Dalitz Plot Analysis of $D^0 \to \pi^-\pi^+\pi^0$ & Measurement of γ using $B^\pm \to D_{\pi^-\pi^+\pi^0} K^\pm$

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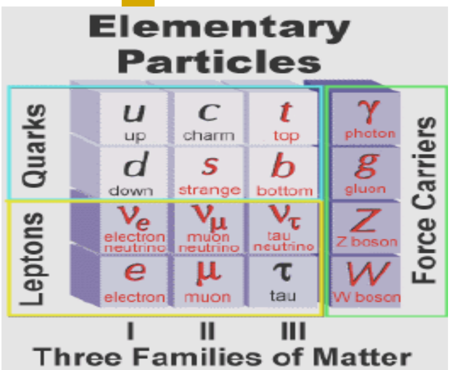
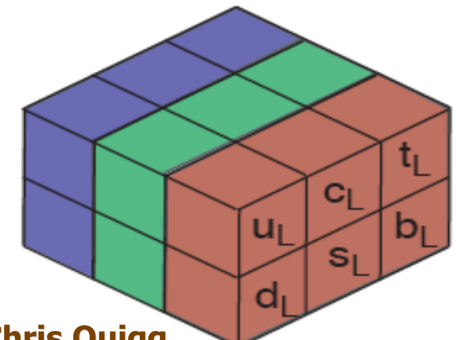
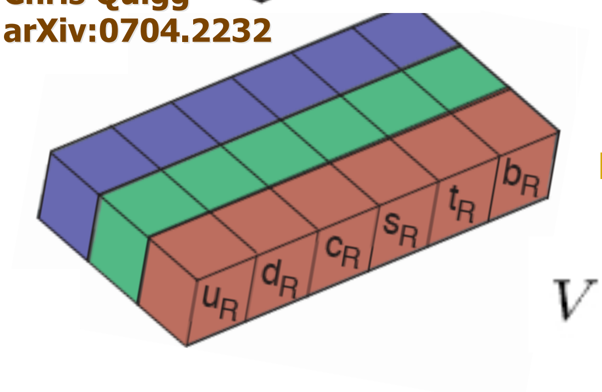
BaBar collaboration

Weak interaction of quarks in SM





Chris Quigg
arXiv:0704.2232

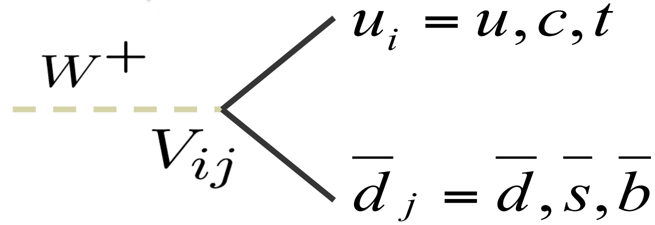


Left handed quarks in doublets $q^i_L = \begin{pmatrix} u^i_L \\ d^i_L \end{pmatrix}$

Right handed quarks in singlets $\Rightarrow$ do not couple to W

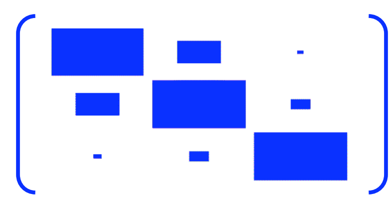
- The electroweak coupling strength of W to left-handed quarks is described by Cabibbo-Kobayashi-Maskawa matrix

$$-\mathcal{L}_{W^\pm} = \frac{g}{\sqrt{2}}\,\overline{u_{Li}}\,\gamma^\mu\,(V_{\rm CKM})_{ij}\,d_{Lj}\,W^+_\mu + \text{h.c.}$$



- 3x3 unitary matrix ==> 4 parameters

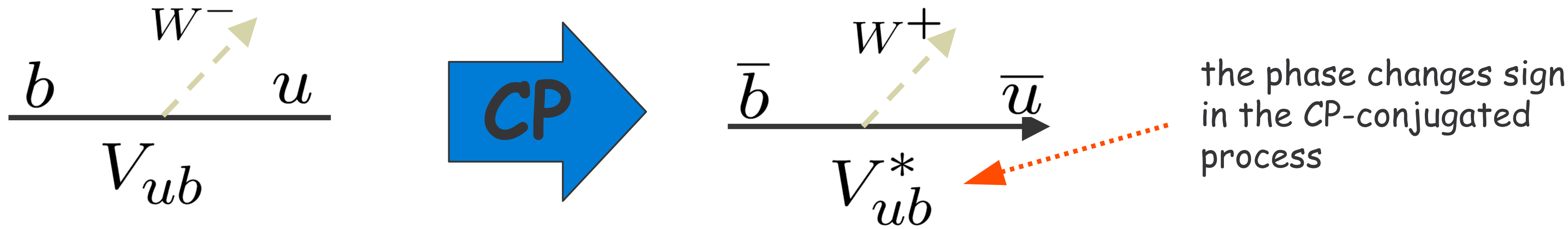
$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} =$$



relative magnitude of the elements

The CKM Matrix

- An irremovable complex phase in V_{CKM} is the origin of CP violation in the SM



- In the Wolfenstein parameterization:

