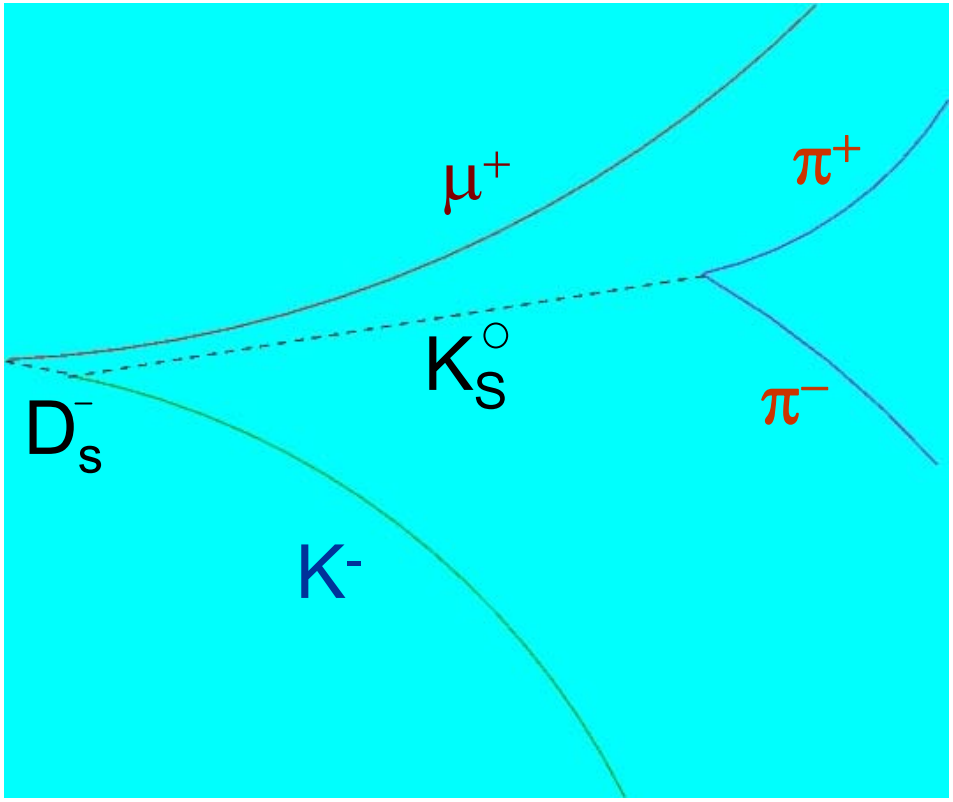
B mixing at DØ: Δm_d & Δm_s



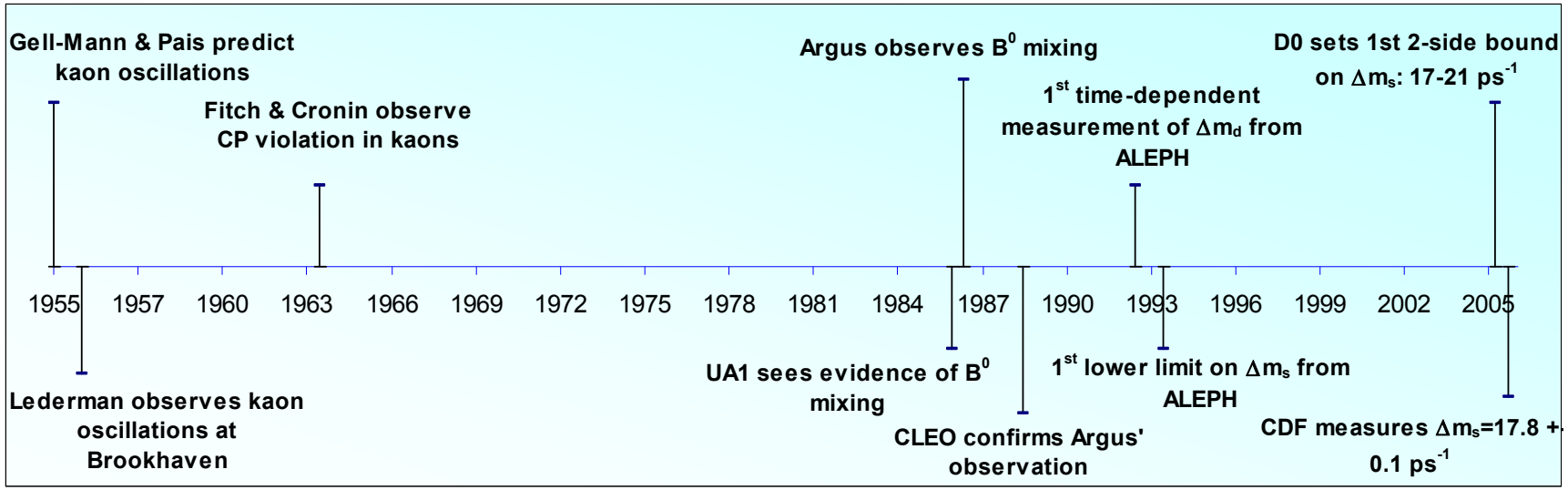
Dan Krop

Indiana University

Overview

- Theory & Motivation
- DØ Luminosity Calibration
- Flavor Tagging
- Measuring Δm_d
- Δm_s: New Channel
- Δm_s: Signal Separation
- Δm_s: Results & Combination

A Brief History of Mixing



- We have known that B_s oscillates since the ARGUS observation of B^0 mixing in 1987.
- It took 19 years to get a measurement of the frequency from 1 experiment.

The CKM Matrix

Charged Current

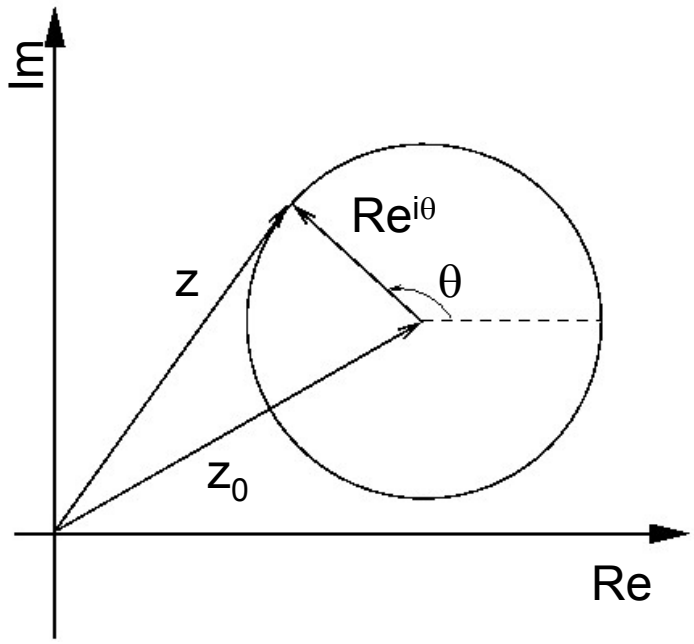
$$J^{\mu}_{W^+} = \frac{1}{\sqrt{2}} \bar{u}^i_L \gamma^\mu V^{ij} d^j_L$$

CKM

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Triangle in Complex Plane

$$z = z_0 + Re^{i\theta}$$



The CKM Matrix

Charged Current

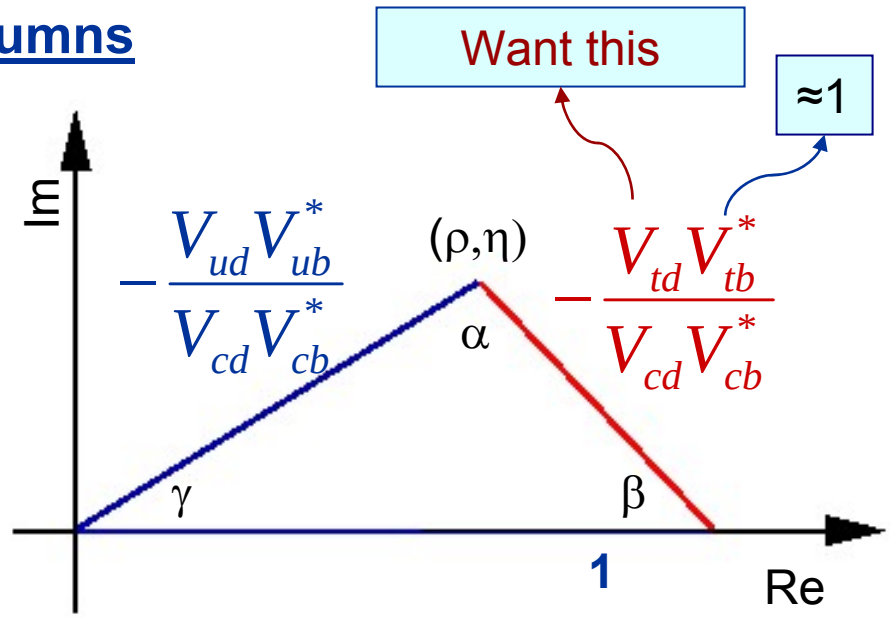
$$J^{\mu}_{W^+} = \frac{1}{\sqrt{2}} \bar{u}^i_L \gamma^\mu V^{ij} d^j_L$$

CKM

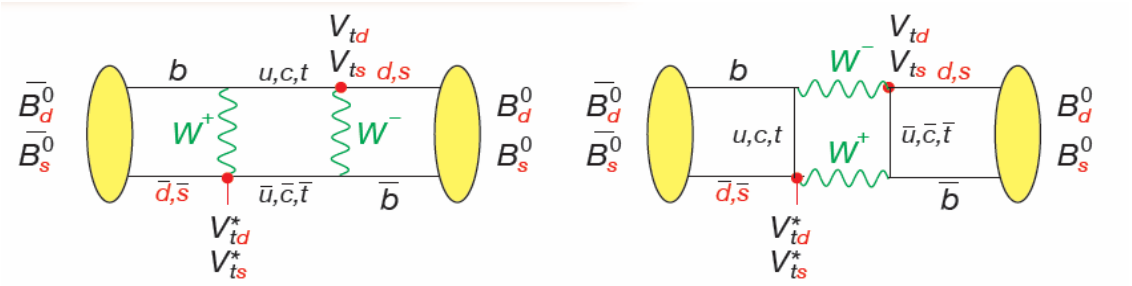
$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Unitarity Constraint on 1st and 3rd Columns

$$-\frac{V_{ud}V^*_{ub}}{V_{cd}V^*_{cb}} = 1 - \frac{V_{td}V^*_{tb}}{V_{cd}V^*_{cb}}$$



Feynman Diagrams



Mixing occurs through box diagrams in SM

15-20% Theory Error

Want high precision

$$\Delta m_d = \frac{G_F^2}{6\pi^2} m_b m_t^2 F\left[\frac{m_t^2}{m_W^2}\right] \eta_{QCD} B_{B_d} f_{B_d}^2 \left|V_{tb}^* V_{td}\right|^2$$

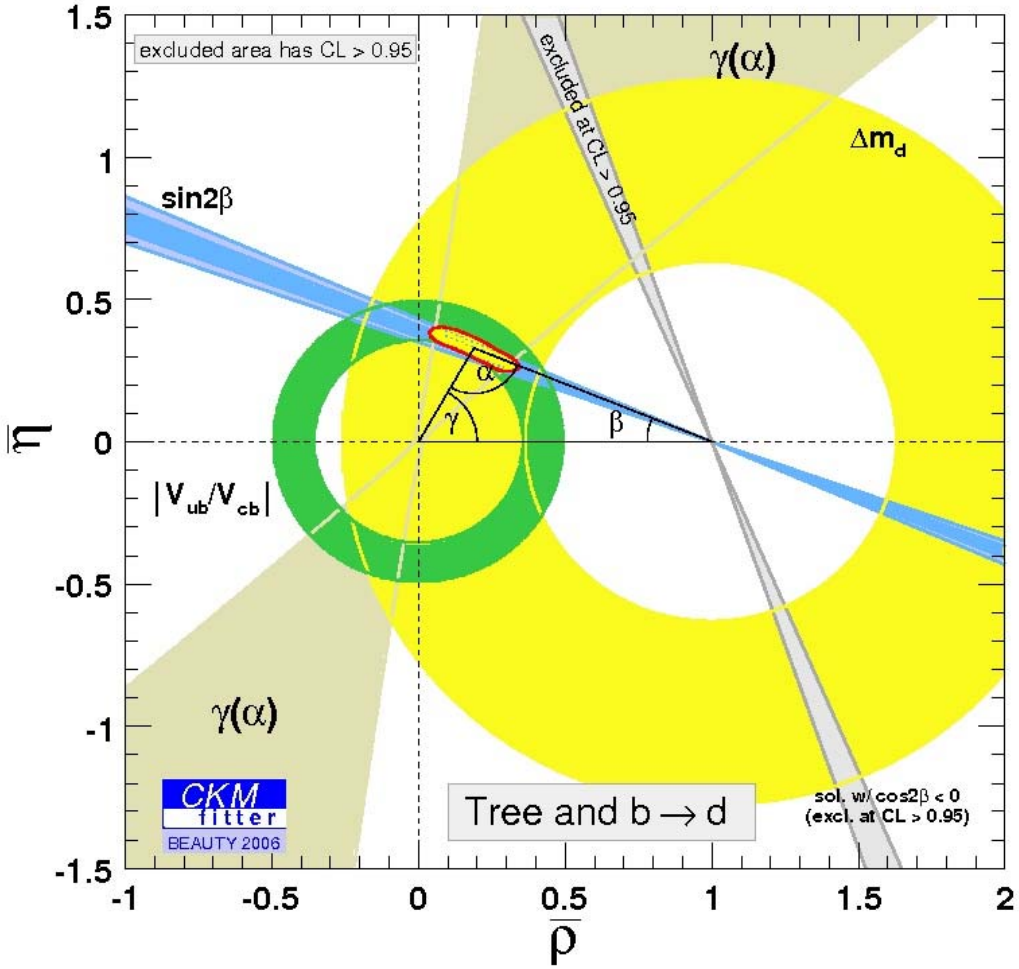
$$\frac{\Delta m_s}{\Delta m_d} = \frac{m_{B_s}}{m_{B_d}} \frac{f_{B_s}^2}{f_{B_d}^2} \frac{B_{B_s}}{B_{B_d}} \left|\frac{V_{ts}}{V_{td}}\right|^2$$

3% Theory Error

Unquenched LQCD, Stewart, LP'05

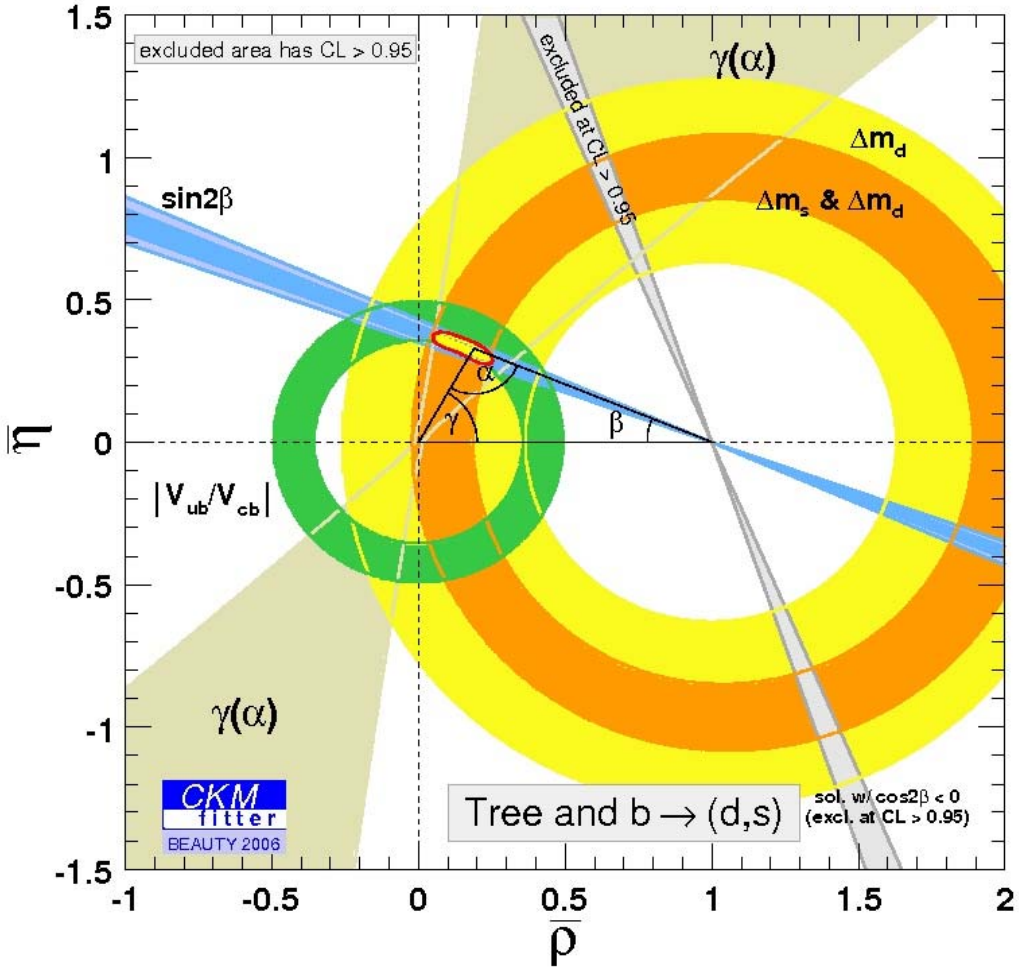
$$\left(1.21 \pm 0.022^{+0.035}_{-0.014}\right)^2$$

Unitarity Triangle



- $|V_{ub}/V_{cb}|$
  - Charmless/charmed
- $\sin(2\beta)$ $[B^0 \rightarrow J/\psi K_S]$
- γ $[B \rightarrow D^{(*)}K]$
- Δm_d

