 = \sqrt{(2E_\nu)^2\,\delta E_\nu^2 + (2p_\nu)^2(\delta p_\nu)^2} > 2p_\nu\,\delta p_\nu$$

$$\therefore \quad |M_1^2 - M_2^2| > 2p_\nu\,\delta p_\nu$$

$$\delta p < \frac{2\pi}{L}$$

uncertainty relation $\delta x \, \delta p \sim 1$

$$\delta x > L$$

→ fuzziness in birthplace greater than oscillation length.

NO OSCILLATION BUT STILL REDUCED PROBABILITY

$$\text{PROB} = \begin{cases} \cos^2\theta & \text{if } \nu_1 \text{ is produced in decay} \\ \sin^2\theta & \text{if } \nu_2 \text{ is produced in decay.} \end{cases}$$

TO OBSERVE OSCILLATION δx MUST BE SMALLER THAN OSCILLATION LENGTH.

(2) DO NOT SET UP TOO FAR AWAY!

MOMENTUM UNCERTAINTY : $\delta p \sim \frac{1}{\delta x}$ $\delta x =$ fuziness in birthplace.

phase difference between two components after distance x.

$$\varphi = \frac{|M_1^2 - M_2^2|}{2p} x$$

phase uncertainty

$$\delta\varphi = \frac{|M_1^2 - M_2^2|}{2p} \frac{\delta p}{p} x$$

loss of coherence when $\delta\varphi \sim 2\pi$ at a distance $L_{washout}$

$$2\pi = \frac{|M_1^2 - M_2^2|}{2p} \frac{\delta p}{p} L_{washout} \quad \therefore \quad L_{washout} = \frac{\delta p}{p} L_{osc}$$

decoherence occurs at $L \sim \frac{p}{\delta p} L_{osc}$. from quantum mechanics.

(3) AVERAGING OVER • SIZE OF NEUTRINO SOURCE AND DETECTOR

• ENERGY RESOLUTION OF DETECTOR

effect is to dampen oscillating part

if detector size > oscillation length

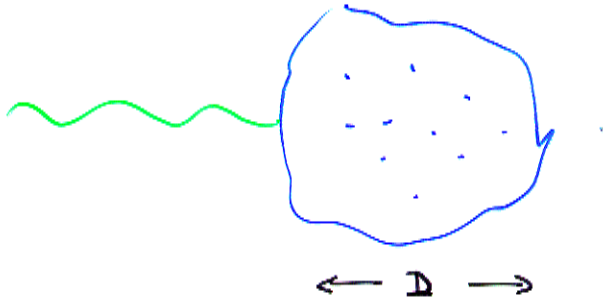
for instance.

$$\frac{\text{distance}}{L_{osc.}} \gg \frac{E}{\Delta E} \quad \ldots$$

IF MANY OSCILLATION TAKE PLACE, NEED TO TIME AVERAGE THUS DESTROYING OSCILLATIONS BUT KEEPING A DEPRESSED PROBABILITY

OSCILLATORY TERM $\rightarrow \frac{1}{2}$

NEUTRINO BEAM IN MATTER



N scattering centers / unit area

slab of width D

$f_p(0)$ = forward scattering amplitude

$$\left[\sigma_t = \frac{4\pi}{p} \, \mathrm{Im}\, f_p(0) \right]$$

$$\Psi(x) = \Psi_{in}(x) + \Psi_{scatt.}(x)$$

$$\approx e^{ipx} + \frac{2\pi i N}{p} f_p(0) D \, e^{ipx}$$

$$\approx e^{ipx + \frac{2\pi i N D}{p} f_p(0)}$$

(if weak, N not too big)

$$= e^{ip(x-D)} \, e^{ipD\, n(p)}$$

INDEX OF REFRACTION: $n(p) \equiv 1 + \frac{2\pi}{p^2} N f_p(0)$

$n(p)$ $\begin{cases} \text{real part changes velocity} \\ \text{imaginary part : extinction} \end{cases}$

electron neutrino :

$$p(n_{\nu_e} - 1) = \frac{2\pi}{p} \left\{ N_e f_p^W(0) + \sum_i N_i f_p^Z(0) \right\}$$