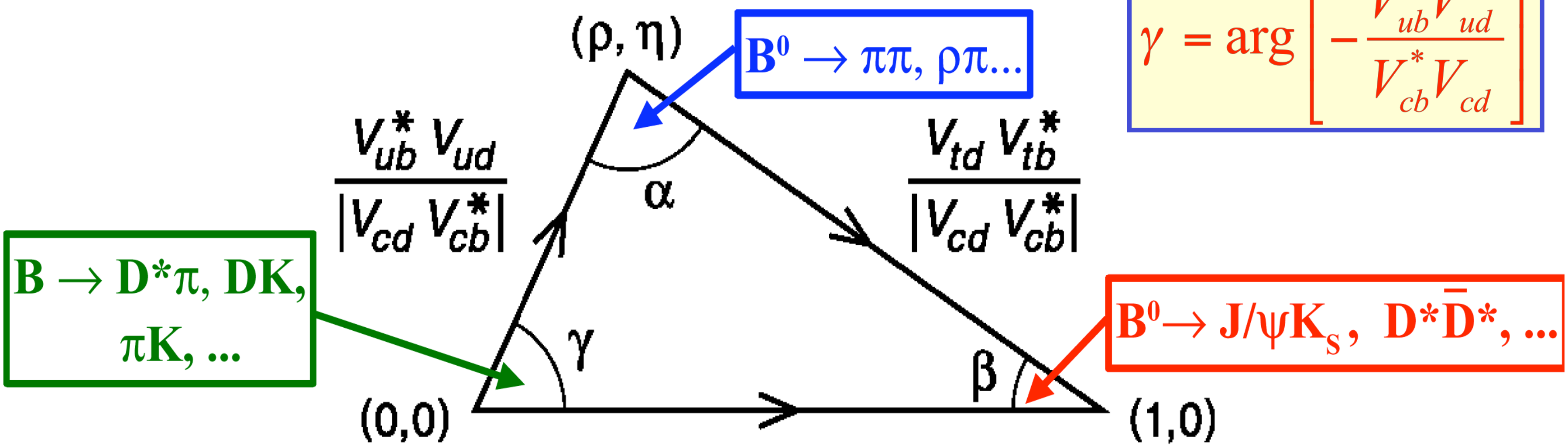
$$V = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

The Unitarity Triangle

- V is unitary: VV⁺=1 $= V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$

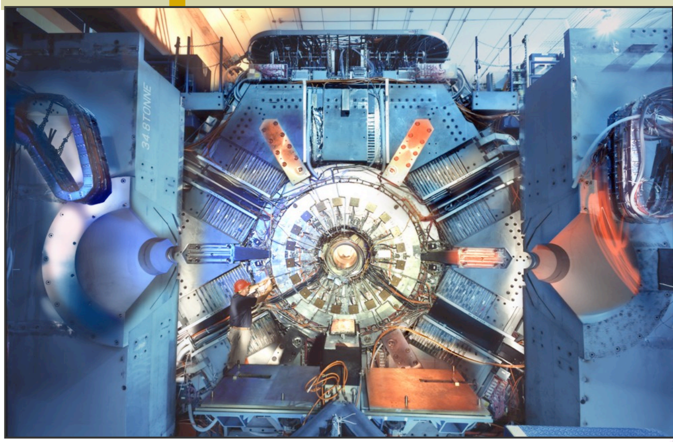
in the complex plane

$$\gamma = \arg\left[-\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}}\right]$$



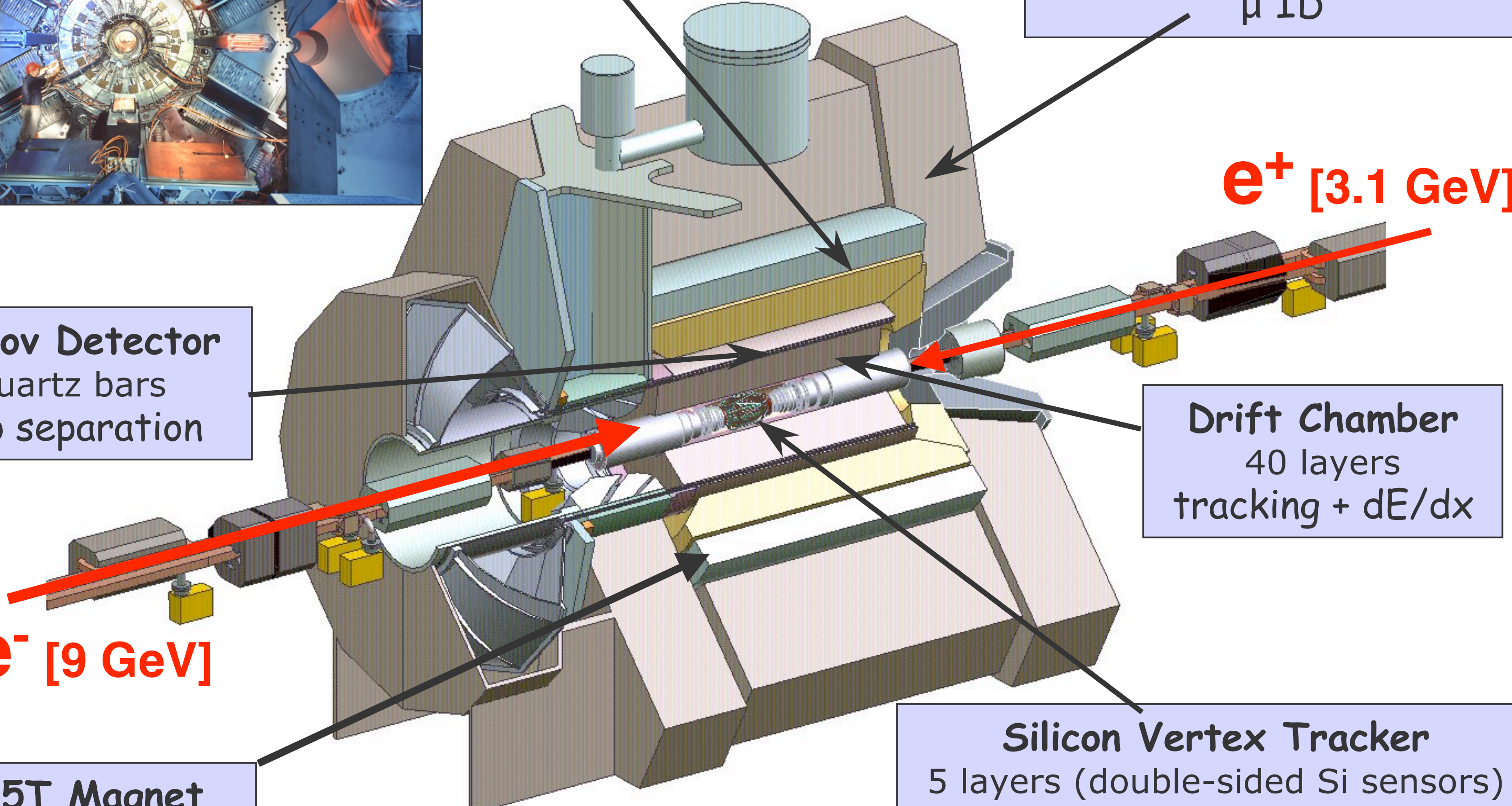
- *Expect* γ to be ~ (60±10)°, *if the Standard Model is consistent*.
- But need to *measure* it directly, need redundant measurements ….
- Several ways to measure γ, no single one of them is “silver bullet” !