Unitarity Triangle



- $|V_{ub}/V_{cb}|$
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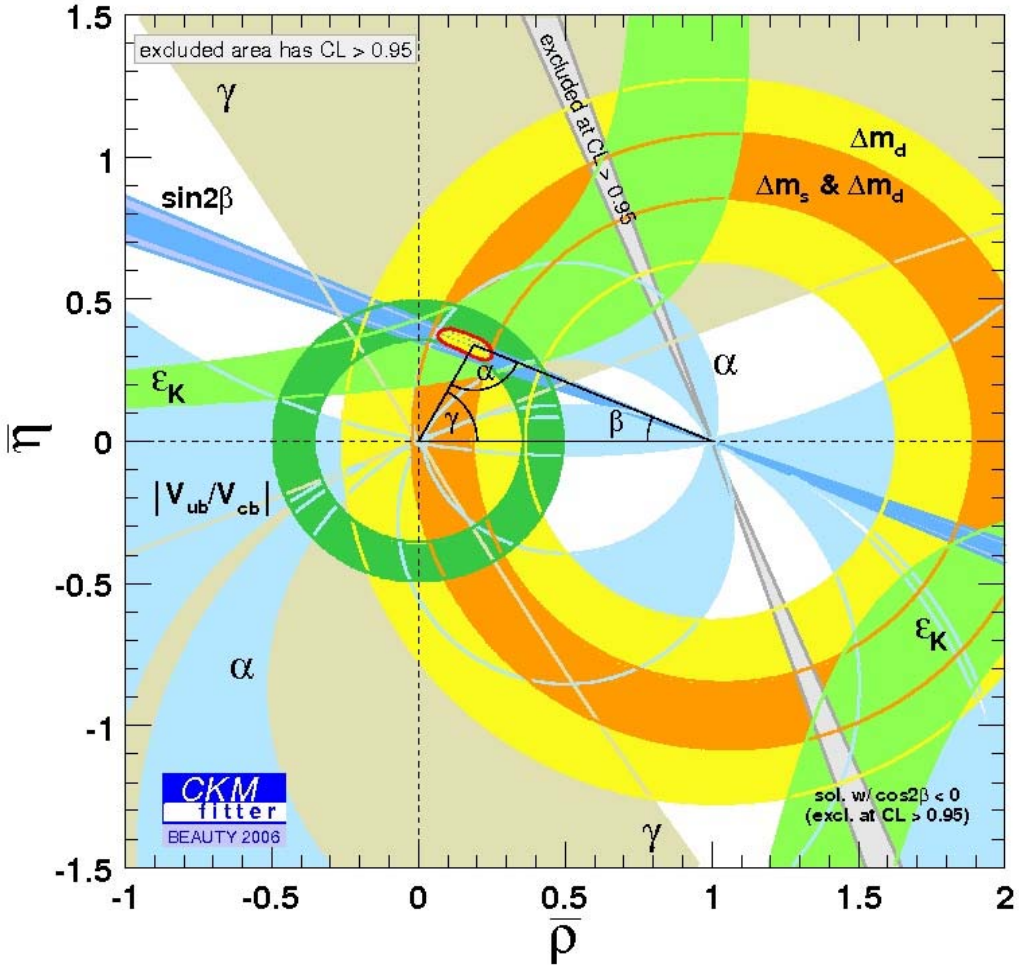
- Δm_d
- Δm_s

Unitarity Triangle



- $|V_{ub}/V_{cb}|$
  - Charmless/charmed
- $\sin(2\beta)$ [$B^0 \rightarrow J/\psi K_S$]
- γ [$B \rightarrow D^{(*)}K$]
- Δm_d
- Δm_s
- ε_K [CPV in kaons]
- α [$B \rightarrow \pi\pi(\rho\rho)$]

Mixing Formalism

$$\hat{H} = \begin{pmatrix} M - \frac{i\Gamma}{2} & M_{12} - \frac{i\Gamma_{12}}{2} \\ M_{12}^* - \frac{i\Gamma_{12}^*}{2} & M - \frac{i\Gamma}{2} \end{pmatrix}$$

Diagonalize

$$|B_L\rangle = p|B\rangle + q|\overline{B}\rangle$$

$$|B_H\rangle = p|B\rangle - q|\overline{B}\rangle$$

Masses & Widths

where $$\frac{q}{p} = \sqrt{\frac{M_{12}^* - \frac{i\Gamma_{12}^*}{2}}{M_{12} - \frac{i\Gamma_{12}}{2}}}$$

$$m_{H,L} = M \pm \Delta m / 2$$

$$\Gamma_{H,L} = \Gamma \pm \Delta\Gamma / 2$$

Mixing Formalism II

Under the assumption of no CP violation in mixing, |q/p|=1, and no lifetime difference, $\Delta\Gamma$=0, we get:

$$P_{u,m}(t) = \frac{1}{2}\Gamma e^{-\Gamma t}\left(1 \pm \cos \Delta m t\right)$$

where

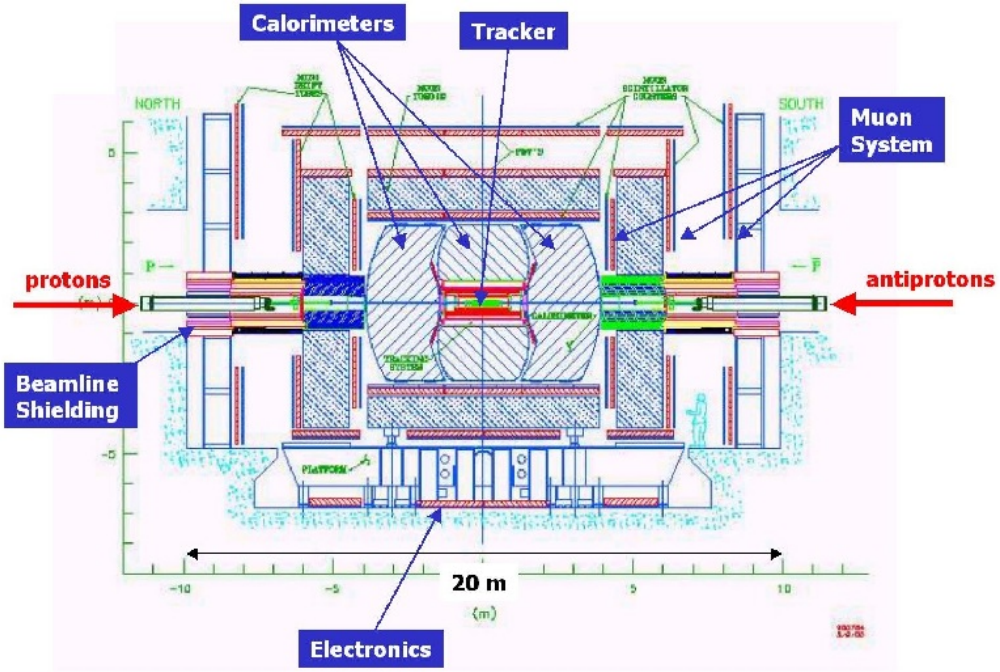
$$P_u^B(t) = \left|\left\langle B \middle| B(t)\right\rangle\right|^2 \qquad P_u^{\overline{B}}(t) = \left|\left\langle \overline{B} \middle| \overline{B}(t)\right\rangle\right|^2$$

$$P_m^B(t) = \left|\left\langle \overline{B} \middle| B(t)\right\rangle\right|^2 \qquad P_m^{\overline{B}}(t) = \left|\left\langle B \middle| \overline{B}(t)\right\rangle\right|^2$$

The Plan

- Use flavor tagger to measure Δm_d.
  - Certify tagger by measuring well-known oscillation frequency.
  - Calibrate tagger by measuring dilution in bins of tagger output variable d_{pr}.
- Apply event-by-event flavor tagging in search for Δm_s.

The DØ Detector

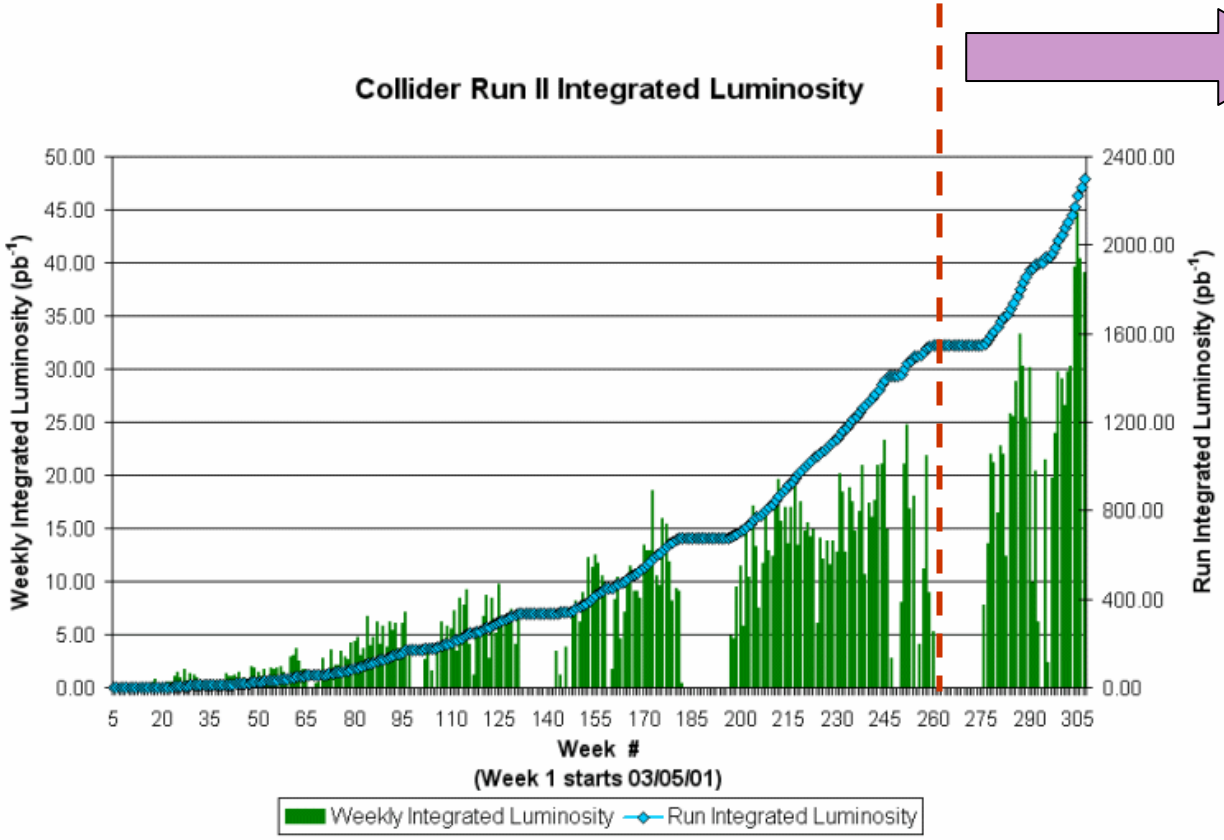


- Multipurpose detector located at Tevatron: $p\bar{p}$ collisions at 1.96 TeV
- Muon detector in 1.8 T toroid extends to $|\eta|<2.0$.
  - Single muon – main trigger for these analyses.

$$\eta = -\ln\left[\tan\left(\frac{\theta}{2}\right)\right]$$

- Silicon & fiber tracker in 2 T solenoid with coverage up to $|\eta|<3.0$
  - $\sigma(\text{IP})=35\ \mu\text{m}$ @ $p_T>5$ GeV
- Calorimeter (EM+hadronic)
  - Used for electron flavor tagging in this analysis.

Data Set

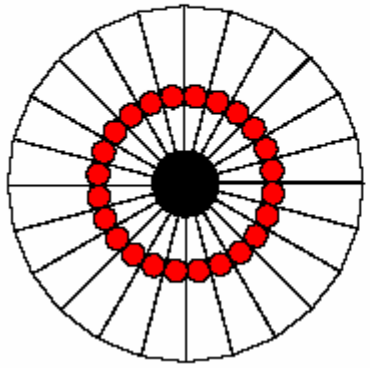


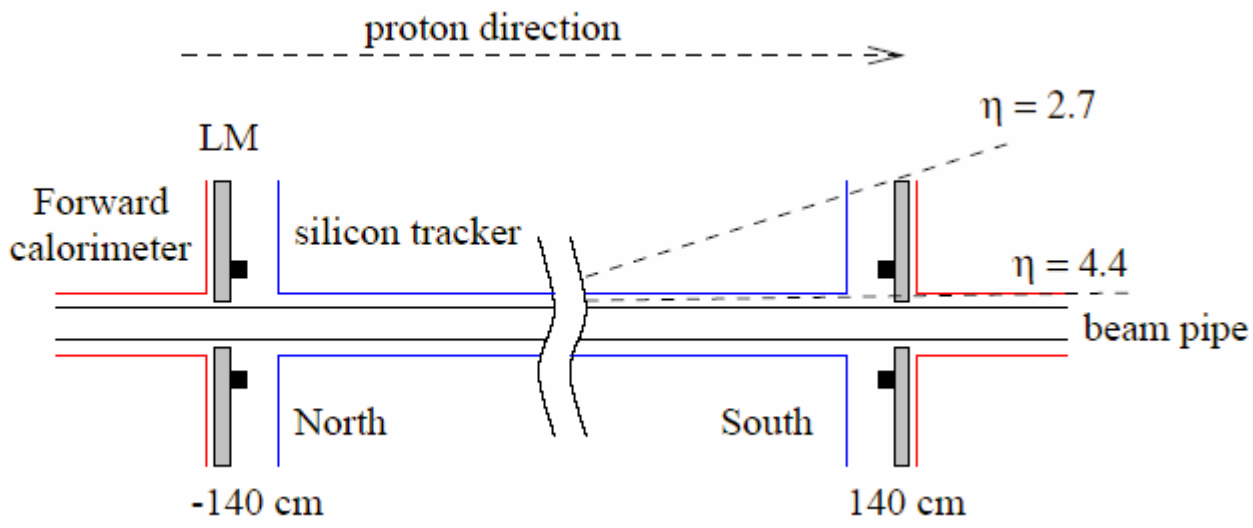
Run IIa: 1.2 fb^{-1}

•Analyses presented today use full Run IIa dataset.

- How do we measure luminosity?

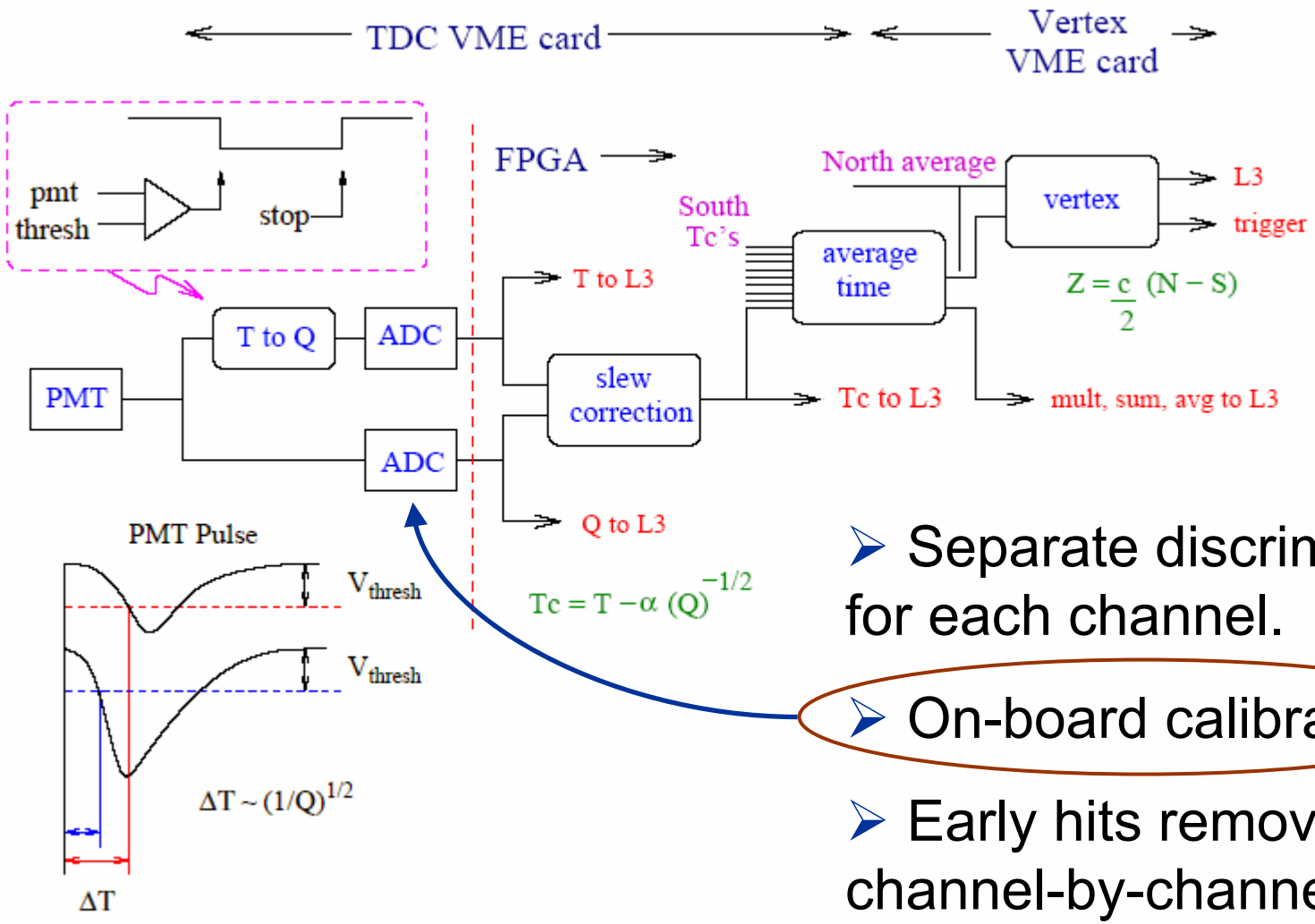
DØ Luminosity Monitor





- Two forward arrays of scintillators.
- Inelastic collisions identified by in-time coincidence of two arrays.