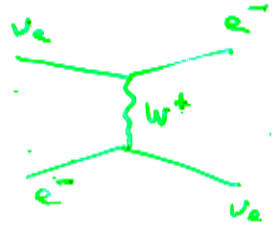
muon and tau neutrinos :

$$p(n_{\nu_\mu, \nu_\tau} - 1) = \frac{2\pi}{p} \left\{ 0 + \sum_i N_i f_p^Z(0) \right\}$$

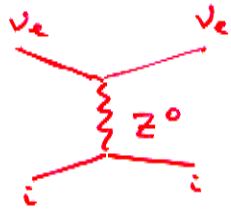
sterile neutrino

$$p(n_{\nu_s} - 1) = \frac{2\pi}{p}\{0 + 0\}$$

$f_p^W(0)$:



$f_p^Z(0)$:



A LITTLE TREK THROUGH STABLE MATTER YIELDS DIFFERENT INDICES OF REFRACTION FOR ν_e AND ν_μ, ν_τ.

$$e^{ipx} \rightarrow e^{ipnx}$$: PHASE SHIFT $px(n-1)$

$$|\nu_e(x)\rangle = e^{ipx + ip(n_{\nu_e}-1)x} \, |\nu_e(0)\rangle$$

$$|\nu_{\mu,\tau}(x)\rangle = e^{ipx + ip(n_{\nu_{\mu,\tau}}-1)x} \, |\nu_{\mu,\tau}(0)\rangle$$

$$|\nu_s(x)\rangle = e^{ipx + 0} \, |\nu_s(0)\rangle$$

FOR LIGHT NEUTRINOS : $p \simeq E$ $\quad x \simeq t$

effect of index of refraction is to

CHANGE ENERGY BY $p(n_f - 1)$ FOR EACH WEAK FLAVOR f

IN THE WEAK EIGENSTATE BASIS

$$i \frac{d}{dt} \begin{pmatrix} |\nu_e(t)\rangle \\ |\nu_x(t)\rangle \end{pmatrix} = H_0 \begin{pmatrix} |\nu_e(t)\rangle \\ |\nu_x(t)\rangle \end{pmatrix} + \begin{pmatrix} p(n_e - 1) & 0 \\ 0 & p(n_x - 1) \end{pmatrix} \begin{pmatrix} |\nu_e(t)\rangle \\ |\nu_x(t)\rangle \end{pmatrix}$$

AFTER A LITTLE ALGEBRA :

$$i \frac{d}{dt} \begin{pmatrix} |\nu_e(t)\rangle \\ |\nu_x(t)\rangle \end{pmatrix} = \frac{\pi}{L_0} \begin{pmatrix} \cos 2\theta + \frac{L_0}{\pi} p n_e & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta + \frac{L_0}{\pi} n_x \end{pmatrix} \begin{pmatrix} |\nu_e(t)\rangle \\ |\nu_x(t)\rangle \end{pmatrix}$$

TAKE OUT THE TRACE WHICH DOES NOT CONTRIBUTE TO OSCILLATIONS, TO GET

$$i \frac{d}{dt} \begin{pmatrix} |\nu_e(t)\rangle \\ |\nu_x(t)\rangle \end{pmatrix} = \frac{\pi}{L_0} \begin{pmatrix} \cos 2\theta + \delta(t) & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta - \delta(t) \end{pmatrix} \begin{pmatrix} |\nu_e(t)\rangle \\ |\nu_x(t)\rangle \end{pmatrix}$$

$$\delta(t) = \frac{L_0}{2\pi} p(n_e - n_x) \equiv \frac{L_0}{L_m}$$

FOR $\nu_e - \nu_\mu$ ONLY CHARGED CURRENT CONTRIBUTES TO DIFFERENCE

$$\delta_{e-\mu}(t) = 2\sqrt{2}\, G_F \frac{p\, N_e(t)}{m_{\nu_e}^2 - m_{\nu_\mu}^2}$$

TIME DEPENDENCE BECAUSE N_e MAY BE VARYING OVER DISTANCES AS FOR EXAMPLE IN THE SUN!