BaBar: B and charm Factory



Electromagnetic Calorimeter
6580 CsI crystals
e+ ID, π^0 and γ reco

Instrumented Flux Return
12-18 layers of RPC/LST
μ ID



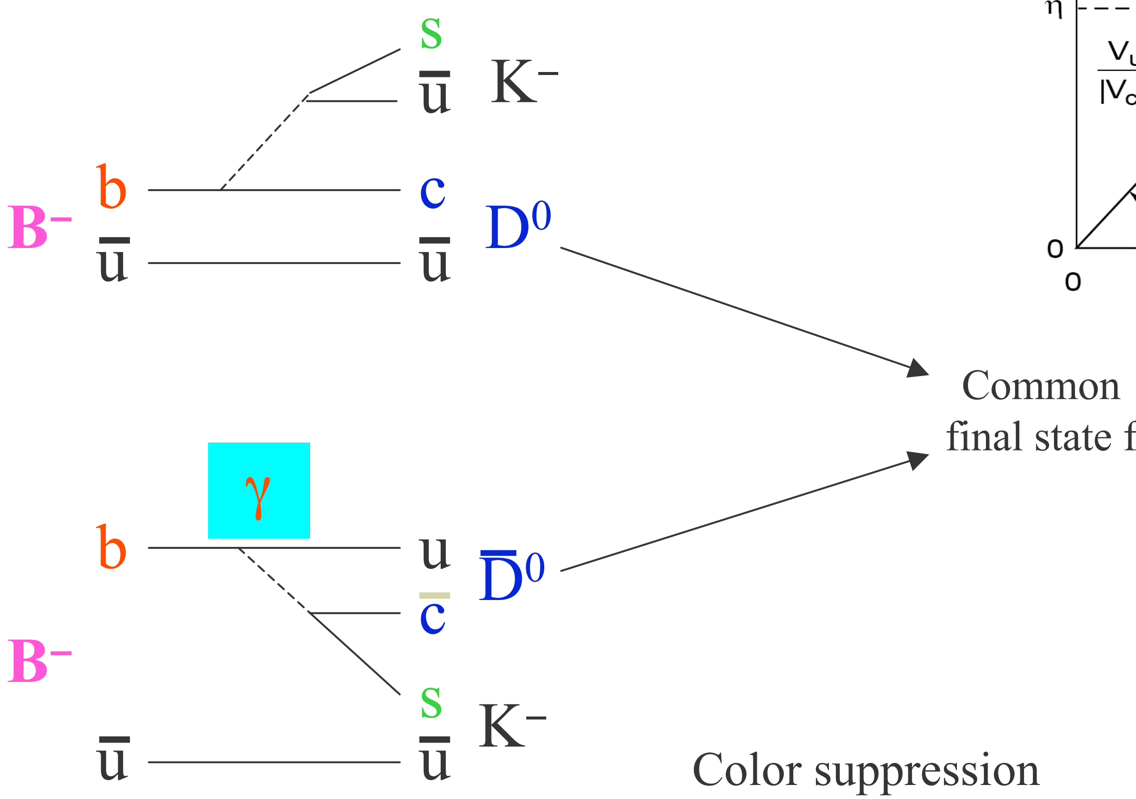
Cherenkov Detector
144 quartz bars
K, π, p separation

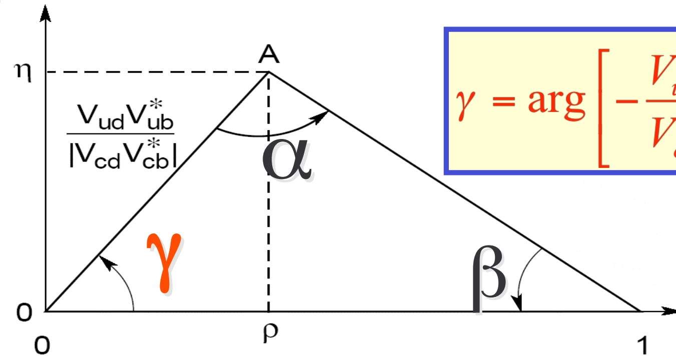
Drift Chamber
40 layers
tracking + dE/dx

1.5T Magnet

Silicon Vertex Tracker
5 layers (double-sided Si sensors)
vertexing + tracking (+ dE/dx)

Extraction of γ with $B \to D^0 K$





$$\gamma = \arg\left[-\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}}\right]$$

Secret to Success: interference between color-allowed D^0K and color-suppressed $\bar{D}^0K$ amplitudes.
Decay time-independent!

Color suppression

$$\text{Magnitude ratio} \equiv r_B \approx \left|\frac{V_{ub}}{V_{cb}} \cdot \frac{V_{cs}}{V_{us}}\right| \cdot \frac{1}{N_{colors}} \approx \frac{\rho - i\eta}{N_{colors}} \approx \frac{0.4}{3} \approx 0.1$$

The bigger the better!
Larger $r_B \Rightarrow$ larger interference term $\Rightarrow$ better constraints on γ.

A Simple Interference Algebra

Amplitude 1 $= A\, e^{i\gamma}$

Amplitude 2 $= B\, e^{i\delta}$

Total amplitude $= Ae^{i\gamma} + Be^{i\delta}$

Decay Rate $= A^2 + B^2 + 2AB\cos(\delta - \gamma)$

Decay Rate of CP-conjugate decay

$= A^2 + B^2 + 2AB\cos(\delta + \gamma)$

If 2 parameters are known (A/B and δ), use the 2 equations to solve for B and γ.

$B \rightarrow DK$, through a slightly more complicated analysis, allows you to measure γ when δ is not known.

Evolution of Methods on γ

- **Gronau, Landon, and Wyler (GLW) Phys. Lett. B 265, 172 (1991)**
  - This was the original $B \to DK$ paper. Reconstruct D in a CP eigenstate.
  - Additional measurements are needed to determine them all: r_B, δ, γ.