Run II VME Readout



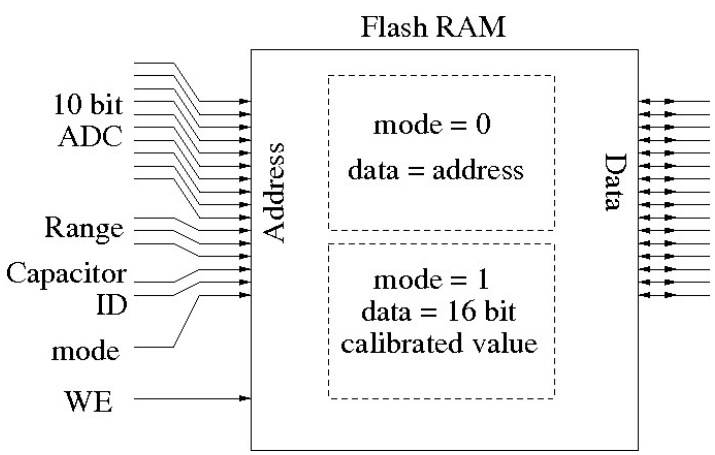
- Separate discriminator for each channel.
- On-board calibration.
- Early hits removed channel-by-channel.

Calibration Procedure

1. Calibrate charge on each card using on-board DAC.
2. Global time calibration:

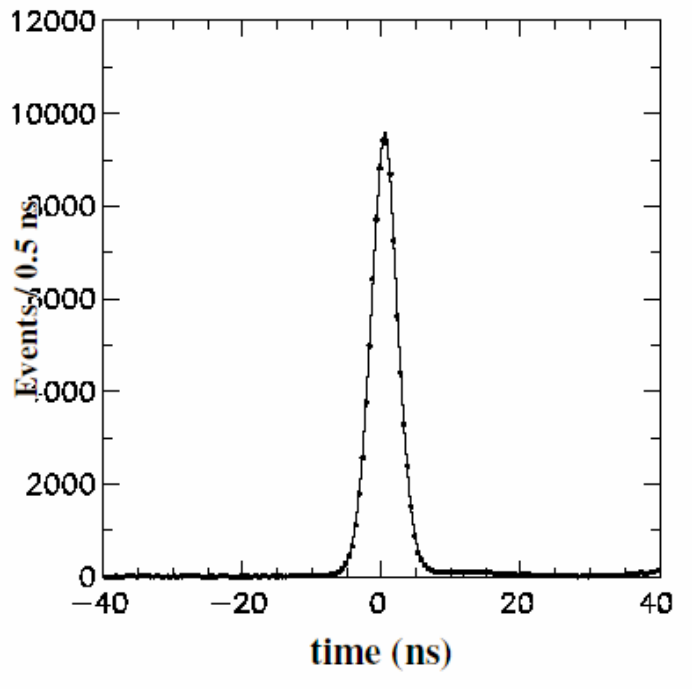
Tracker vertex position

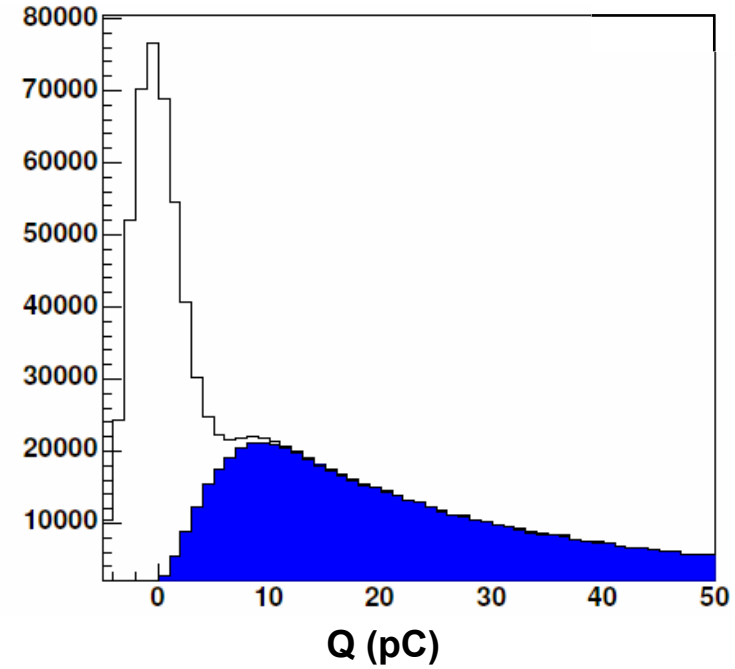
$$\chi^2 = \sum_{Events}\left(\langle T_N\rangle - \langle T_S\rangle - \frac{2z_v}{c}\right)^2 + \frac{\lambda}{N_{Hits}}\sum_{Events}\sum_{Counters} T$$



- Calibration constants stored on flash RAM sitting on "CAFÉ" ADC cards on TDC boards.
- CAFÉ's have 8 ranges, 4 integration capacitors per range → large dynamic response.
- 2 CAFÉ cards per LM channel:
  - time & charge (slewing correction).

Lumonisty Calibration Results





- $\sigma(T)=2$ ns.
  - Hits within ±6.4 ns (3.2σ) defined as “in-time”.
- Blue curve→discriminator fired (3 pC threshold).
  - MIP peak seen at ≈ 10 pC.

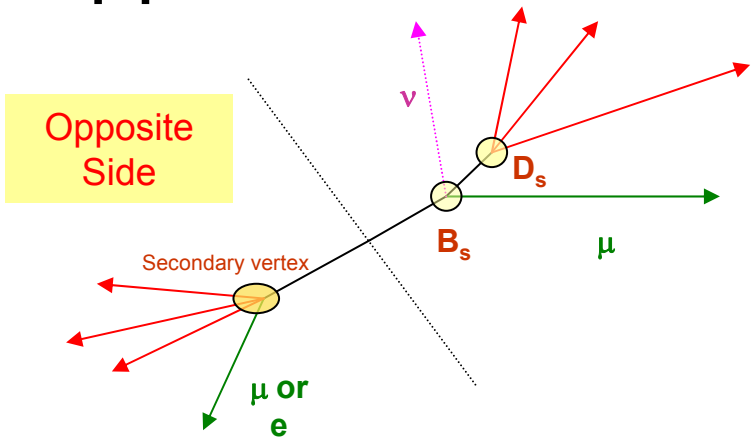
Part I:
Flavor Tagging & Measuring Δm_d

Initial State Flavor Tagging

- We use opposite-side flavor tagging.

<u>Efficiency</u>

$$\varepsilon = \frac{N_{tagged}}{N_{total}}$$

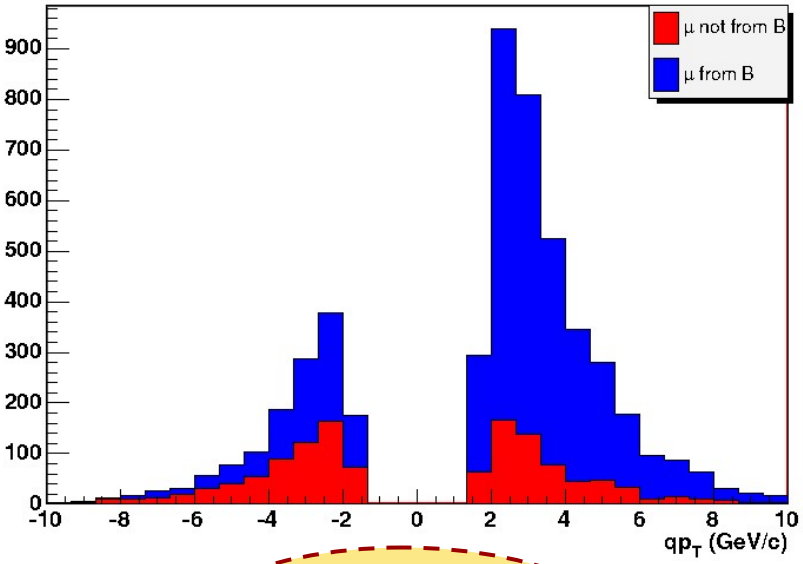


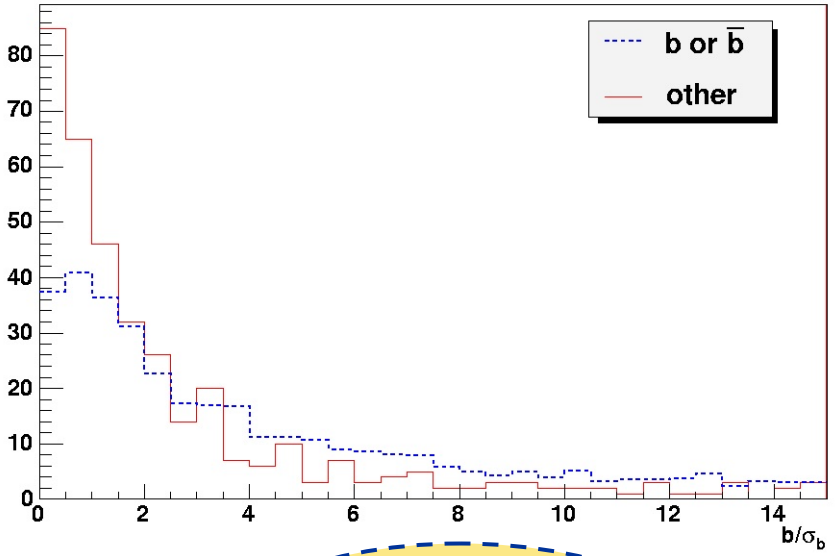
<u>Dilution</u>

$$D = \frac{N_{correct} - N_{incorrect}}{N_{correct} + N_{incorrect}}$$

- 2 taggers are used. Data/MC crosscheck.
  - Derived from $B^+ \rightarrow X\mu D^0 \rightarrow K\pi$ data; no correlations taken into account.
  - Derived from $B^\pm \rightarrow J/\psi K^\pm$ MC; correlations taken into account.

Tagging Discriminant Correlations





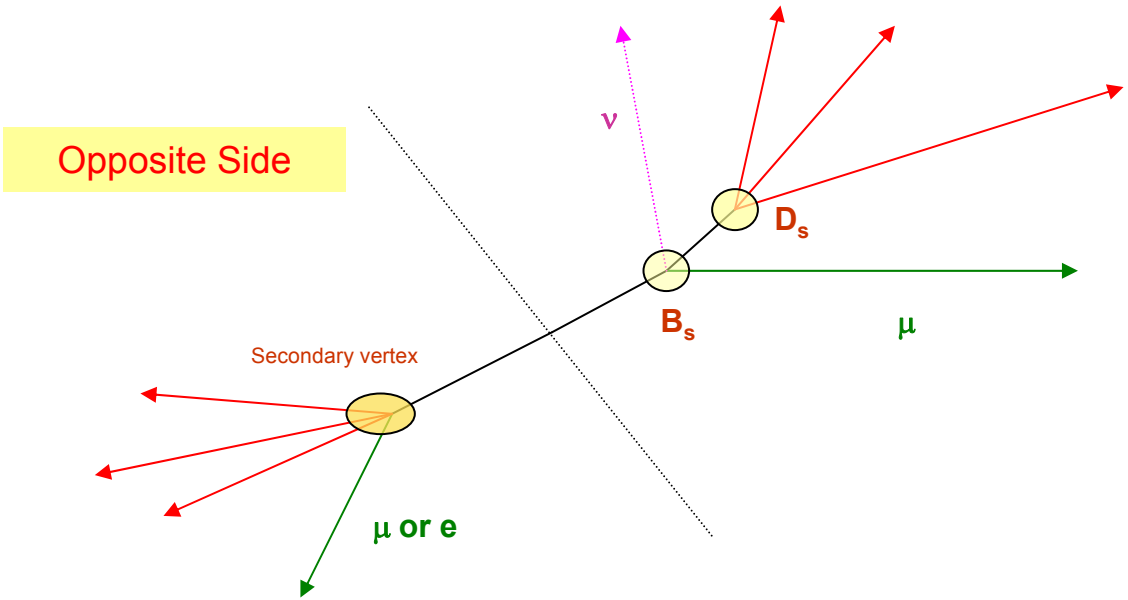
qp_T shape different for B and non-B muons

b/σ_b different for B and non-B muons

Put Correlations into Tagging Algorithm

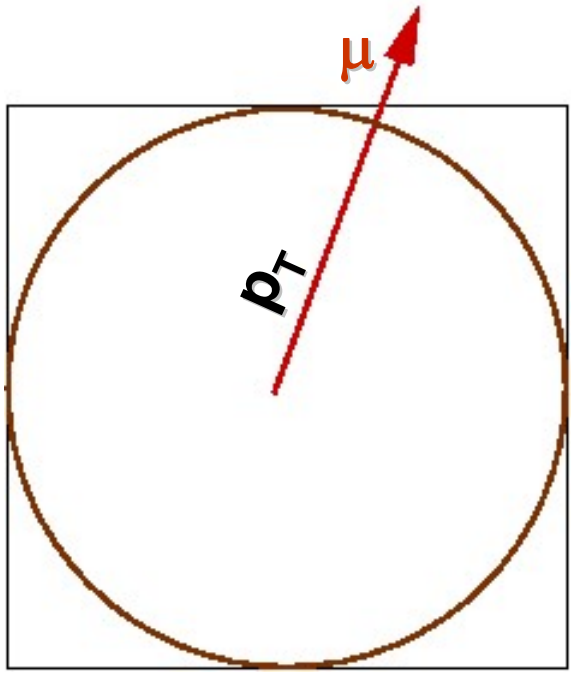
$\rho(qp_T$,imp. sig.) improves tagger

Flavor Tagging Discriminants



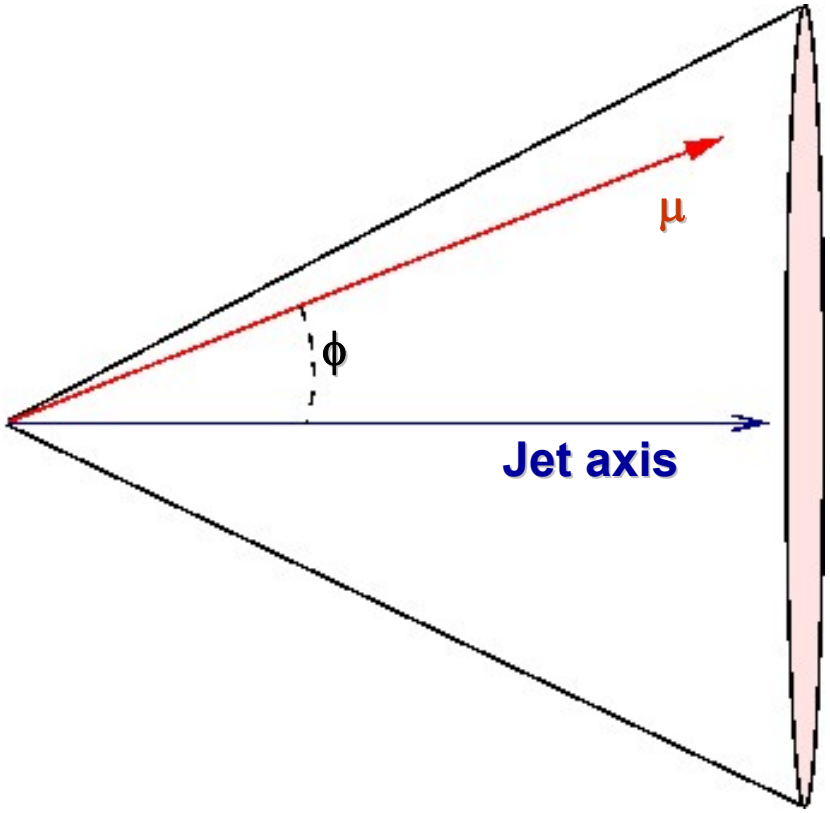
- All discriminants are kinematic variables associated with opposite-side objects.
  - Muons
  - Electrons
  - Secondary Vertices