IF $m_{\nu_\mu} > m_{\nu_e}$ $\quad \delta_{e-\mu} < 0$

for some t_R: $\quad \cos 2\theta = -\delta(t_R)$

t_R : determined by critical electron density

$$N_e^{crit} = \frac{(m_{\nu_\mu}^2 - m_{\nu_e}^2)\cos 2\theta}{2\sqrt{2}\, G_F E_\nu}$$

* analysis different for $\nu_e - \nu_{sterile}$ because of neutral currents.

ANALYTIC SOLUTION DIFFICULT BECAUSE $\delta(t)$ IS TIME-DEPENDENT

FIND EIGENSTATES AND MIXING ANGLES AT TIME t

AT $t = t_R$ MIXING MATRIX IS PURELY OFF-DIAGONAL

$$\begin{pmatrix} 0 & \sin 2\theta \\ \sin 2\theta & 0 \end{pmatrix}$$

NO MATTER HOW SMALL θ IS, GET MAXIMAL MIXING AT THAT TIME : THIS IS THE MSW EFFECT!

INSTANTANEOUS EIGENVALUES:

$$E_\pm = \pm \frac{\pi}{L_0} \sqrt{1 + 2\delta(t)\cos 2\theta + \delta^2(t)}$$

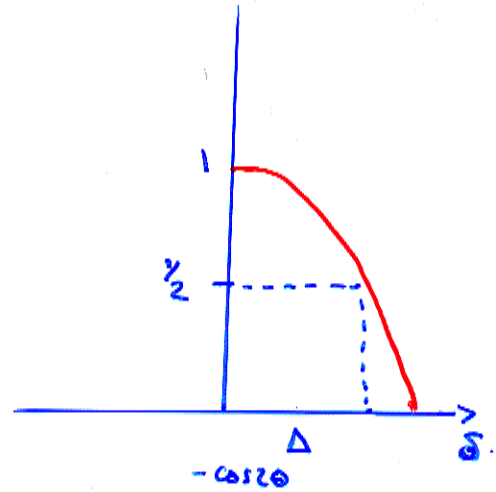
INSTANTANEOUS MIXING ANGLE

$$\sin^2 2\theta_m(t) = \frac{\sin^2 2\theta}{1 + 2\delta(t)\cos 2\theta + \delta^2(t)}$$

at $\delta(t) = -\cos 2\theta \rightarrow \sin^2 2\theta_m(t_R) = \dfrac{\sin^2 2\theta}{1-\delta^2} = 1.$

$\theta_m = 45°.$

$$\sin^2 2\theta_m(t) = \frac{1}{1 + \left(\frac{\delta + \cos 2\theta}{\sin 2\theta}\right)^2}$$



Region of enhancement

$$-\Delta - \cos 2\theta < \delta < \Delta - \cos 2\theta$$

$$\Delta = 2\sqrt{2}\, G_F \frac{E}{|\delta M^2|} \delta N_e$$

$$= \sin 2\theta$$

$$\boxed{\frac{\delta N_e}{N_e} = - \tan 2\theta}$$

$\longrightarrow$ width of resonance layer : Δr

At resonance; oscillation length is

$$L_{res} = \frac{L_0}{\sin 2\theta} = \frac{4\pi E_\nu}{|M_1^2 - M_2^2| \sin 2\theta}$$

Two length scales:

$L_{res} \ll \Delta r$: many oscillations in resonance layer

NOT VIOLENT : incoming states keep their identities.