Main Drawback:

$BF(B \to DK) \sim 10^{-4}$, $BF(D \to f_{CP}) \sim 10^{-2}$
Small… $\Rightarrow$ strongly statistics limited

- **Atwood, Dunietz, and Soni (ADS), Phys. Rev. Lett. 78, 3257 (1997)**
  - Noted the sizable interference between the DCS and CF decays of D, and proposed to use them, to realize the interference.
  - Method can't be used standalone either, since there is only one 2-body DCS mode, $D^0 \to K^+\pi^-$, while at least 2 modes are needed. Need additional input of strong phase difference in D decays.

No significant signal with current data

- **Giri, Grossman, Soffer, Zupan (GGSZ) Phys. Rev. D68, 054018 (2003)**
  - Outlines the method for using multi-body D decays with model-dependent and –independent analysis

Will elaborate on this later

- **BaBar, hep-ex/0507101 and Belle, hep-ex/0504013 (2005)**
  - The experimental measurements of γ using $B \to DK$, $D \to K_S\pi^+\pi^-$
- **Bondar, A. Poluektov, ph/0510246 (2005)**
  - MC study of the model-independent (binned Dalitz plot) measurement of γ

Discrete Ambiguities

- The observables are $\cos(\delta + \gamma)$ and $\cos(\delta - \gamma)$, which are invariant under
  - ✓ S_{ex} : $\delta \leftrightarrow \gamma$
  - ✓ $S_{\pm}$: $\delta \rightarrow -\delta$, $\gamma \rightarrow -\gamma$
  - ✓ S_{π} : $\delta \rightarrow \delta + \pi$, $\gamma \rightarrow \gamma + \pi$

- If δ_f and $\delta_{f'}$ are different enough, S_{ex} is resolved, since you can't simultaneously satisfy both $\delta_f \leftrightarrow \gamma$ and $\delta_{f'} \leftrightarrow \gamma$

While measuring γ, one encounters two devils: statistics and ambiguity, and they often feed each other.

2-body vs Multi-body D^0 Final States

Advantages of multi-body final states:

- Effectively, provide many final states, due to the variation of r_f and δ_f. This helps to resolve ambiguities down to an irreducible 2-fold ambiguity :)
- Add statistics – access to modes for which the 2-body final-state technique for measuring γ is not applicable :)

Disadvantages:

- More complicated analysis :(
- New systematic errors (how well do we understand the D final-state phase-space distribution?) unless using model-independent analysis approach :(

Overall:

- A-priori, both kinds of states are approximately equally useful in measuring γ. Measurement is statistically limited, need all the modes we can get. In practice, some modes will turn out to be more useful than others.

Analysis Steps for $B^{\pm}\rightarrow D_{\pi^-\pi^+\pi^0} K^{\pm}$

Step 1: Obtain $D^0 \rightarrow \pi^+\pi^-\pi^0$ Dalitz Plot parameterization using $D^{*+}\rightarrow D^0\pi^+$ (and c.c) sample

Step 2: Fit $B^-\rightarrow D_{\pi\pi\pi^0}K^-$ (and c.c) sample to obtain signal yield and branching-ratio asymmetry

Step 3: Fit for CP parameters using results of Steps 1 and 2 on $B^-\rightarrow D_{\pi\pi\pi^0}K^-$ sample

Step 1

3-Particle Phase Space

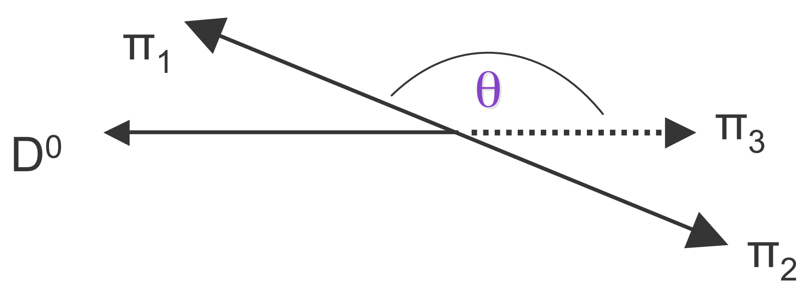
- **2 Observables**

From four vectors	12
Conservation laws	-4
Final state particle masses	-3
Free rotation in decay plane	-3
Σ	2

- **Usual choice**

  Invariant mass squared m^2_{12}

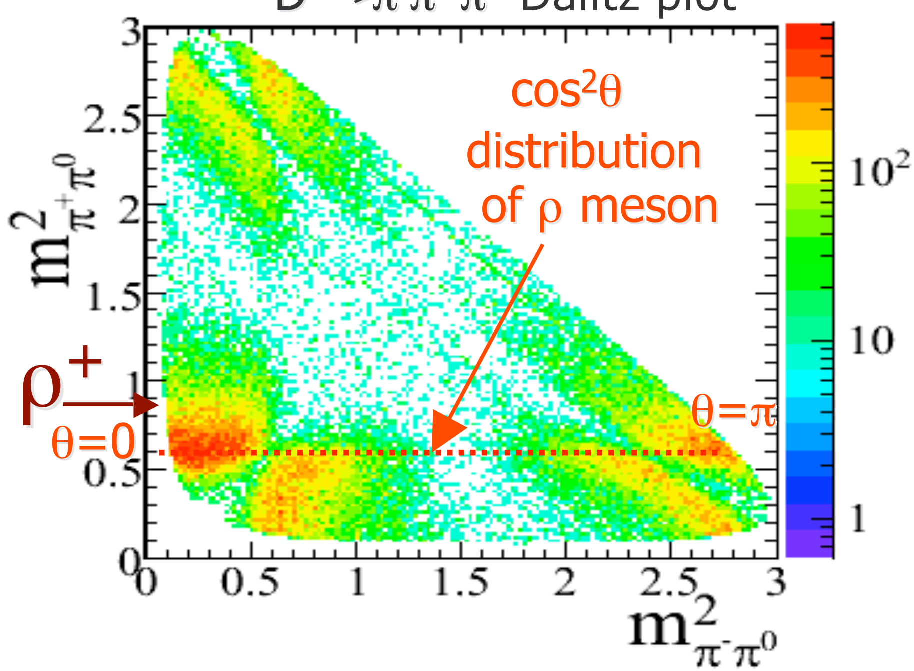
  Invariant mass squared m^2_{13}



$\{ \pi_1 , \pi_2, \pi_3 \} = \{ \pi^+, \pi^0, \pi^- \}$

- Dalitz plot provides info on angular distr.
- Also about dynamical amplitudes involved.
- Flat if no dynamics involved.