Flavor Tagging Discriminants



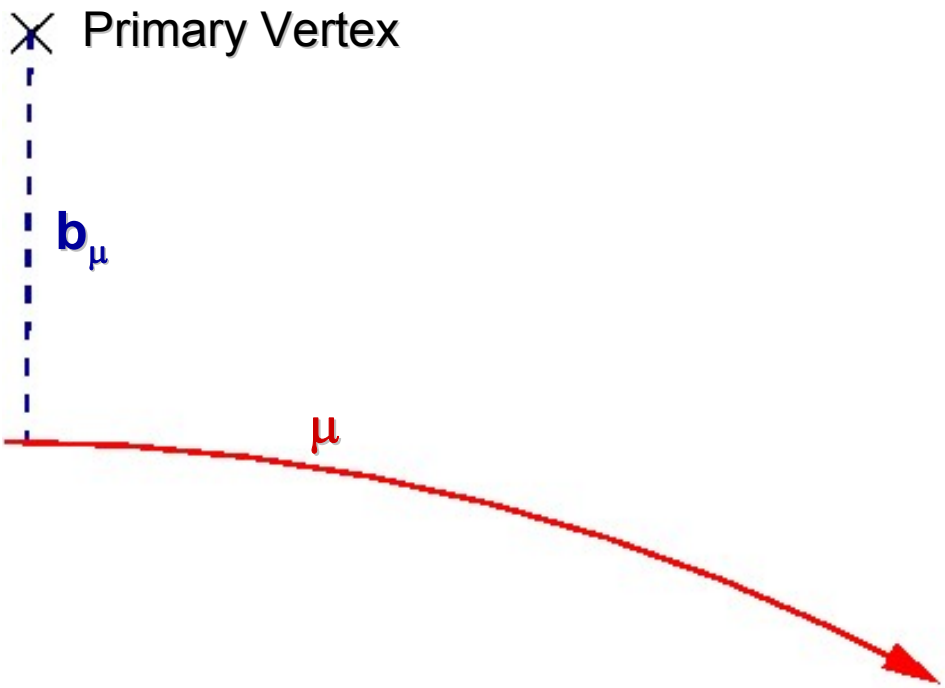
- $p_T(\mu)$

Flavor Tagging Discriminants



- $p_T(\mu)$
- $p_T^{rel}(\mu) = |p_\mu| \sin\phi$

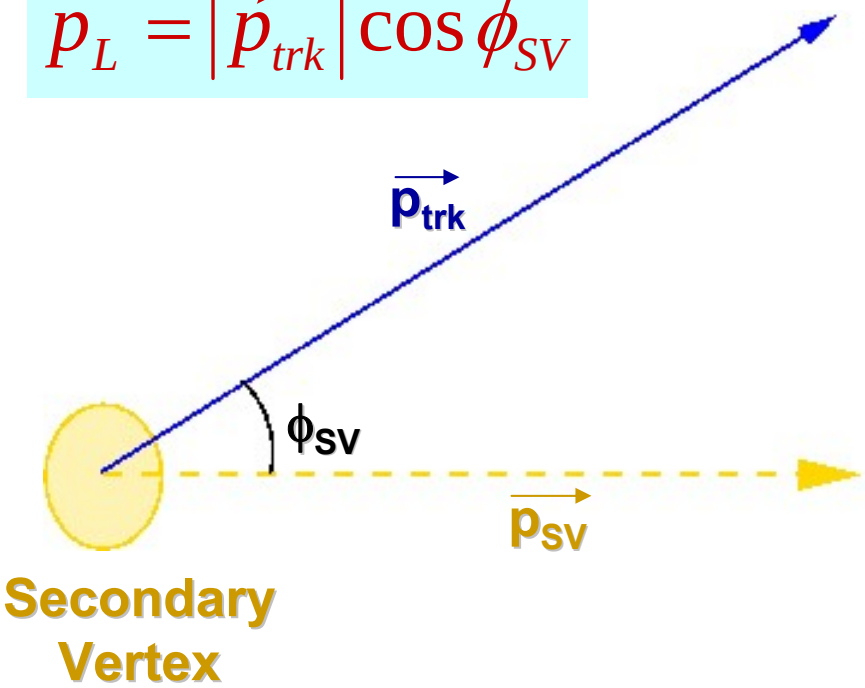
Flavor Tagging Discriminants



- $p_T(\mu)$
- $p_T^{rel}(\mu) = |p_\mu| \sin\phi$
- $b_\mu / \sigma(b)$

Flavor Tagging Discriminants

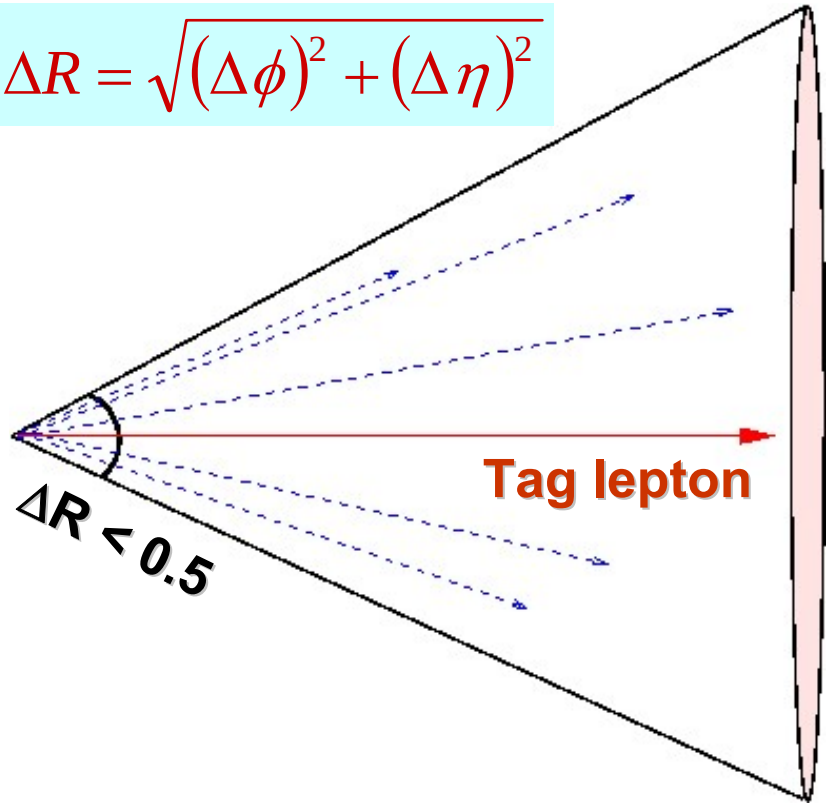
$$p_L = \left|\vec{p}_{trk}\right| \cos\phi_{SV}$$



- $p_T(\mu)$
- $p_T^{rel}(\mu) = \left|p_\mu\right| \sin\phi$
- $b_\mu / \sigma(b)$
- $Q_{SV} = \dfrac{\sum \left(q^i p_L^i\right)^{0.6}}{\sum \left(p_L^i\right)^{0.6}}$

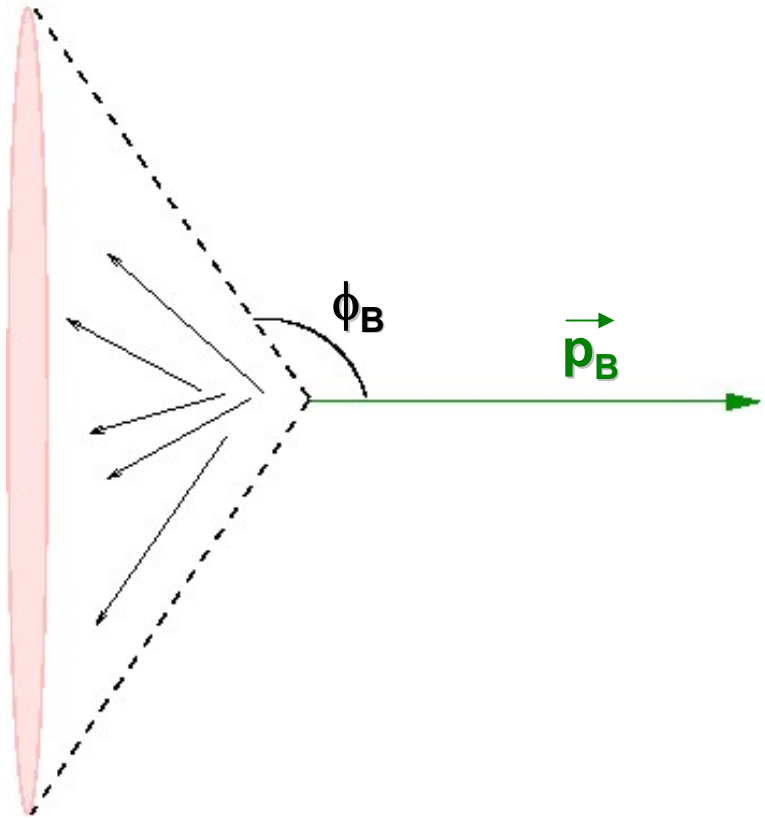
Flavor Tagging Discriminants

$$\Delta R = \sqrt{(\Delta\phi)^2 + (\Delta\eta)^2}$$



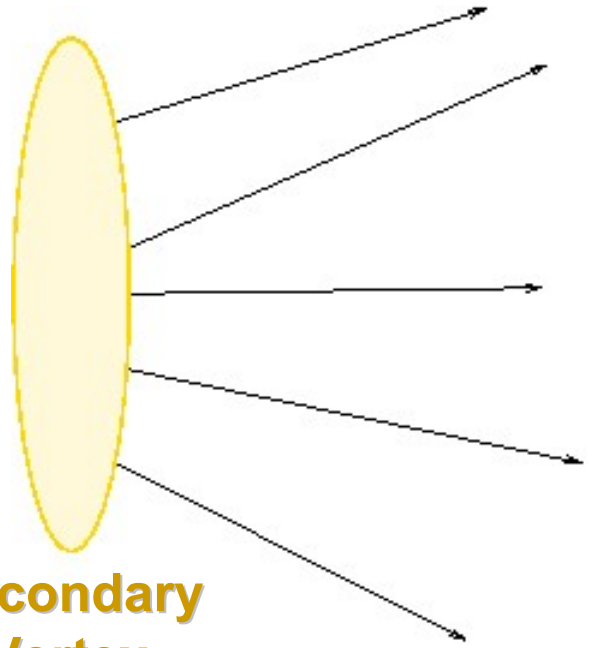
- $p_T(\mu)$
- $p_T^{rel}(\mu) = |p_\mu| \sin\phi$
- $b_\mu / \sigma(b)$
- $Q_{SV} = \dfrac{\sum (q^i p_L^i)^{0.6}}{\sum (p_L^i)^{0.6}}$
- $Q_{Jet}^l = \dfrac{\sum_{\Delta R<0.5} q^i p_T^i}{\sum_{\Delta R<0.5} p_T^i}$

Flavor Tagging Discriminants



- $p_T(\mu)$
- $p_T^{rel}(\mu) = |p_\mu| \sin\phi$
- $b_\mu / \sigma(b)$
- $Q_{SV} = \dfrac{\sum \left(q^i p_L^i\right)^{0.6}}{\sum \left(p_L^i\right)^{0.6}}$
- Q_{Jet}^l
- $Q_{EV} = \dfrac{\sum_{\cos\phi_B < 0.8} q^i p_T^i}{\sum_{\cos\phi_B < 0.8} p_T^i}$

Flavor Tagging Discriminants



- $p_T(\mu)$
- $p_T^{rel}(\mu) = |p_\mu| \sin\phi$
- $b_\mu / \sigma(b)$
- $Q_{SV} = \dfrac{\sum \left(q^i p_L^i\right)^{0.6}}{\sum \left(p_L^i\right)^{0.6}}$
- Q_{Jet}^l
- $Q_{EV} = \dfrac{\sum_{\cos\phi_B < 0.8} q^i p_T^i}{\sum_{\cos\phi_B < 0.8} p_T^i}$
- $p_T(SV) = \left(\sum_{SV} \vec{p}_i\right)_T$

Likelihood-Based Flavor Tagging

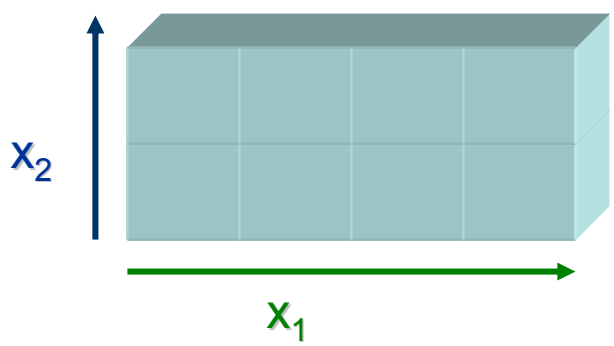
- Define the predicted dilution for a given event as

$$d_{pr} = \frac{L(\overline{B}^0;\vec{x}) - L(B^0;\vec{x})}{L(\overline{B}^0;\vec{x}) + L(B^0;\vec{x})}$$

B^+ MC $\leftrightarrow \mathcal{L}(B^0)$ $\{B^+ = \bar{b}u$ & $B^0 = \bar{b}q_{d,s}\}$

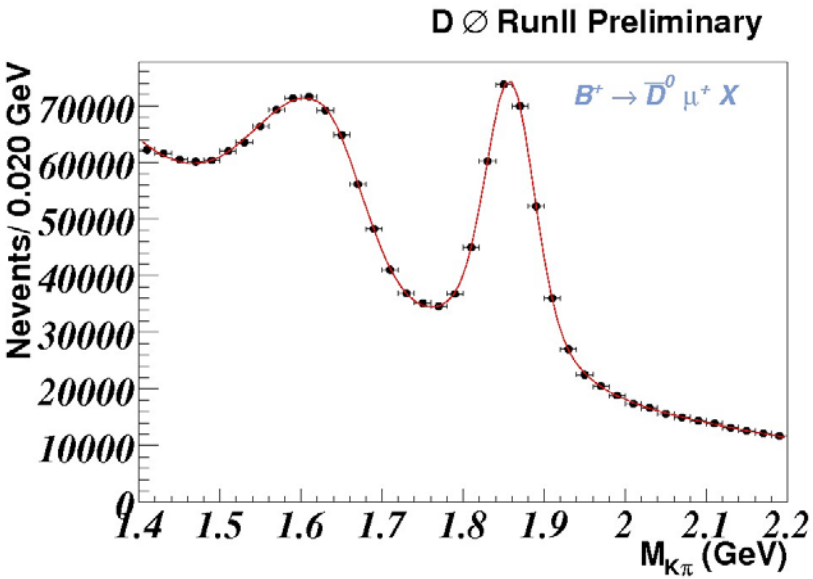
B^- MC $\leftrightarrow \mathcal{L}(\overline{B}^0)$ $\{B^- = b\bar{u}$ & $\overline{B}^0 = b\bar{q}_{d,s}\}$

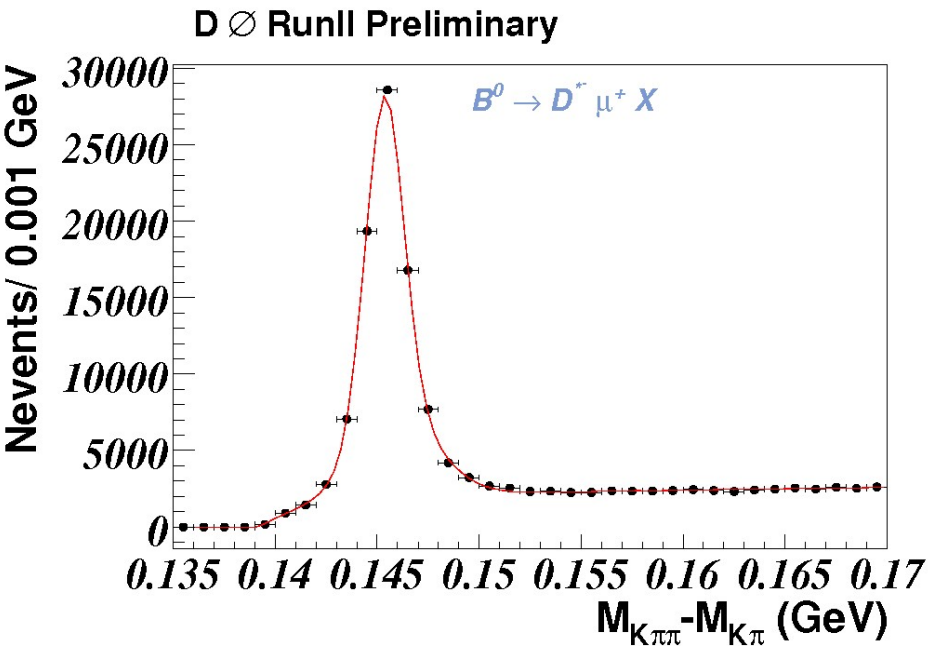
- MC tagger is multidimensional ⇒ $\vec{x}$ is a vector of kinematic discriminants.



- In practice, simply an n-dimensional histogram

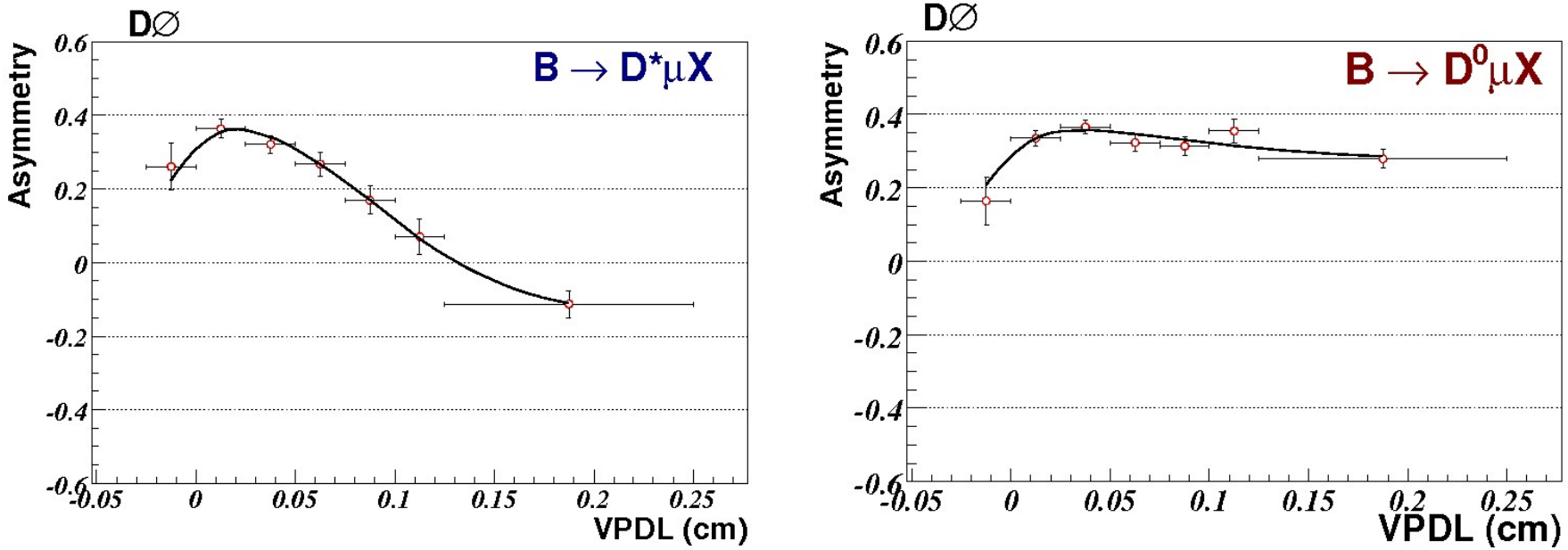
Δm_d: Data Samples





- 2 samples fitted simultaneously for Δm_d.
  - $B^0 \to D^* \mu X$; $D^* \to D^0 \pi$; $D^0 \to K\pi$
  - $B^+ \to D^0 \mu X$; $D^0 \to K\pi$

Δm_d: Fitted Asymmetries



- Binned χ^2 fit to asymmetry:

$$A = \frac{N_{unmixed} - N_{mixed}}{N_{unmixed} + N_{mixed}}$$

Δm_d: Results

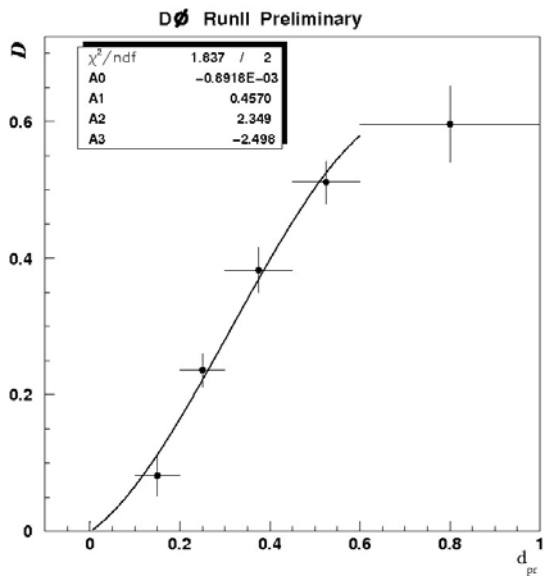
	MC Tagger	Data Tagger
$\mathcal{D}_u$	0.363±0.012	0.419±0.012
$\mathcal{D}_d$	0.395±0.022	0.443±0.022
$\varepsilon\mathcal{D}^2$	(1.90±0.41)%	(2.48±0.21)%
Δm_d (ps^{-1})	0.486±0.021(stat.)	0.506±0.020(stat.) ±0.016(syst.)