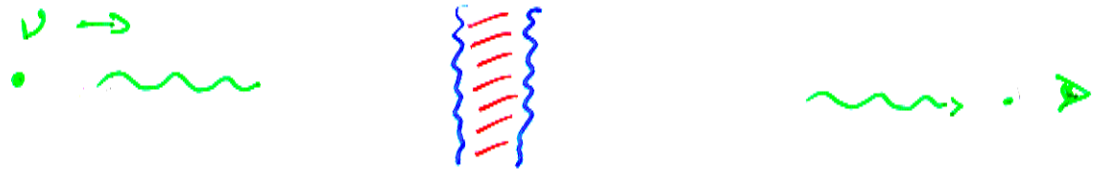
ADIABATIC

$L_{res} \gg \Delta r$: hardly has time to oscillate and part of the wave gets lost through oscillation

incoming state gets broken up

NON- ADIABATIC

Gelb+Rosen



ADIABATIC EVOLUTION

$$|\nu_e\rangle = \cos\theta_b |\nu_1\rangle_I + \sin\theta_b |\nu_2\rangle_I \qquad t=0$$

$$|\nu_e(t_{in})\rangle = \cos\theta_b\, e^{-\int_0^{t_{in}} E_1\, dt'} |\nu_1(t_{in})\rangle_I + \sin\theta_b\, e^{-\int_0^{t_{in}} E_2\, dt'} |\nu_2(t_{in})\rangle_I \qquad t=t_{in}$$

BEAM IS MSW'ed

IF ADIABATIC :

$$|\nu_e(t_{out})\rangle = \cos\theta_b\, e^{-\int_0^{t_{out}} E_1\, dt'} |\nu_1(t_{out})\rangle + \sin\theta_b\, e^{-\int_0^{t_{out}} E_2\, dt'} |\nu_2(t_{out})\rangle \qquad t=t_{out}$$

IF NON-ADIABATIC

$$|\nu_e(t_{out})\rangle = \cos\theta_b\, e^{-\int E_1 dt'} \left[A\, |\nu_1(t)\rangle_I + B\, |\nu_2\rangle_I \right]$$
$$+ \sin\theta_b\, e^{-\int E_2\, dt'} \left[-B^*\, |\nu_1(t)\rangle_I + A^*\, |\nu_2\rangle_I \right]$$

$$|A|^2 + |B|^2 = 1$$

$|B|^2 = P_{jump}$ = probability that entering adiabatic state jumps into orthogonal adiabatic state

APPLICATION TO THE SUN

is $|\delta(t)| < 1$ ever satisfied?