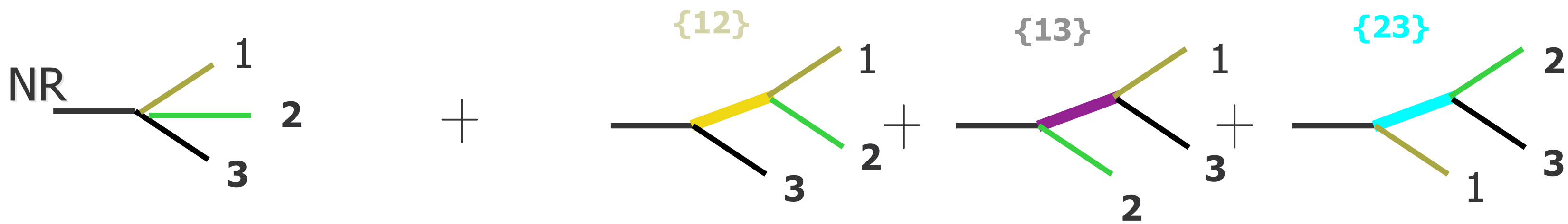
$D^0 \to \pi^- \pi^+ \pi^0$ Dalitz plot



- Dalitz applied this method first to K_L-decays
  - To resolve τ/θ puzzle with only few events
  - goal was to determine spin and parity
- And he never called them Dalitz plots !

Isobar Model Formalism

three-body decay D→ABC decaying through an r=[AB] resonance



D decay three-body amplitude $\mathcal{A}_D(s_{12}, s_{13}) = a_0 e^{i\delta_0} + \sum_r a_r e^{i\delta r} \mathcal{A}_r(s_{12}, s_{13})$

NR term(direct 3 body decay)

a_0, δ_0, a_r, δ_r : Free parameters of fit

$$\mathcal{A}_r(s_{12}, s_{13}) = F_D^J F_r^J \times M_r^J \times BW_r^J$$

Relativistic Breit-Wigner

$$BW_r^J(s) = \begin{cases} \dfrac{1}{M_r^2 - s - iM_r\Gamma_r(\sqrt{s})} \\ \dfrac{1}{M_r^2 - s - i\ (\ \rho_1 g_1^2 + \rho_2 g_2^2)} \end{cases}$$

$f_0(980)$
$a_0(980)$

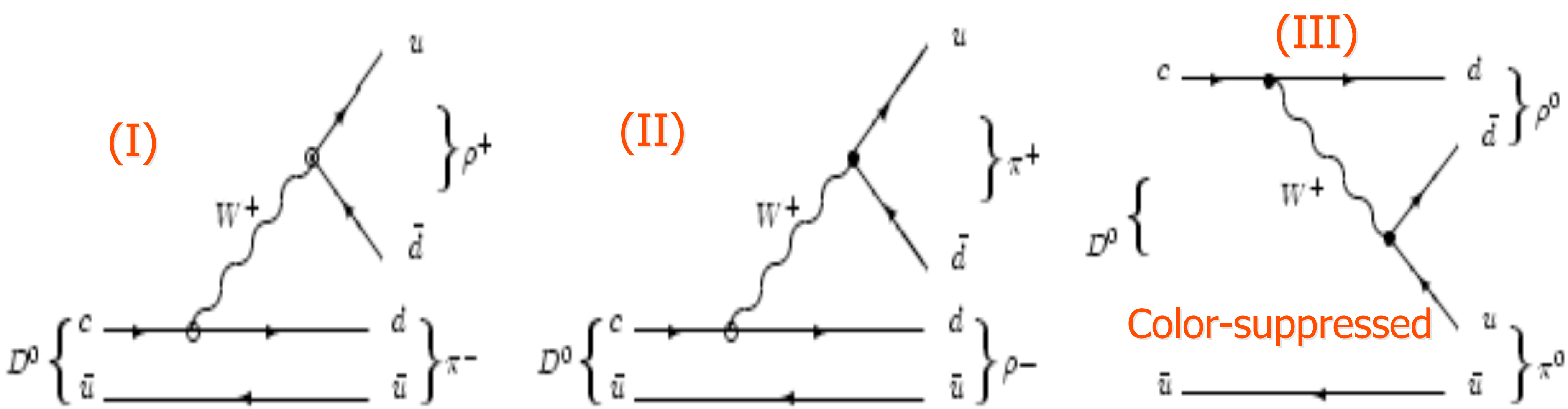
Angular distribution

D and *r* Blatt-Weisskopf form factors

Step 1

$D^0 \to \pi^- \pi^+ \pi^0$ Dalitz Plot Amplitudes

Interference between three types of singly Cabibbo-suppressed amplitudes



$$\mathcal{A}[D^0 \to \pi^- \pi^+ \pi^0] \equiv f_{D^0}(m^2_{\pi^+\pi^0}, m^2_{\pi^-\pi^0})$$
$$\bar{\mathcal{A}}[\bar{D}^0 \to \pi^+ \pi^- \pi^0] \equiv f_{D^0}(m^2_{\pi^-\pi^0}, m^2_{\pi^+\pi^0})$$

$m^2_{\pi^+\pi^0} + m^2_{\pi^-\pi^0} + m^2_{\pi^+\pi^-} =$
$m^2_{\pi^+} + m^2_{\pi^-} + m^2_{\pi^0} + m^2_{D^0}$

PDF for signal events = $|\, f \,|^2$

Assumes no D-mixing, no CP violation in D decays!

Step 1

$D^0 \to \pi^- \pi^+ \pi^0$ Event Reconstruction

$D^0 \to \pi^- \pi^+ \pi^0$ Reconstruction

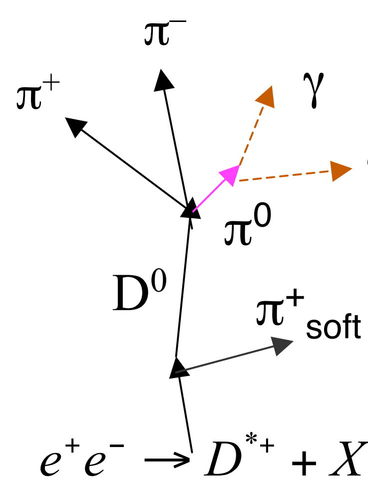
- π^- and π^+ tracks are fit to a vertex
- Mass of π^0 candidate is constrained to m_{π^0} at $\pi^-\pi^+$ vertex
- P_{CM} (D^0) > 2.77 GeV/c

Background Sources

- Charged track combinatoric
- Mis-reconstructed π^0
- Real D^0, fake π_s
- $K\pi\pi^0$ reflection in sideband

D* Reconstruction

- D^{*+} candidate is made by fitting the D^0 and π_s^+ to a vertex constrained in x and y to the measured beam-spot.
- $|m_{D^*} - m_{D^0} - 145.5| < 0.6$ MeV/c^2
- Vertex χ^2 probability > 0.01
- Choose the best candidate per event with the smallest χ^2 for the decay chain (multiplicity = 1.03).