- Δm_d agrees w/ W.A. (0.507±0.004 ps^{-1}) for both taggers.
- Consistent between B^+ & B^0 samples.
  - Taggers independent of reco side.

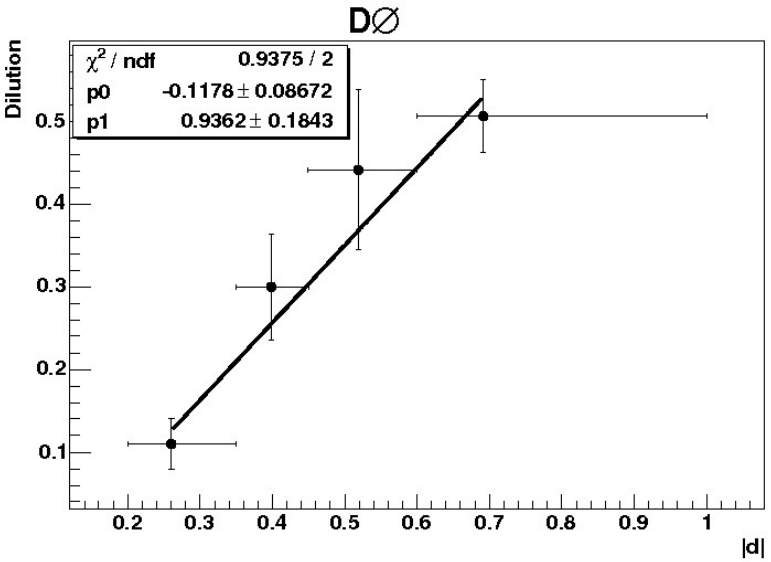
Tagger Calibration

- Measure dilution in Δm_d in bins of tagger output d_{pr}.
  - Get $\mathcal{D}(d_{pr})$ and use in event-by-event Δm_s analysis.
- Multidimensional tagger is more linear, as expected.

Data-Based



MC-Based



Part II:
Searching for Δm_s

New B_s Mixing Channel

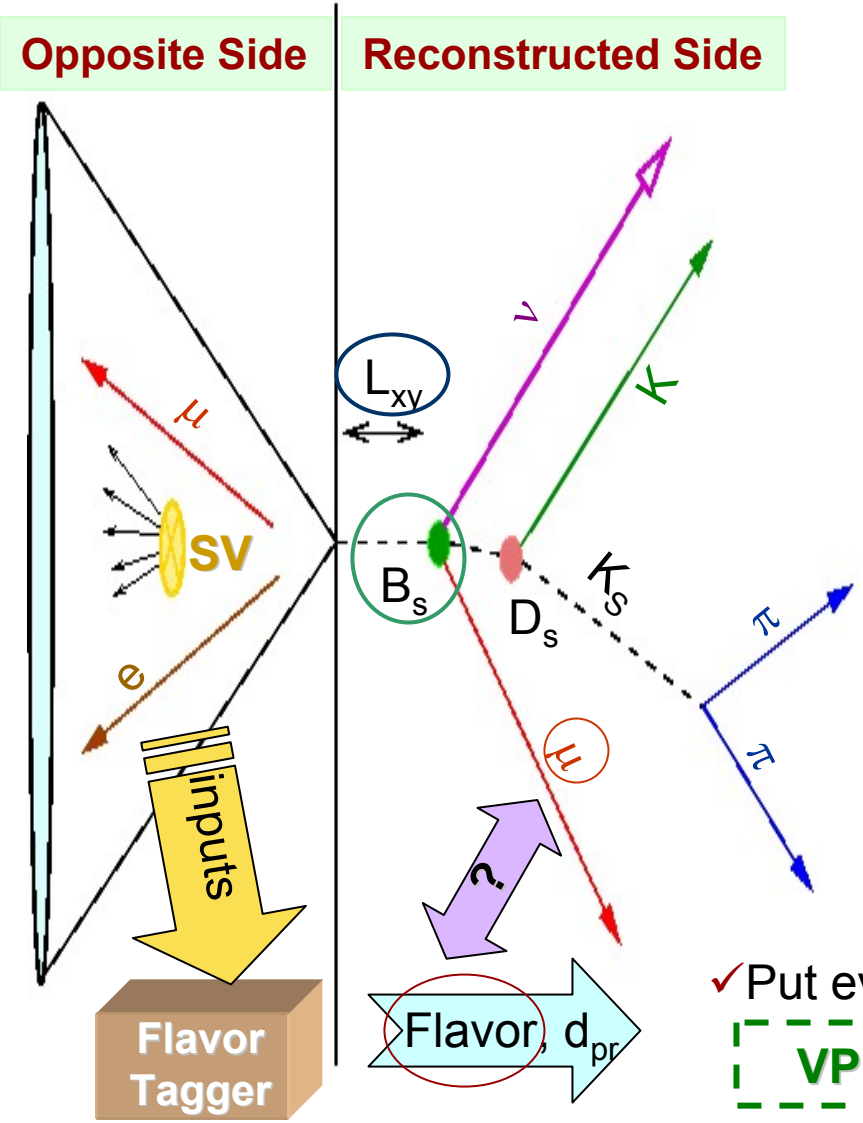
- We have been studying semileptonic decays involving a D_s so far at DØ:
  - semi-muonic ($B_s \rightarrow D_s \mu X$) w/ $D_s \rightarrow \phi\pi$
  - semi-electronic ($B_s \rightarrow D_s e X$) w/ $D_s \rightarrow \phi\pi$
  - semi-muonic ($B_s \rightarrow D_s \mu X$) w/ $D_s \rightarrow K^* K$
- There are 3 main decay modes of the D_s:

1. $D_s \rightarrow \phi\pi$; BR=1.8%
2. $D_s \rightarrow K^* K$; BR=3.3%
3. $D_s \rightarrow K^0_S K$; BR=1.2%

We will add this mode (semi-muonic).

Reco efficiency is lower because $c\tau(K_s)$=2.7 cm

Δm_s: Step-by-step



- ✓ Reconstuct Signal B_s.
- ✓ Flavor at decay given by q(μ).
- ✓ Flavor at production given by opposite-side (OS) flavor tagger.
  - Uses μ, e, and secondary vertices as OS object.
- ✓ Classify event as oscillated or non-oscillated. Is flavor(t_0) = flavor(t_{decay})?
- ✓ Measure *Visible Proper Decay Length* (VPDL) of B candidate,

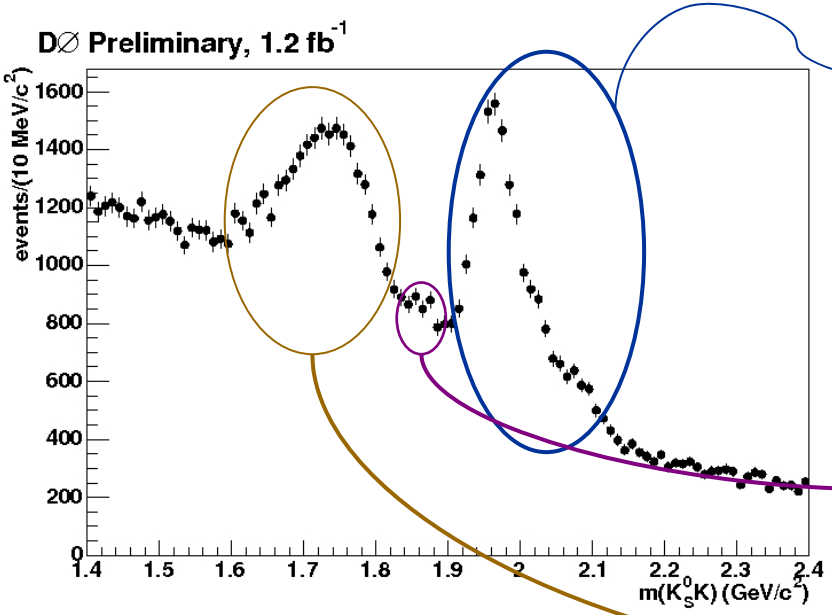
$$VPDL = \left(\vec{L}_{xy} \cdot \vec{p}_{xy}^{D_s\mu}\right) \Big/ \left(p_T^{\mu D_s}\right)^2 \cdot M_{B_s}$$

- ✓ Put event into mixing likelihood. Inputs needed:

VPDL, σ_{VPDL}, d_{pr} (from flavor tagger), $m(D_s)$

Total (Untagged) Data Sample

- $m(K_sK)$ spectrum shows multiple peaks.



$D_s \rightarrow K_sK$ & $D^- \rightarrow K_s\pi$ & $\Lambda_c \rightarrow K_sp$

$D^- \rightarrow K_sK$

$D^- \rightarrow K_s\pi X$

Mass Fitting Procedure

- We have developed a 2-dimensional mass fitting technique to separate the signal D_s candidates from the reflections.
- By Taylor expanding, we can write the mass of a misidentified system as

$$M^2(\lambda) = M_X^2 + \left(\frac{2}{1-\lambda}\right)\left(M_K^2 - M_{trk}^2\right),$$

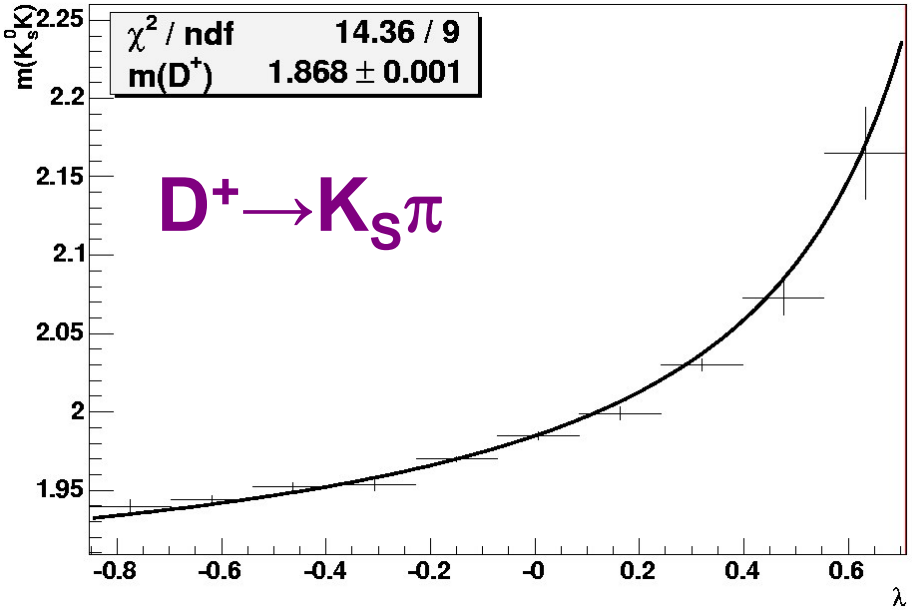
$$\lambda = \frac{P_{K_s} - P_{trk}}{P_{K_s} + P_{trk}}$$

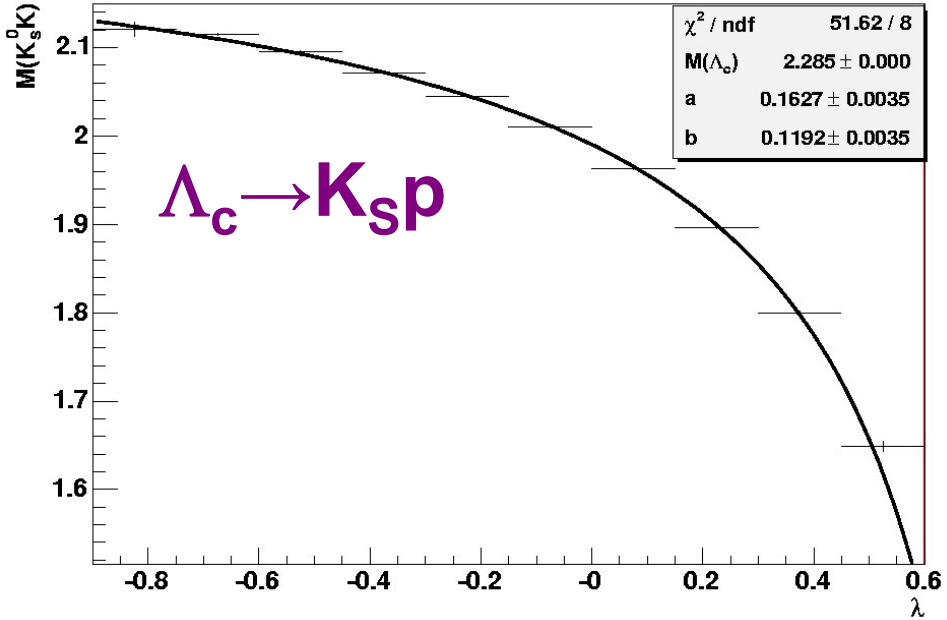
Momentum asymmetry between track & K_S

for decays $X \rightarrow K_S + track$ where M_{trk} is the true track mass.

(e.g. $\boldsymbol{D^- \rightarrow K_S \pi}$ **&** $\boldsymbol{\Lambda_c \rightarrow K_S p}$)

Validity of Analytical Form





- Taylor expansion doesn't work as well for Λ_c because of large proton mass.
  - Put in ad-hoc correction terms:

$$M^2(\lambda) = M^2_{\Lambda_c} + \left(\frac{2}{1-\lambda}\right)\left(M^2_K - M^2_p\right) + a\left(\frac{1+\lambda}{1-\lambda}\right) + b$$

Correction terms

Mass Likelihood

- We end up with a 2-dimensional joint PDF for the mass.

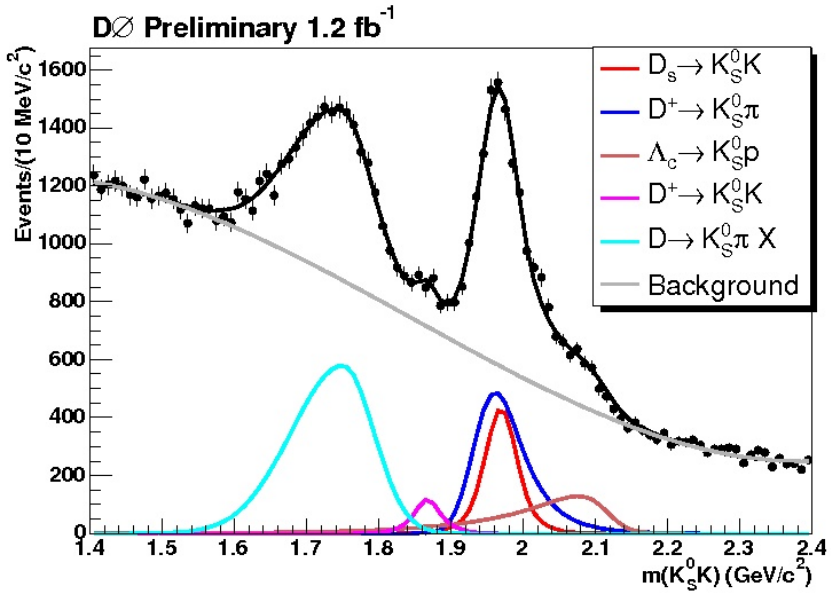
$$L_i^{kin}(m \mid \lambda) = P_i(\lambda) \cdot \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{M(K_s^0 K) - M_i(\lambda)}{\sigma} \right)^2 \right]$$

Taken from MC

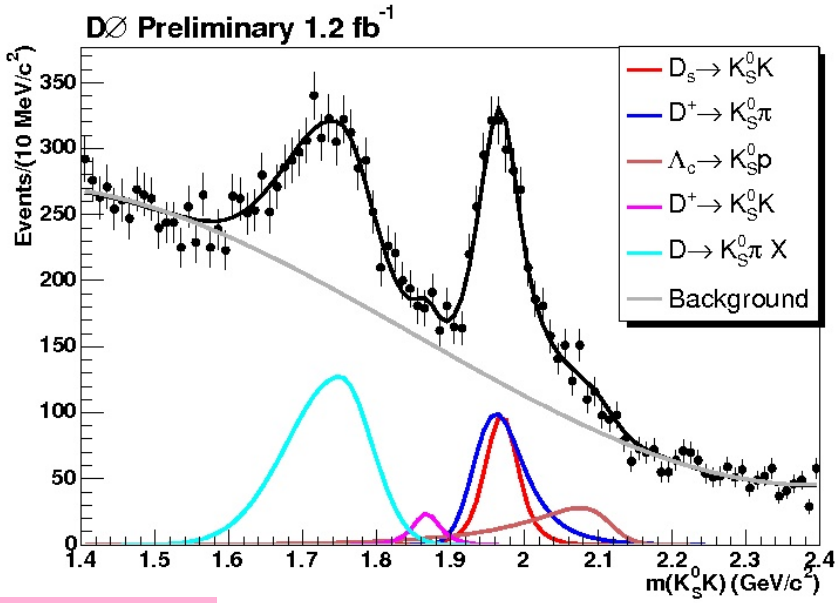
- $D^+ \rightarrow K_S \pi$ & $\Lambda_c \rightarrow K_S p$ have $M_i(\lambda)$.
- Other components' M_i don't depend on λ– free parameters.
  - No misidentified track.