for neutrinos produced at the Sun's core

$$.23 < E_\nu < 14 \quad \text{MeV}$$

$N_e(x)$: roughly constant at core $\sim N_c$

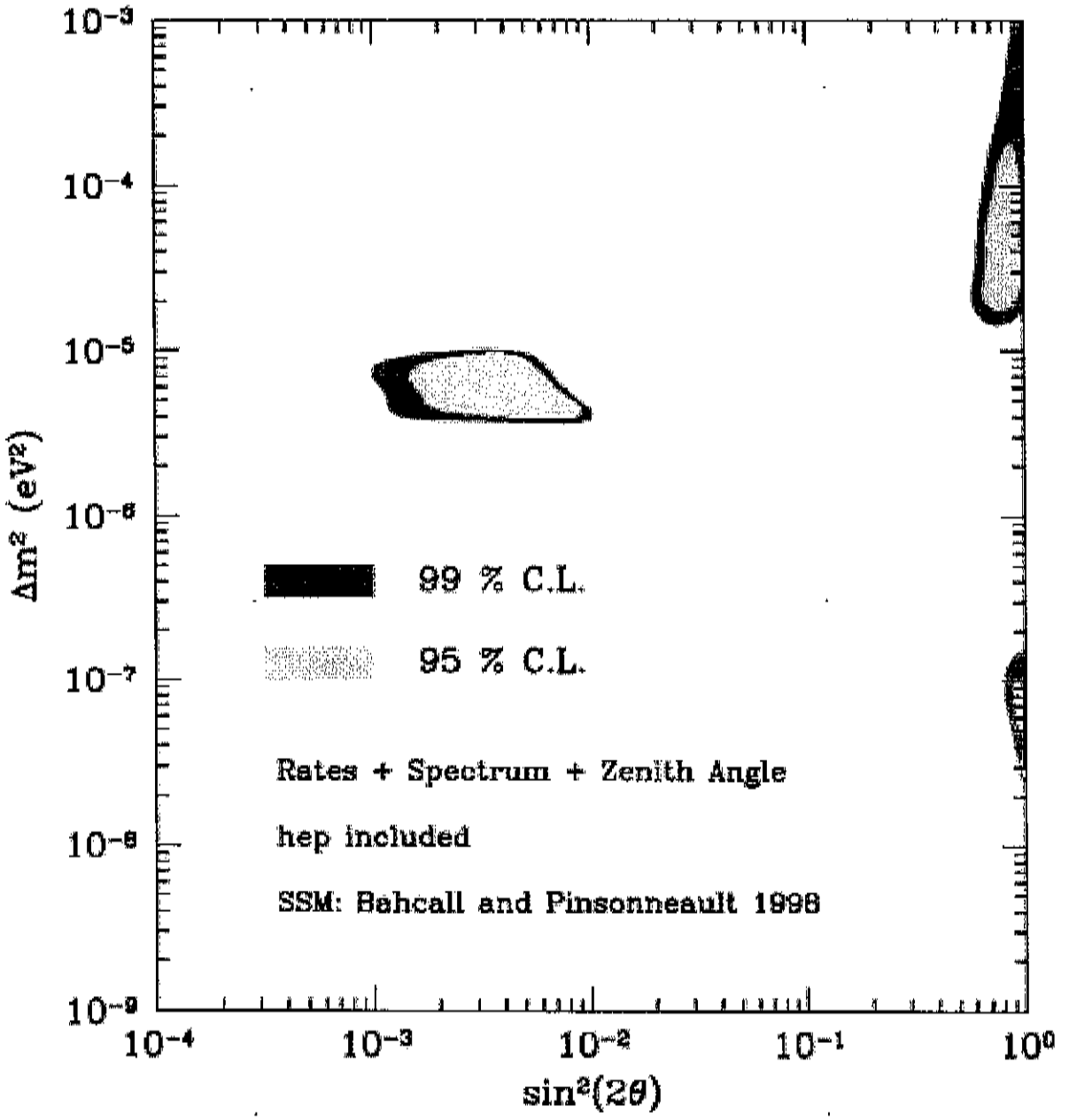
decrease exponentially outside the core $N_e(r) \simeq N_c e^{-r/r_0}$

$$r_0 \sim \frac{1}{10} R \sim 7 \times 10^7 \text{ m.}$$

$$E_\nu > E_\nu^{min} = \frac{m_{\nu_\mu}^2 - m_{\nu_e}^2}{2\sqrt{2}\, G_F N_c} \cos 2\theta = 4 \times 10^4 \, |\Delta m^2| (eV^2) \cos 2\theta_0$$

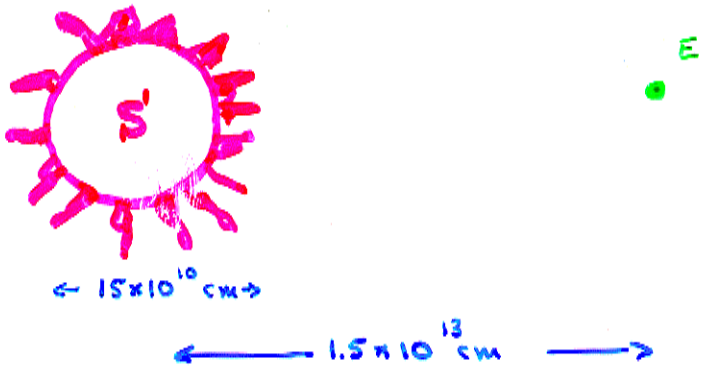
NON-ADIABATIC SMALL ANGLE : small res. layer