Event Selection

P_{CM} (D^0) > 2.77 GeV/c
$|m_{D^*} - m_{D^0} - 145.5|$ < 0.6 MeV/c^2

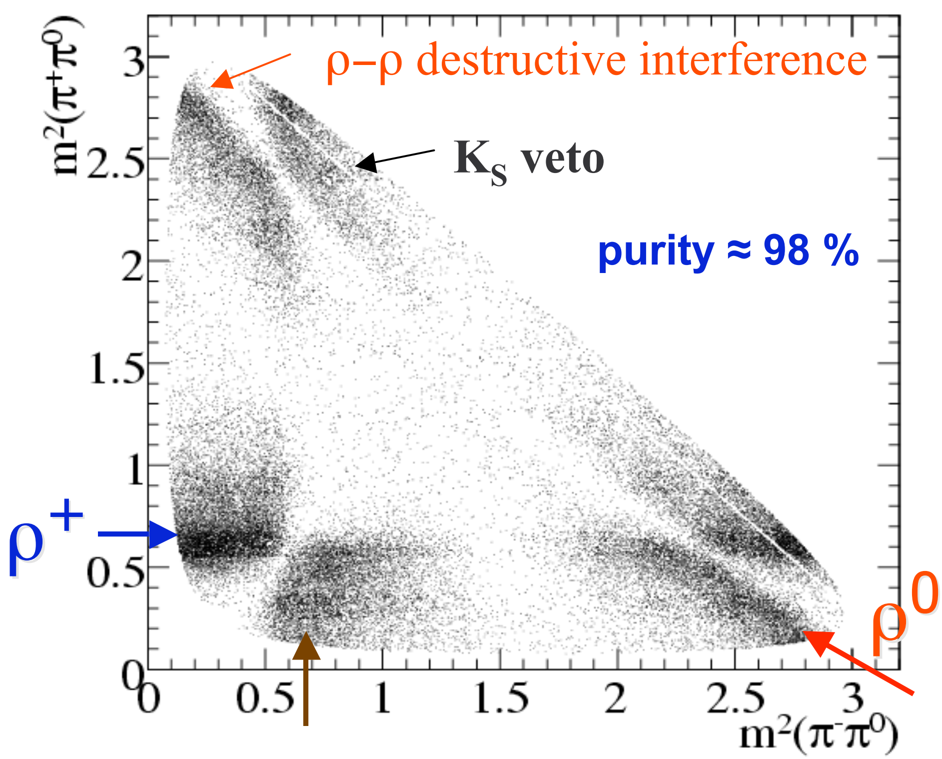
Phys. Rev. D74, 091102 (2006)

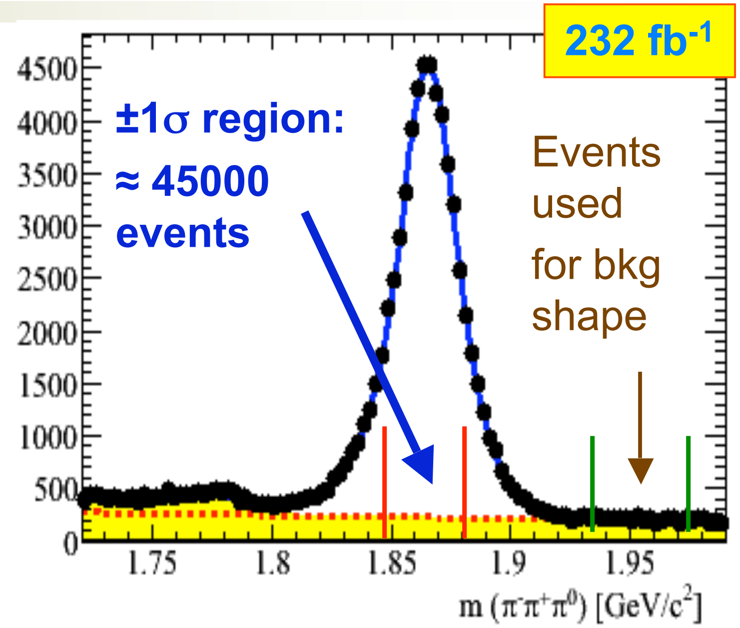
Step 1

Dalitz Plot Analysis of $D^0 \to \pi^- \pi^+ \pi^0$

Motivation: **CKM angle γ using $B^\pm \to D[\to \pi^- \pi^+ \pi^0] K^\pm$**

- Three $I = 1$ particles in the final state
- Gives rise to a rich interference structure
- The three ρ regions are clearly enhanced in the DP, and ρ-ρ destructive interference is evident





The 3 destructively interfering $\rho\pi$ amplitudes suggest an $I = 0$, $\Delta I = 1/2$ dominated final state.
C. Zemach, Phys. Rev. 133, B1201 (1964).