Yields

Total Sample



Flavor Tagged Sample



S/B(3σ) =0.16

Component	N(Untagged)	N(Tagged)
$D_s \to K_S K$	2603 ± 110	593 ± 67
$D^+ \to K_S \pi$	4481 ± 106	914 ± 64
$\Lambda_c \to K_S p$	2244 ± 86	490 ± 42
$D^{(0,-,*)} \to K_S \pi X$	8314 ± 187	1828 ± 112

Unbinned Likelihood

- We minimize $\mathcal{L}$ = -2 ln$\mathcal{P}$,

where $\mathcal{P}=\Pi(P_{sig}*f_{sig} + (1-f_{sig})*P_{bg})$.

- For a given event the probability is given by:

$$P_i = P_i^{x^M}(x^M, \sigma_{x^M}, d_{pr}) P_i^{\sigma_{x^M}} P_i^{d_{pr}} P_i^m(m,\lambda) P_i^{\lambda} P_i^y$$

VPDL(x^M) PDF:

Cond. Variables – σ_x,d_{pr}

Mass PDF:

Cond. Variable – λ

Selection Variable PDF

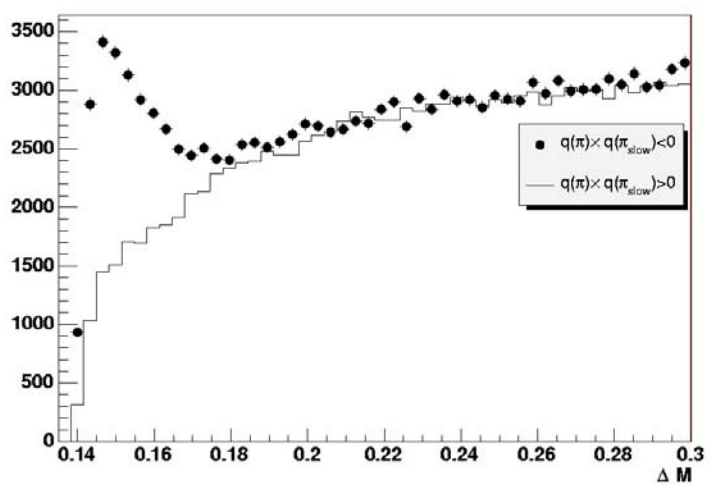
Output variable of flavor tagger

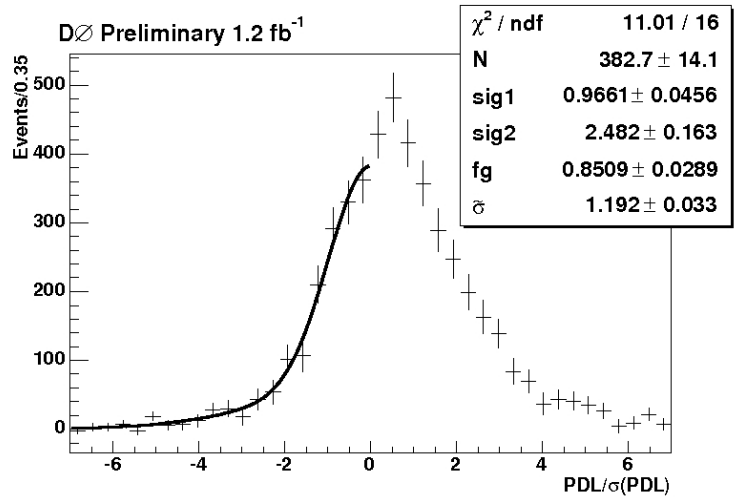
Resolution Scale Factor

- The per-event VPDL error, σ_x, needs to be scaled by a scale factor.
- We determine this scale factor using $D^{*+} \to D^0\pi^+; D^0 \to K_s^0\pi^-\mu^+X$ decays.
  - decay chain has a K_S.
  - easy to estimate background through charge correlation of pion from D^* and pion from D^0
    - $q(\pi_{slow})\times q(\pi)<0$: right sign correlation (SIGNAL)
    - $q(\pi_{slow})\times q(\pi)>0$: wrong sign correlation (BG)

Scale Factor Results

- Fit the negative tail of the pull, $PDL(D^*)/\sigma_{PDL}(D^*)$, to a double Gaussian.
  - negative side of distribution dominated by reconstruction errors→pull should have width of 1 if errors are correct.



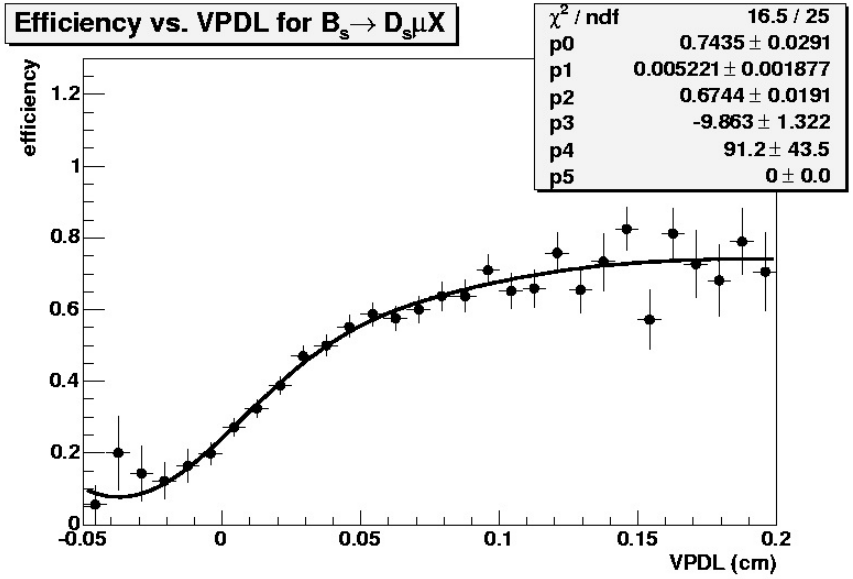


SF=0.967 for 85% of events

SF=2.482 for 15% of events

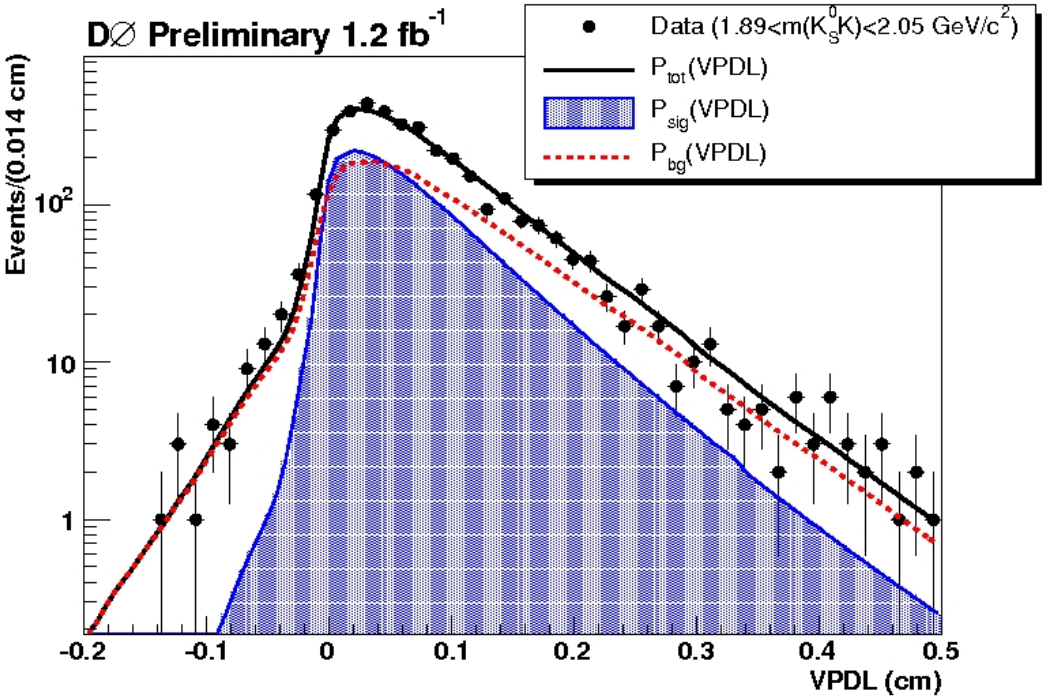
Efficiency vs. VPDL

- We use lifetime-dependent cuts to improve S/B.
- Use MC to get the efficiency as a function of VPDL for all sources.



- ε_{max} = 74.4% compare to $\varepsilon_{max}(\phi\pi\mu)$ = 98.8%
  - effect of long-lived K_S

Lifetime Fit



- $c\tau_{Bs}$=498 ± 39 μm.
- 1.5σ from WA.
- Fixed to WA as systematic error.

- Fit for lifetime of sample to set it as an input to oscillation fit and to verify fitting procedure.

Amplitude Method

- Add a term to the oscillation probability,

$$P_{u,m}(t)=0.5\times\Gamma\exp(-\Gamma t)[1\pm\mathcal{A}\mathcal{D}(d_{pr})\cos(\Delta m_s t)]$$

- Scan Δm_s and fit for $\mathcal{A}$.
- The 95% CL limit is given by the value of Δm_s where fitted $\mathcal{A}$ could fluctuate to $1\rightarrow\mathcal{A}(\Delta m_s)+1.645\sigma_{\mathcal{A}}(\Delta m_s)=1$.
- The sensitivity is given by the value of Δm_s where $\mathcal{A}$=0 could fluctuate to $1\rightarrow 1.645\sigma_{\mathcal{A}}=1$.

Expected Sensitivity

- Using $Sig(\Delta m_s) = \sqrt{\frac{\varepsilon D^2}{2}} \frac{S}{\sqrt{S+B}} e^{-(\Delta m_s \sigma_t)^2/2}$

we compute an expected upper limit on the sensitivity by assuming errors are linear w.r.t. Δm_s and $\sigma_t(\phi\pi)=\sigma_t(K_sK)$.

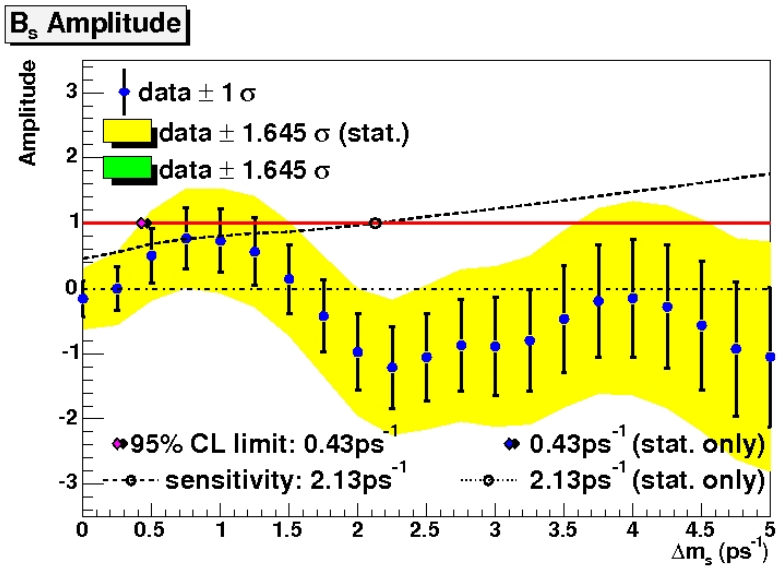
- Time resolution is actually worse in K_sK.

$$Sen(K_sK) < \frac{S_{K_sK}}{\sqrt{S_{K_sK}+B_{K_sK}}} \frac{\sqrt{S_{\phi\pi}+B_{\phi\pi}}}{S_{\phi\pi}} \bullet Sen(\phi\pi)$$

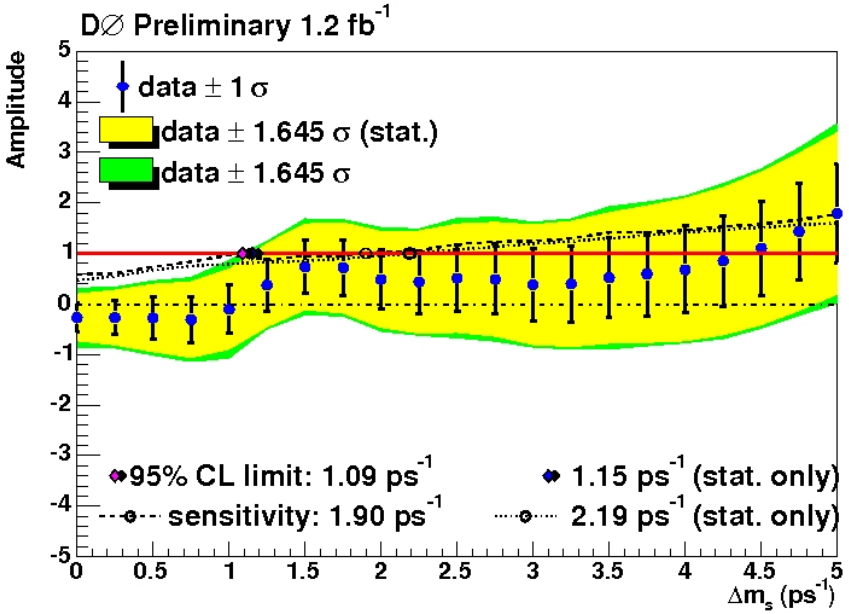
$$= 0.18 \bullet 14.1 ps^{-1} = 2.5 ps^{-1}$$

Fits for Δm_s

Flavor Blind

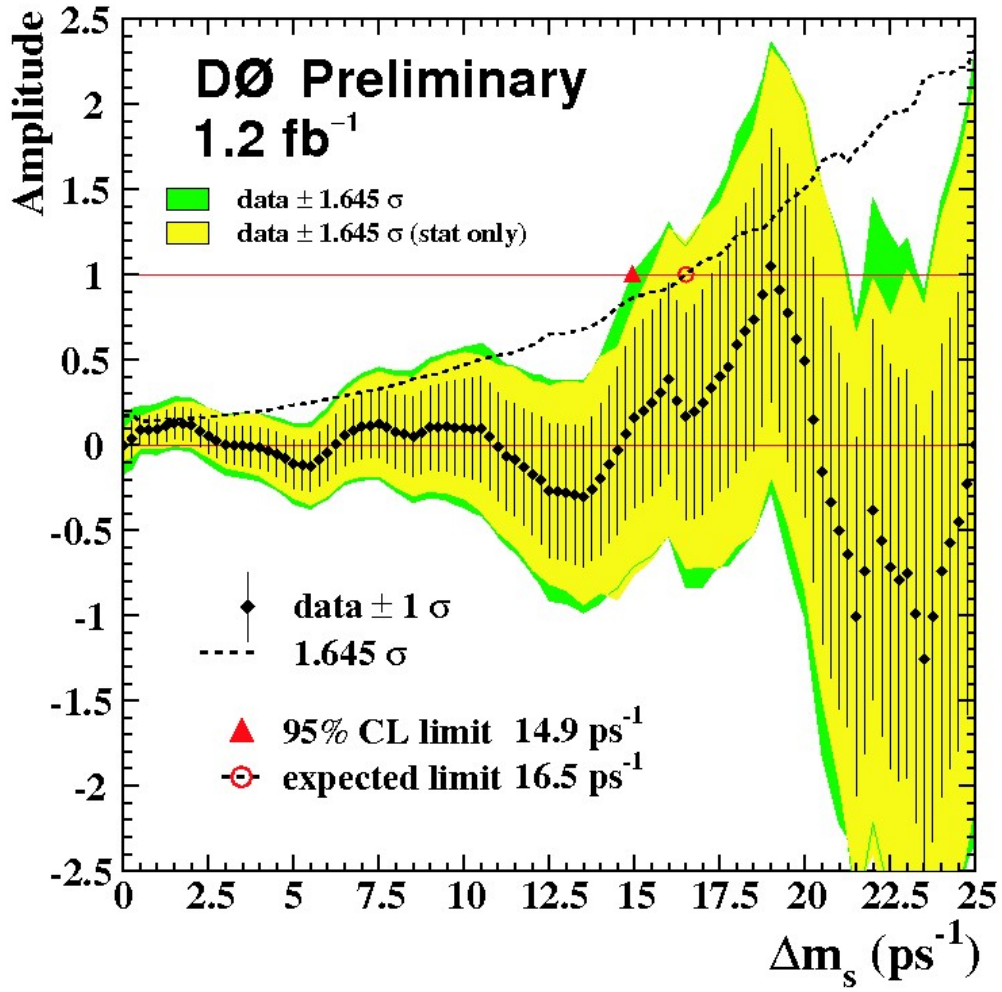


Unblinded



- Sensitivities between blinded and unblinded result are similar and conform to simple expectation given on previous slide.