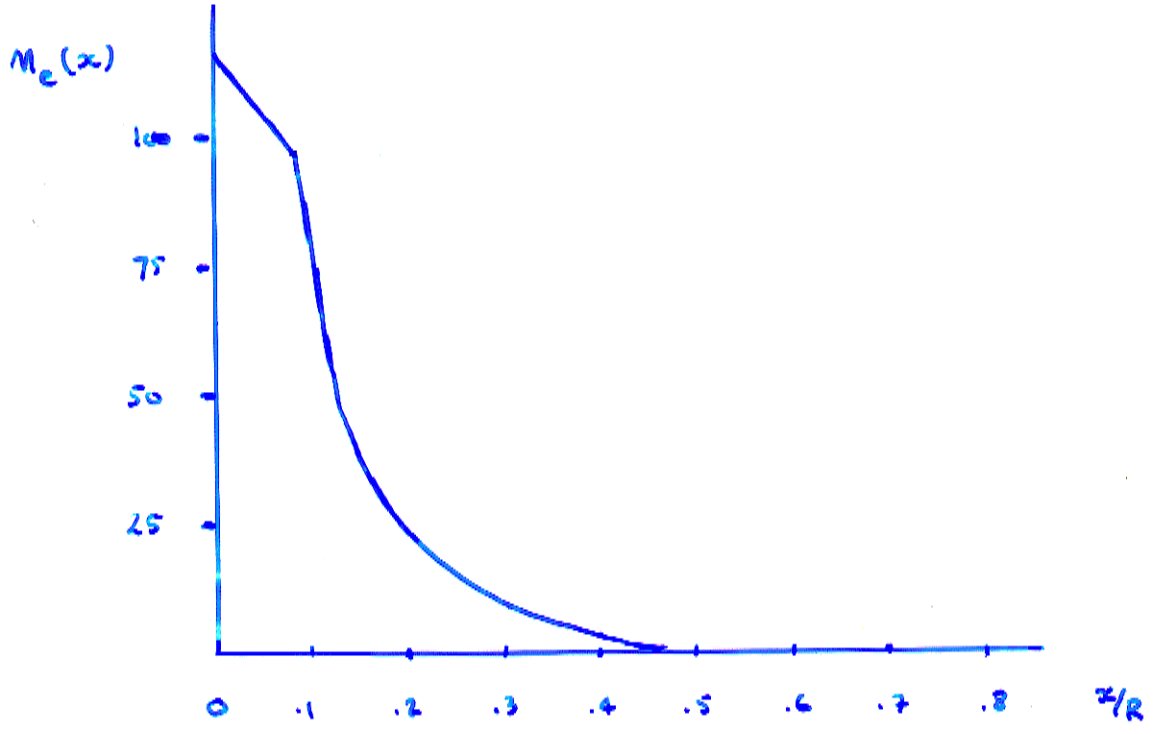
ADIABATIC LARGE ANGLE : large res. layer


10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
Δm^2 (eV^2)
99 % C.L.
95 % C.L.
Rates + Spectrum + Zenith Angle
hep included
SSM: Bahcall and Pinsonneault 1998
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
$\sin^2(2\theta)$