hep-ex / 0703037

Step 1

Fit Results

hep-ex/0703037

ρ^+ : 68 %
ρ^- : 35 %
ρ^0 : 26 %

Small contributions from higher ρ, f_0, f_2 and σ states

State	Amplitude a_r	Phase ϕ_r	Fraction $f_r(\%)$
$\rho^+(770)$	1	0	67.8±0.0±0.2
$\rho^0(770)$	0.588±0.006±0.001	16.2±0.6±0.3	26.2±0.5±0.4
$\rho^-(770)$	0.714±0.008±0.003	-2.0±0.6±0.5	34.6±0.8±0.1
$\rho^+(1450)$	0.21±0.06±0.10	-146±18±8	0.11±0.07±0.06
$\rho^0(1450)$	0.33±0.06±0.04	10±8±6	0.30±0.11±0.07
$\rho^-(1450)$	0.82±0.05±0.04	16±3±3	1.79±0.22±0.12
$\rho^+(1700)$	2.25±0.18±0.14	-17±2±2	4.1±0.7±0.7
$\rho^0(1700)$	2.51±0.15±0.13	-17±2±2	5.0±0.6±0.9
$\rho^-(1700)$	2.00±0.11±0.07	-50±3±3	3.2±0.4±0.6
$f_0(980)$	0.052±0.004±0.006	-59±5±3	0.25±0.04±0.04
$f_0(1370)$	0.22±0.03±0.03	156±9±6	0.37±0.11±0.09
$f_0(1500)$	0.20±0.02±0.02	12±9±4	0.39±0.08±0.07
$f_0(1710)$	0.39±0.05±0.06	51±8±7	0.31±0.07±0.08
$f_2(1270)$	0.30±0.01±0.06	-171±3±2	1.32±0.08±0.08
σ (400, 600)	0.24±0.02±0.04	8±4±3	0.82±0.10±0.10
Non-Res	0.57±0.07±0.08	-11±4±2	0.84±0.21±0.12

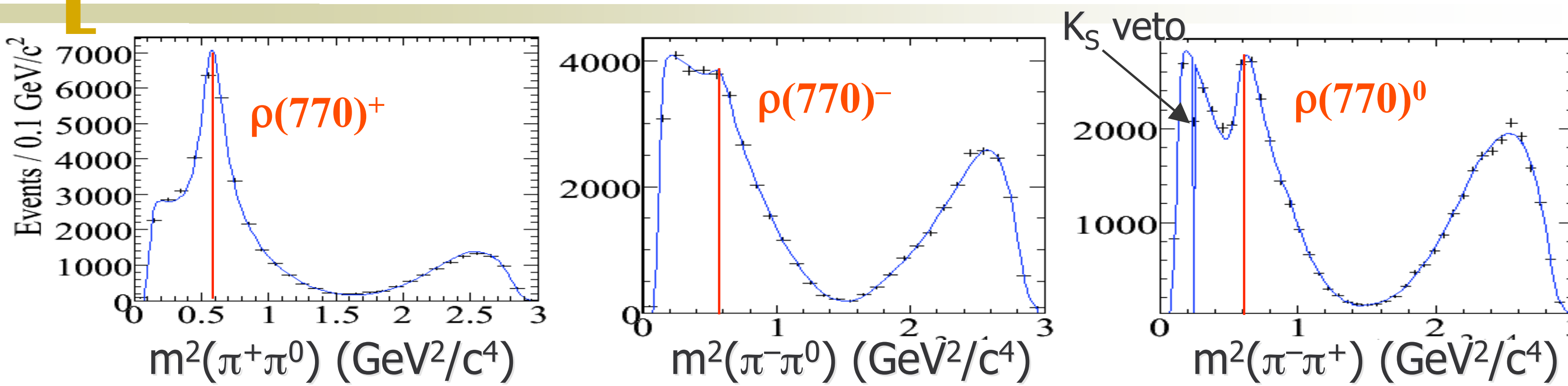
Systematic errors:

- σ and $\rho(1700)$ parameters
- reconstruction & PID eff
- Form factor variation
- Flavor mistags

The distribution is marked by 3 destructively interfering $\rho\pi$ amplitudes, suggesting an $I = 0$, $\Delta I = 1/2$ dominated final state. C. Zemach, Phys. Rev. 133, B1201 (1964).

Step 1

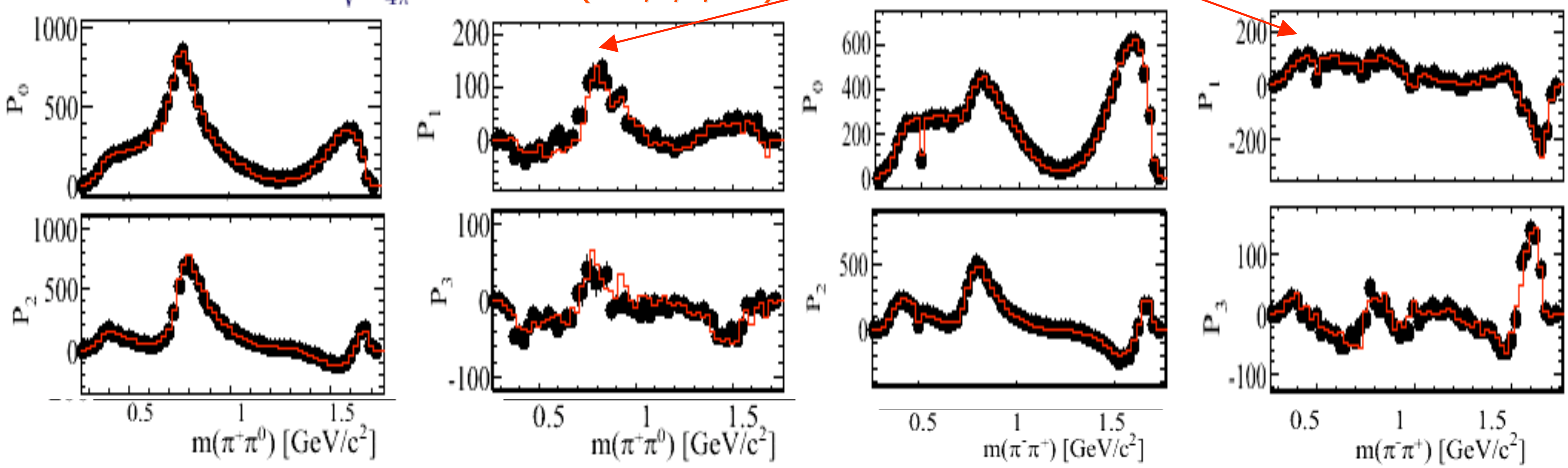
Mass-projections & Angular Moments



Each event is weighted by the spherical harmonic functions $Y_l^0(\cos\theta_H) = \sqrt{\frac{2l+1}{4\pi}}\, P_l(\cos\theta_H)$ (l=0,1,2,.....).

Excellent agreement between data & fit.

Large interference between S and P waves.