Combined DØ Result



4 modes in result:

1. $\phi\pi\mu$
2. $\phi\pi e$
3. $K^*K\mu$
4. $K_S K\mu$

95% C.L. = 14.9 ps^{-1}

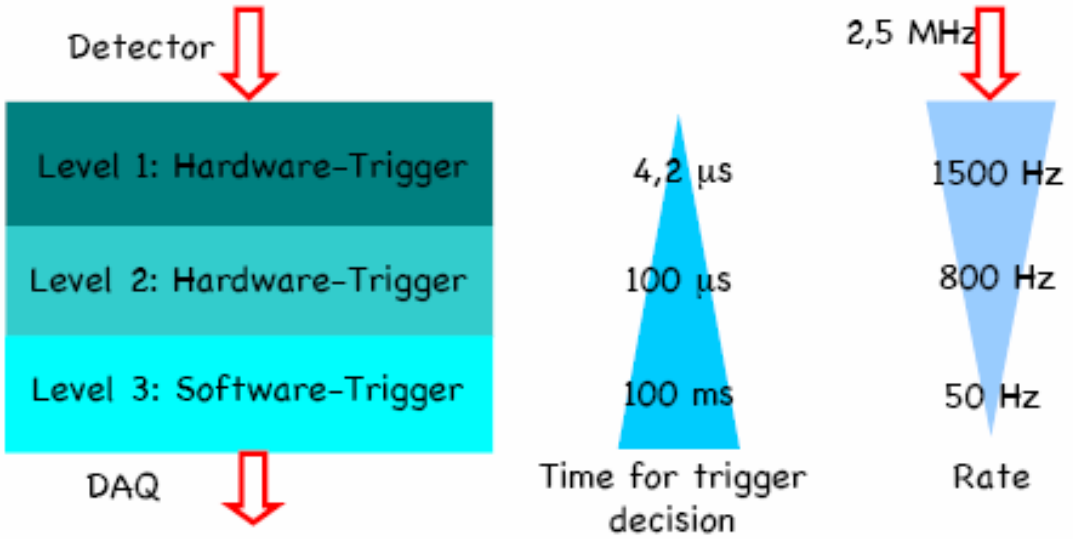
Sensitivity = 16.5 ps^{-1}

Conclusions and Outlook

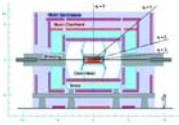
- Calibrated DØ luminosity → important part of Run II LM upgrade.
- We have developed OS flavor tagging and measured Δm_d.
- We have searched for B_s oscillations in a sample of 593 tagged $B_s \rightarrow D_s \mu X$ ($D_s \rightarrow K_S K$) events
  - 95% C.L. = 1.09 ps^{-1}; Sensitivity = 1.90 ps^{-1}
- The combined DØ result is now
  - 95% C.L. = 14.9 ps^{-1}; Sensitivity = 16.5 ps^{-1}
- Future improvements: adding new data, more channels, new silicon, bandwidth upgrade.

BACKUPS

Trigger Rates



Compare: at luminosity: $60*10^{30}$ $cm^{-2}s^{-1}$
→µ-rate from b-pair-production: ≈ 17 kHz



Muon Flavor Tagging

- If event contains μ+SV ⇒ Tag(μ+SV)
  - Tag(μ+SV)={Q_{jet}^{μ}; p_T^{rel}; Q_{SV}}
- If event contains μ without SV ⇒ Tag(μ)
  - Tag(μ)={Q_{jet}^{μ}; p_T^{rel}; p_T; imp. Sig}

Binnings

Nbins(Q_J^{μ})=6: {-1, -0.5, -0.25, 0, 0.25, 0.5, 1}
Nbins(Q_{SV})=6: {-1, -0.5, -0.25, 0, 0.25, 0.5, 1}
Nbins(p_T^{rel})=3: {0, 1.5, 3.5, >3.5}
Nbins(p_T)=3: {0, 5, 10, >10}
Nbins(imp. sig.)=2: {0, 2, >2}

Secondary Vertex Flavor Tagging

- Tag(SV without μ)={Q_{EV}; Q_{SV}; p_T^{SV}}

Binnings

Nbins(Q_{EV})=6: {-1, -0.5, -0.25, 0, 0.25, 0.5, 1}

Nbins(Q_{SV})=6: {-1, -0.5, -0.25, 0, 0.25, 0.5, 1}

Nbins(p_T^{SV})=3: {0, 5, 10, >10}

Base Selections I

μ

- >3 hits in tracker
- must pass through at least 2 layers of muon detector
- p_T > 2.0 GeV
- |p| > 3.0 GeV

K^0_S

- >3 hits in tracker
- q(track 1)*q(track 2)<0
- 460 MeV < m($\pi\pi$) < 525 MeV
- m(e^+e^-) > 25 MeV
- p_T > 650 MeV
- d_{xy} > 0.3 cm

K

- >3 hits in tracker
- Jet(K)=Jet(μ)
- p_T > 1.5 GeV

Base Selections

D_S

- $\chi^2 < 16$
- l_{xy} sig. > 4
- $\cos(\theta) > 0.9$

$$Iso = \frac{p(\mu D_s)}{p(\mu D_s) + \sum_{\Delta R<0.5} p_i}$$

B_S

- $\chi^2 < 9$
- 2.6 GeV < m(B) < 5.4 GeV
- if $l_{xy}(B) > l_{xy}(D)$: l_{xy} sig.($B \rightarrow D$) < 2
- if $\cos(\theta) < 0.95$: l_{xy} sig.(B) < 4
- $q(\mu)*q(K)<0$
- Iso > 0.3

Likelihood Ratio Selections I

- We define signal and background regions,
  - S: $1.90 < M(D_s) < 2.02$ GeV, $q(\mu)\times q(K)<0$
  - B: $1.90 < M(D_s) < 2.02$ GeV, $q(\mu)\times q(K)>0$,

and construct likelihood ratios using the discriminants:

$$y = \frac{PDF_{bg}(x)}{PDF_{sig}(x)}$$

- $p_T(K)$
- $p_T(K_s)$
- $l_{xy}(K_s)$
- $m(\pi_1,\pi_2)$
- $\chi^2(D_s)$
- Iso(B)
- $m(\mu D_s)$

Likelihood Ratio Selections II

- All ratios are combined using

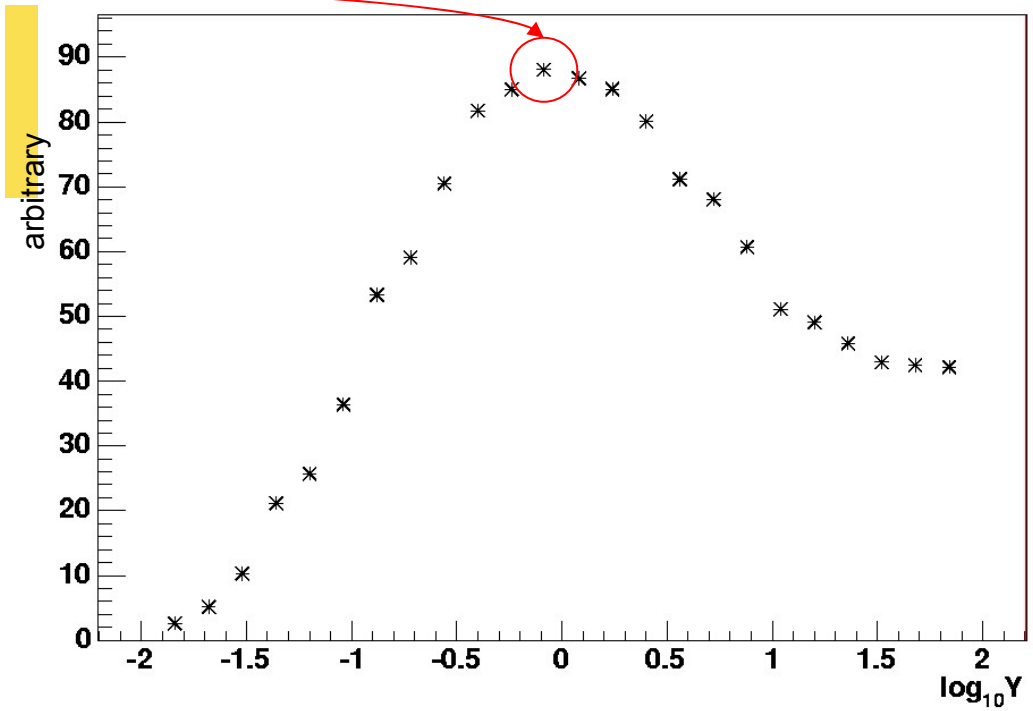
$$Y = \prod_i y_i$$

  - final cut is on $\log_{10}Y$

maximize S/B

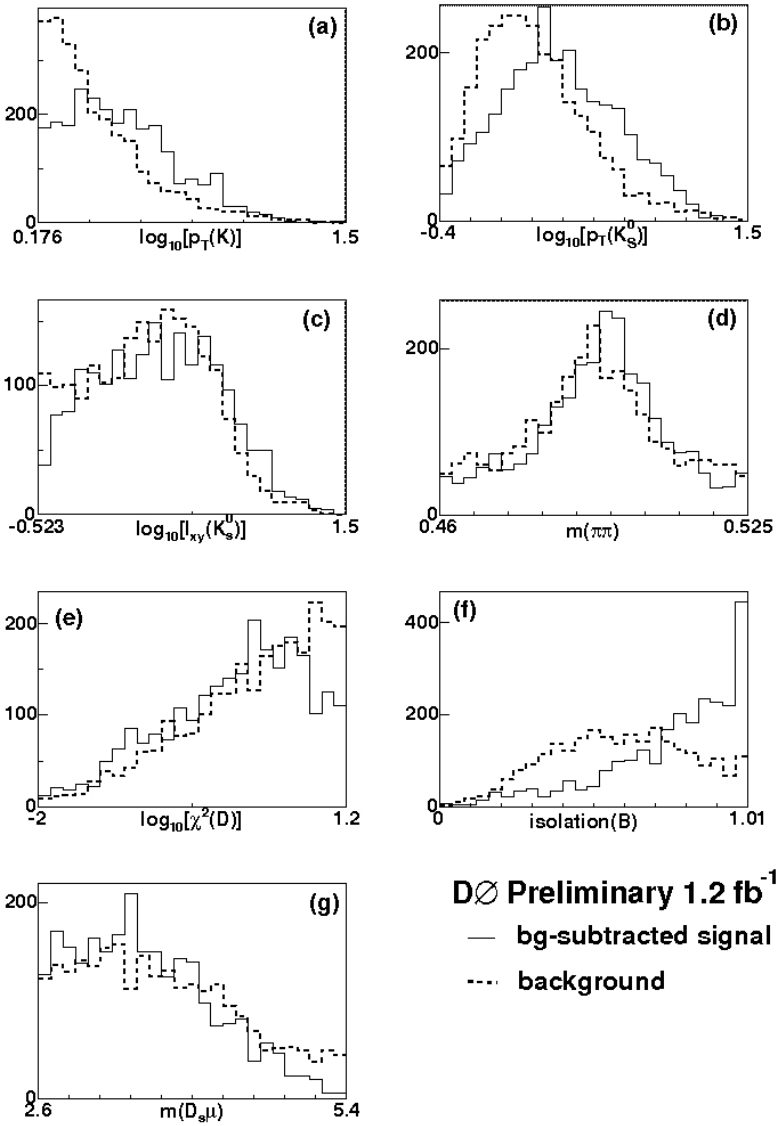
Final Cut

$\log_{10}Y < -0.08$

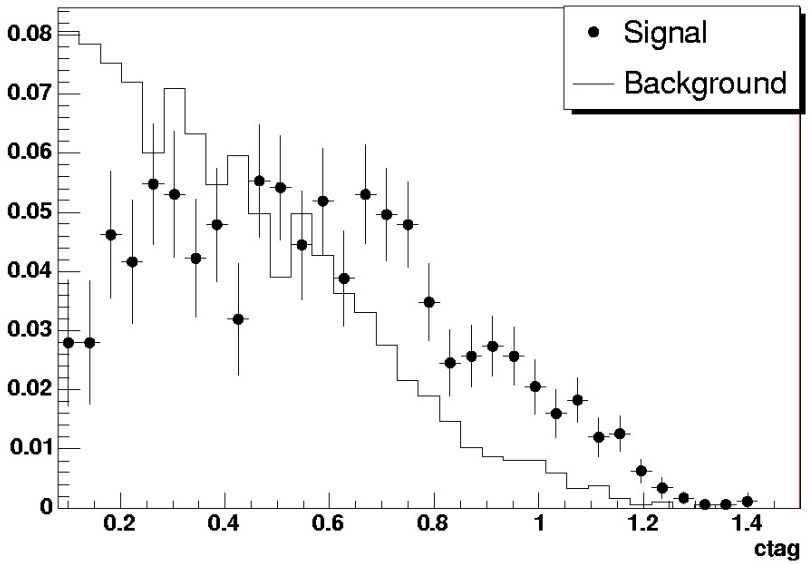


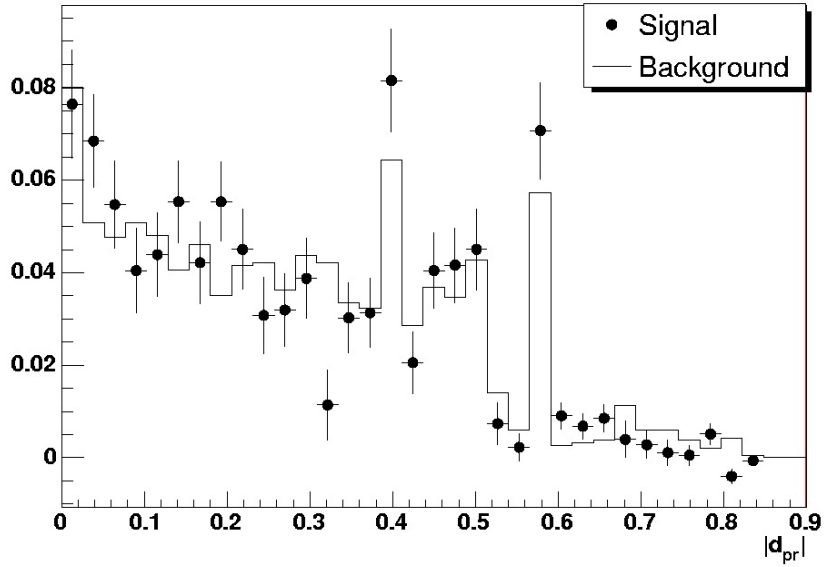
S/B increases from 0.06 to 0.17 through application of Likelihood Ratio Selections.

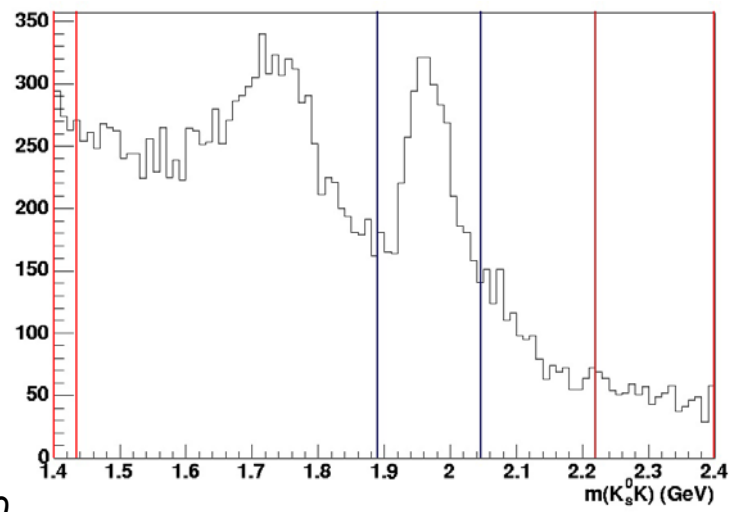
Likelihood Ratio Discriminants

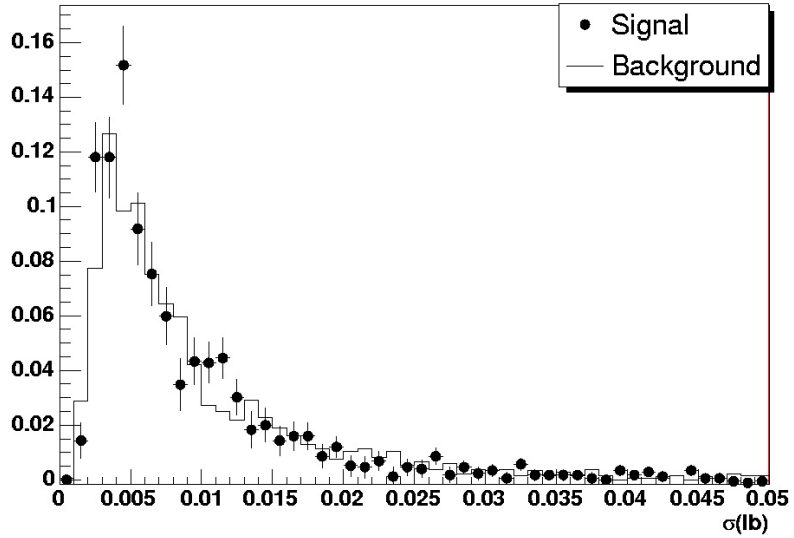


PDF's


Signal
Background
ctag

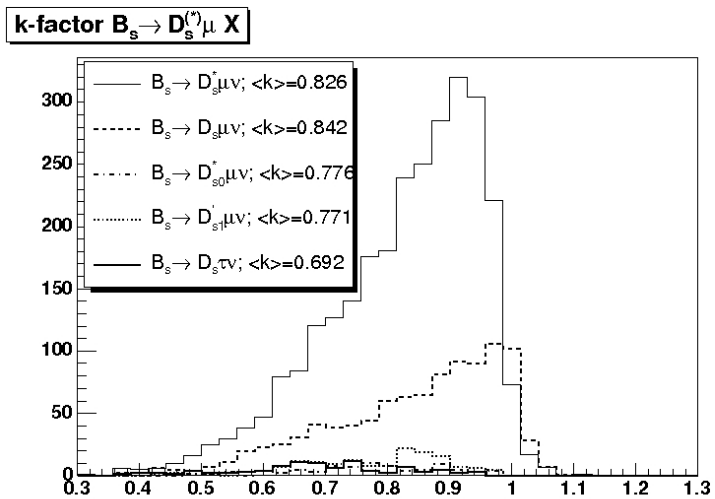

Signal
Background
$|d_{pr}|$


$m(K_s^0K)$ (GeV)


Signal
Background
σ(lb)

K-Factors

- We must correct the measured decay length for the effect of the missing neutrino.