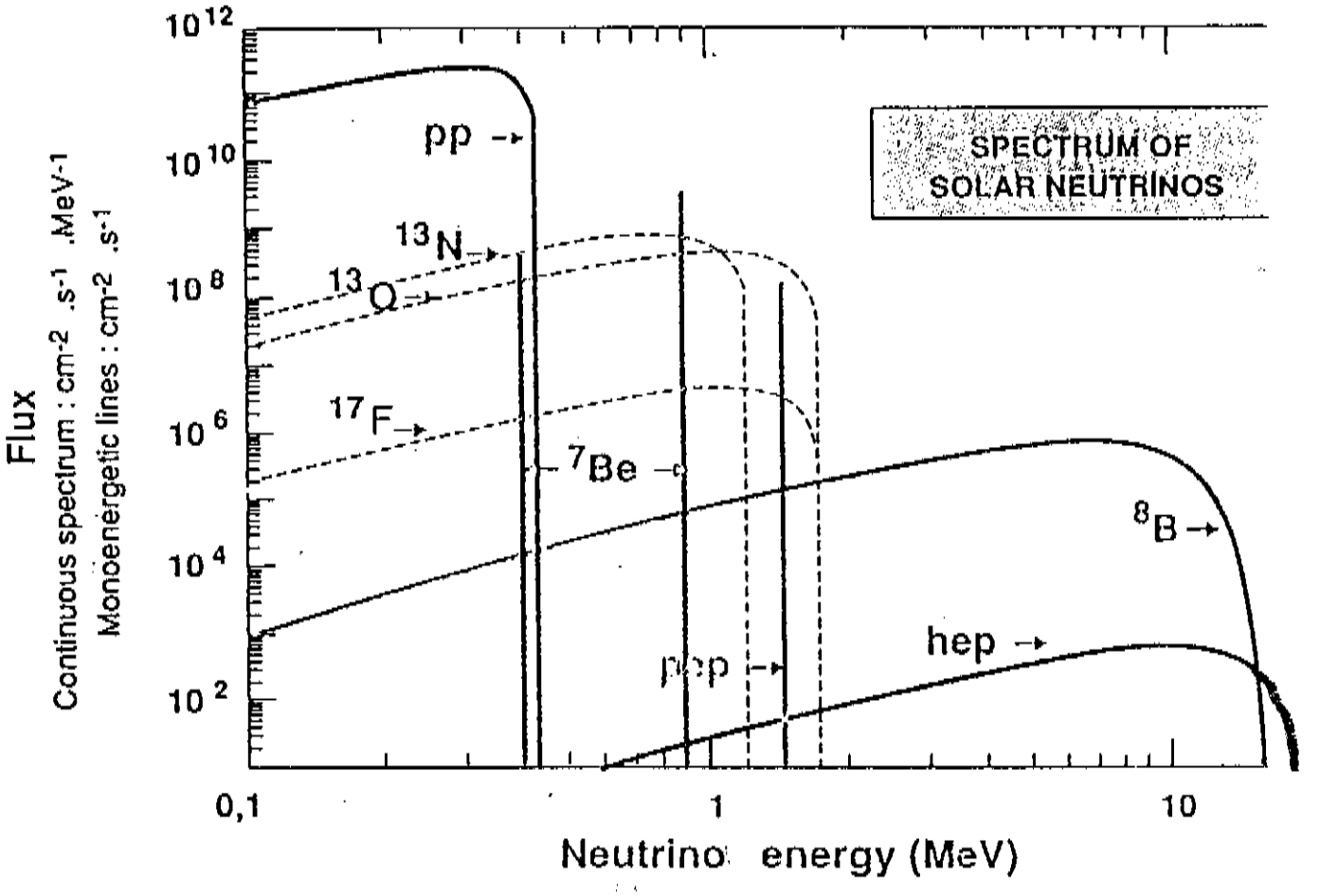
Bahcall

SOLAR DATA





RMP 54, 767 (1982)


SPECTRUM OF
SOLAR NEUTRINOS
Flux
Continuous spectrum : cm-2 .s-1 .MeV-1
Monoenergetic lines : cm-2 .s-1
10 12
10 10
10 8
10 6
10 4
10 2
pp
13N
13O
17F
7Be
8B
pep
hep
0,1
1
10
Neutrino energy (MeV)