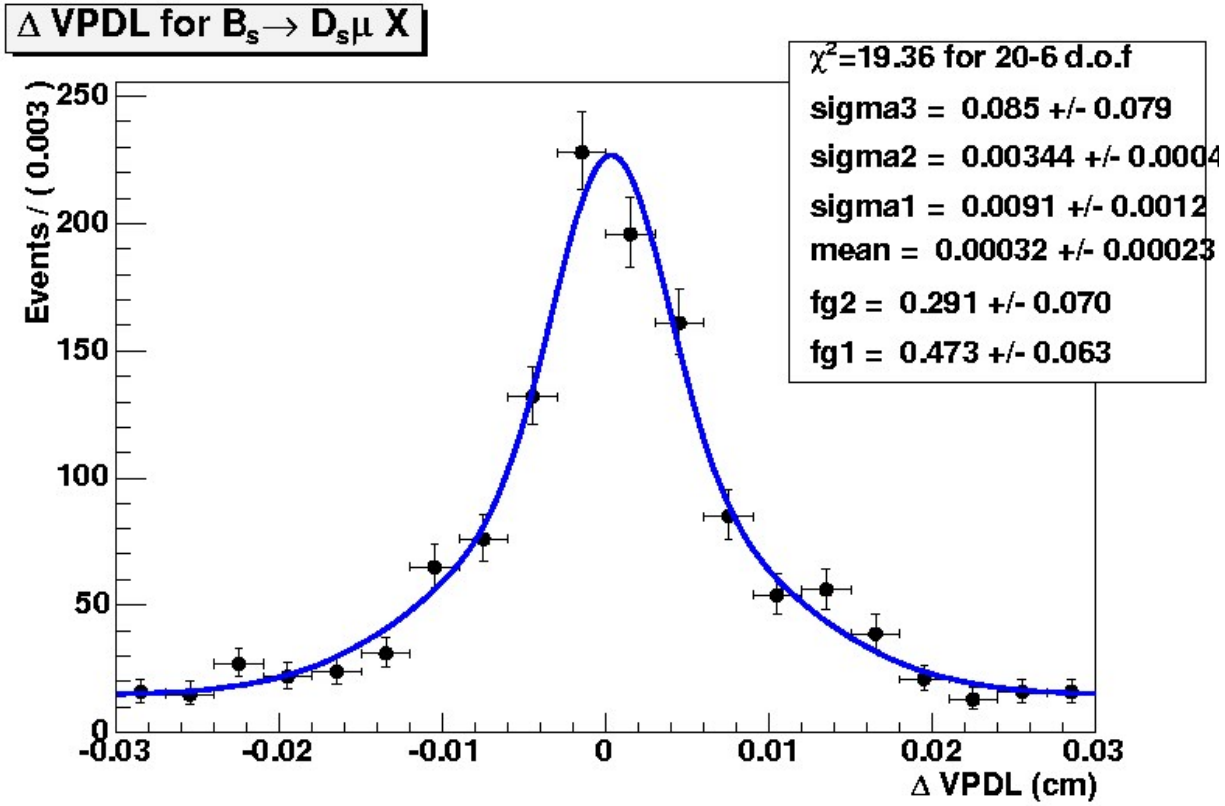


$$VPDL = \left(\vec{L}_{xy} \cdot \vec{p}_{xy}^{\mu D_s}\right) \Big/ \left(p_T^{\mu D_s}\right)^2 \cdot M_{B_s}$$

$$K = p_T^{D_s \mu} \Big/ p_T^{B_s} \qquad c\tau_{B_s} = VPDL \cdot K$$

- Obtain K-factors from MC in 4 bins of $M(\mu D_s)$.

VPDL Resolution



σ(weighted)=254 μm

Background Probability

The following components were used for the background x^M PDF:

1. Quasi-vertices distributed around the primary vertex.
   → *$c\bar{c}$ fake vertices*
2. Negative exponential to account for outliers in the negative x^M tail.
3. Long-lived background insensitive to tagging.
   → *B decay product combinatorics*
4. Non-oscillating long-lived background sensitive to tagging.
   → *Charged B mesons*
5. Long-lived background sensitive to tagging and oscillating at Δm_d.
   → *Neutral B mesons*

Amplitude Method: Introduction

- Time-dependent oscillation signal.

Fourier transform

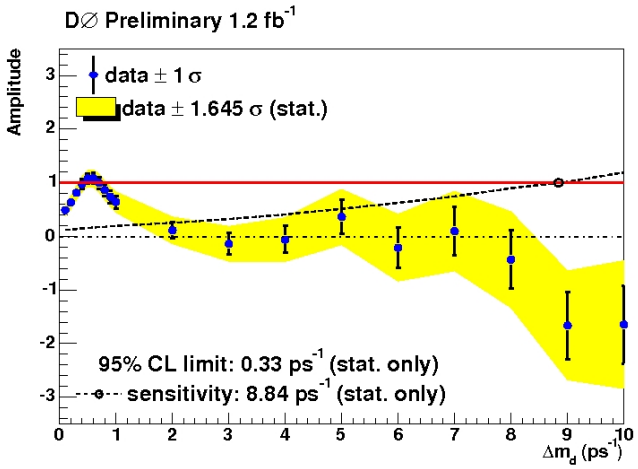
$$FT[f(t)] = g(\nu) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(t) e^{-i\nu t} dt$$

$f(t)$	$\mathrm{Re}[g(\nu)]$
$\cos(\omega t)$	$\sqrt{\frac{2}{\pi}}\delta(\nu - \lvert\omega\rvert)$
$\Gamma \exp(-\Gamma t)$	$\sqrt{\frac{2}{\pi}}\frac{\Gamma^2}{\Gamma^2+\nu^2}$
$\Gamma \exp(-\Gamma t)\cos(\omega t)$	$\frac{1}{\sqrt{2\pi}}\left(\frac{\Gamma^2}{\Gamma^2+(\omega-\nu)^2} + \frac{\Gamma^2}{\Gamma^2+(\omega+\nu)^2}\right)$
$\frac{1}{\sqrt{2\pi}\sigma}\exp\left(\frac{t^2}{2\sigma^2}\right)$	$\exp\left(-\frac{\nu^2\sigma^2}{2}\right)$

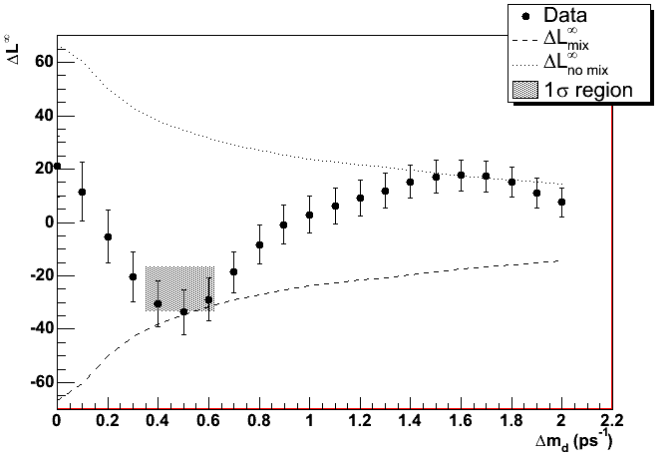
Breit-Wigner

Fits for Δm_d

- We have 3 components oscillating at Δm_d:
  - *$D^+ \rightarrow K_S K^+, D^+ \rightarrow K_S \pi^+$, & long-lived background component (#5 on slide 38).*



Convert to $\Delta\mathcal{L}^\infty$



Verifies correct dilution, signal separation, ability to detect oscillation signal.

$\Delta m_d = 0.50 \pm 0.13 \rightarrow$ agrees w/ W.A.

Sample Composition

$B_s \rightarrow D_s \mu \nu$	20.9%
$B_s \rightarrow D^*_s \mu \nu$	57.6%
$B_s \rightarrow D^*_{s0} \mu \nu$	1.45%
$B_s \rightarrow D_{s1} \mu \nu$	3.32%
$B_s \rightarrow D_s \tau \nu$	1.99%
$B_s \rightarrow D_s D_s X$	0.99%
$B_s \rightarrow D_s D X$	1.6%
$B^0 \rightarrow D_s D X$	6.35%
$B^- \rightarrow D_s D X$	5.77%

83% signal

Effect of increased $p_T(\mu)$ from trigger turn-on is considered in systematic errors.

Systematics Table

TABLE I: Systematic uncertainties on the amplitude. The shifts of both the measured amplitude, $\Delta\mathcal{A}$,and its statistical uncertainty, $\Delta\sigma$, are

Osc. frequency (ps^{-1})		0.00	0.50	1.00	1.50	2.00	2.50	3.00	3.50	4.00	4.50	5.00
$\mathcal{A}$		−0.284	−0.282	−0.188	0.771	0.432	0.539	0.358	0.543	0.668	1.130	1.934
Stat. uncertainty		0.281	0.420	0.489	0.528	0.595	0.678	0.754	0.829	0.908	0.980	1.036
Dilution	$\Delta\mathcal{A}$	−0.013	−0.034	−0.006	−0.022	+0.002	+0.002	−0.003	+0.008	+0.049	+0.069	+0.085
	$\Delta\sigma$	−0.002	−0.003	−0.004	−0.003	−0.003	−0.004	−0.005	−0.008	−0.009	−0.009	−0.010
Scale Factor	$\Delta\mathcal{A}$	−0.002	+0.009	+0.027	+0.024	+0.007	−0.014	−0.029	−0.038	−0.038	−0.030	−0.018
	$\Delta\sigma$	−0.000	+0.000	−0.000	−0.000	−0.000	−0.001	−0.002	−0.004	−0.003	−0.002	−0.000
$\mathrm{Br}(B_s \to D_s mu) = 5.5\%$	$\Delta\mathcal{A}$	+0.014	+0.018	−0.002	+0.020	+0.011	+0.014	+0.009	+0.017	+0.021	+0.039	+0.069
	$\Delta\sigma$	+0.010	+0.015	+0.019	+0.020	+0.023	+0.026	+0.030	+0.033	+0.036	+0.039	+0.041
$Br(B_s \to D_s D_s) = 5.5\%$	$\Delta\mathcal{A}$	+0.012	+0.016	−0.003	+0.024	+0.014	+0.017	+0.011	+0.020	+0.025	+0.044	+0.079
	$\Delta\sigma$	+0.012	+0.018	+0.021	+0.023	+0.026	+0.030	+0.033	+0.037	+0.040	+0.044	+0.047
$Br(B_s \to D_s D_s) = 23\%$	$\Delta\mathcal{A}$	−0.008	−0.009	−0.003	+0.014	+0.010	+0.010	+0.007	+0.009	+0.012	+0.020	+0.034
	$\Delta\sigma$	+0.005	+0.008	+0.009	+0.009	+0.010	+0.012	+0.013	+0.014	+0.016	+0.017	+0.018
$c\bar{c}$: 4.62%	$\Delta\mathcal{A}$	−0.002	+0.001	+0.005	+0.018	+0.010	+0.007	+0.002	+0.002	+0.002	+0.007	+0.019
	$\Delta\sigma$	+0.003	+0.005	+0.006	+0.006	+0.007	+0.008	+0.009	+0.010	+0.012	+0.013	+0.014
$c\tau_{B_s} = 438\mu m$	$\Delta\mathcal{A}$	−0.016	−0.006	−0.079	+0.049	−0.049	+0.031	−0.024	+0.024	−0.014	+0.034	+0.150
	$\Delta\sigma$	−0.002	+0.014	+0.009	+0.013	+0.020	+0.028	+0.036	+0.046	+0.055	+0.065	+0.069
$p_{T_\mu} > 6$ GeV/c	$\Delta\mathcal{A}$	−0.035	−0.032	−0.082	+0.011	−0.070	+0.011	−0.034	+0.004	−0.038	−0.013	+0.058
	$\Delta\sigma$	−0.016	−0.008	−0.017	−0.014	−0.012	−0.008	−0.005	+0.000	+0.005	+0.010	+0.011
D_s yield $\pm 1.15\sigma$	$\Delta\mathcal{A}$	+0.071	+0.121	−0.035	+0.136	+0.019	+0.115	+0.066	+0.140	+0.079	+0.151	+0.363
	$\Delta\sigma$	−0.028	−0.026	−0.037	+0.067	+0.082	+0.098	+0.117	+0.134	+0.149	+0.166	+0.176
D^+ yield $\pm 1\sigma$	$\Delta\mathcal{A}$	+0.038	+0.079	−0.043	+0.082	−0.095	+0.083	+0.027	+0.084	+0.032	+0.097	+0.267
	$\Delta\sigma$	−0.018	−0.012	−0.021	+0.040	−0.016	+0.063	+0.076	+0.089	+0.102	+0.116	+0.123
k-factor $\pm 2\%$	$\Delta\mathcal{A}$	−0.027	−0.006	−0.108	+0.038	−0.087	+0.030	+0.005	+0.046	+0.050	+0.149	+0.258
	$\Delta\sigma$	−0.003	+0.012	+0.005	+0.011	+0.020	+0.028	+0.026	+0.047	+0.055	+0.063	+0.071
k-factor smoothed	$\Delta\mathcal{A}$	−0.017	−0.009	−0.086	+0.034	−0.055	+0.032	−0.022	+0.028	−0.010	+0.046	+0.153
	$\Delta\sigma$	−0.003	+0.011	+0.006	+0.010	+0.017	+0.025	+0.032	+0.042	+0.050	+0.060	+0.064
Reco k-factor	$\Delta\mathcal{A}$	−0.018	−0.009	−0.089	+0.040	−0.031	+0.059	−0.014	+0.048	+0.045	+0.109	+0.166
	$\Delta\sigma$	−0.003	+0.012	+0.009	+0.015	+0.024	+0.034	+0.044	+0.057	+0.068	+0.079	+0.086
BG Scale Factor = 2.0	$\Delta\mathcal{A}$	−0.018	−0.021	−0.109	+0.030	−0.062	+0.026	−0.028	+0.018	−0.021	+0.027	+0.139
	$\Delta\sigma$	−0.003	+0.015	+0.008	+0.012	+0.018	+0.026	+0.034	+0.044	+0.052	+0.061	+0.065
$frNeg + 1\sigma$	$\Delta\mathcal{A}$	−0.017	−0.010	−0.086	+0.035	−0.056	+0.028	−0.022	+0.026	−0.012	+0.035	+0.143
	$\Delta\sigma$	−0.004	+0.011	+0.006	+0.010	+0.016	+0.023	+0.031	+0.040	+0.048	+0.057	+0.061
$f_{cc}(bg) \pm 1\sigma$	$\Delta\mathcal{A}$	−0.021	−0.003	−0.090	+0.040	−0.063	+0.034	−0.019	+0.027	−0.012	+0.035	+0.144
	$\Delta\sigma$	−0.003	+0.011	+0.006	+0.010	+0.016	+0.023	+0.031	+0.040	+0.048	+0.057	+0.061
frBd $\pm 1\sigma$	$\Delta\mathcal{A}$	−0.060	−0.007	−0.115	+0.045	−0.060	+0.029	−0.022	+0.026	−0.011	+0.034	+0.142
	$\Delta\sigma$	−0.003	+0.012	+0.006	+0.009	+0.016	+0.023	+0.031	+0.041	+0.048	+0.057	+0.060
frMix $\pm 1\sigma$	$\Delta\mathcal{A}$	−0.067	+0.059	−0.111	+0.041	−0.076	+0.048	−0.006	+0.038	−0.001	+0.044	+0.150
	$\Delta\sigma$	−0.003	+0.011	+0.005	+0.009	+0.017	+0.022	+0.029	+0.038	+0.046	+0.055	+0.059
Background Mass Shape	$\Delta\mathcal{A}$	−0.034	−0.023	−0.064	+0.030	−0.047	+0.016	−0.021	+0.053	+0.020	+0.030	+0.100
	$\Delta\sigma$	+0.001	+0.015	+0.009	+0.012	+0.017	+0.031	+0.041	+0.048	+0.049	+0.050	+0.050
$N(D^+ \to K^0_S K)/N(D^+ \to K^0_S \pi) \pm 0.035$	$\Delta\mathcal{A}$	+0.005	+0.022	−0.076	+0.057	−0.074	+0.044	−0.009	+0.042	−0.002	+0.049	+0.178
	$\Delta\sigma$	−0.010	+0.002	−0.005	+0.021	+0.003	+0.037	+0.047	+0.058	+0.067	+0.078	+0.083
Total syst.	σ^{sys}_{tot}	0.216	0.186	0.338	0.260	0.236	0.300	0.211	0.343	0.208	0.270	0.467
Total	σ_{tot}	0.357	0.449	0.589	0.580	0.626	0.720	0.753	0.860	0.885	0.962	1.081