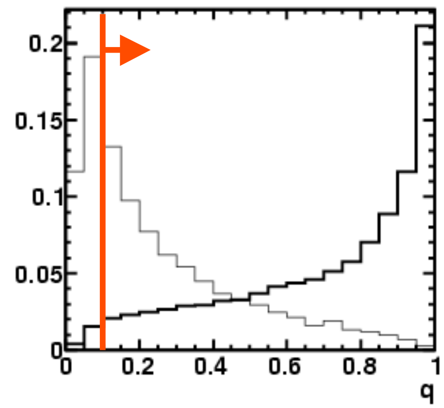
Step 2

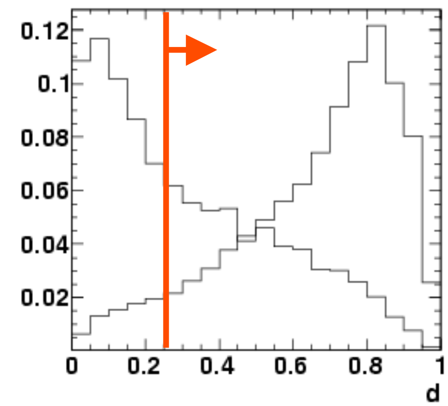
Event Selection for $B^{\pm}\to D_{\pi^-\pi^+\pi^0} K^{\pm}$

Based on BR and asymmetry analysis

Phys. Rev. D72, 071102 (2005)

- $5.272 < m_{ES} < 5.3$ GeV (Avoids DP-m_{ES} correlations in bkg)
- $1.83 < m_D < 1.895$ GeV (Avoids DP-m_D correlations in bkg)
- Kaon, pion identification
- $K_S \to \pi\pi$ veto ($D^0 \to K_S\pi^0$ is a CF decay unrelated to GGSZ method)
- $q > 0.1$ (continuum NN)
- $d > 0.25$ (fake D^0 NN)
- $\varepsilon = 11.4\%$





Step 2

Event Types in $B^{\pm}\rightarrow D_{\pi^{-}\pi^{+}\pi^{0}} K^{\pm}$

1. DK_D: Correctly reconstructed signal ("signal")
2. DK_{bgd}: Mis-reconstructed signal events
3. $D\pi_D$: Correctly-reconstructed $D\pi$ with π misidentified as K
4. $D\pi_{badD}$: $D\pi$ events with a fake D candidate. K candidate is usually a true kaon picked at random from the event
5. DKX: $B\rightarrow DK$ with $D\rightarrow$ non-$\pi\pi\pi^0$. The K is good
6. $D\pi X$: $B\rightarrow D\pi/\rho$ with $D\rightarrow$ non-$\pi\pi\pi^0$. K candidate is usually a true kaon picked at random from the event
7. BBC_D: Combinatoric BB events with a good D candidate
8. BBC_{badD}: Combinatoric BB events with a fake D candidate
9. qq_D: Continuum with a good D candidate
10. qq_{badD}: continuum with a fake D candidate

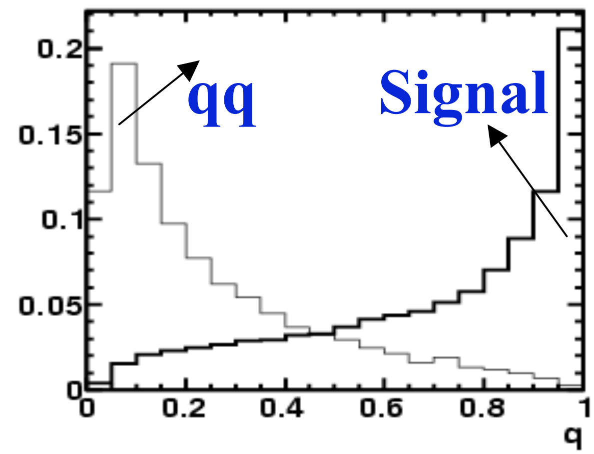
Step 2

BR & Asymmetry for $B^{\pm}\rightarrow D_{\pi^-\pi^+\pi^0} K^{\pm}$

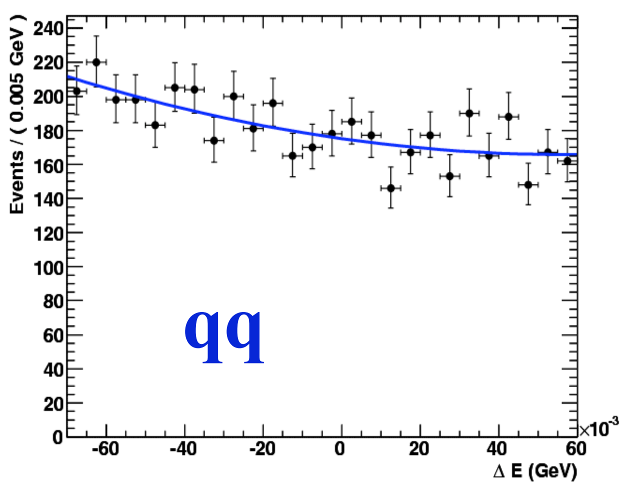
Fit $B^-\rightarrow D_{\pi\pi\pi^0}K^-$ sample with ΔE, q, d

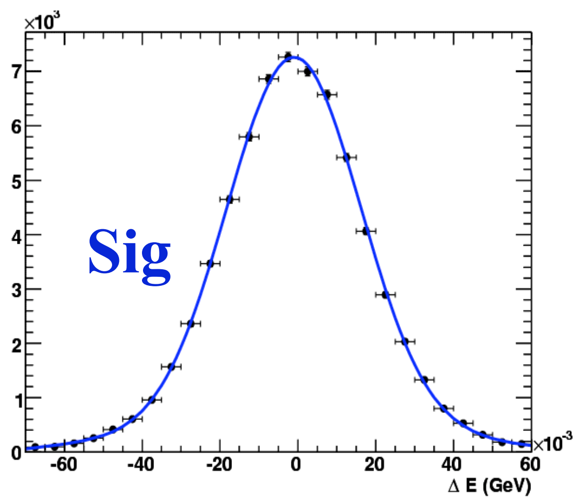
Obtain signal yield & asymmetry

Nsig	170 ± 29
Asym	-0.02 ± 0.15



ΔE PDFs are Gaussian and 2nd-order polynomial:





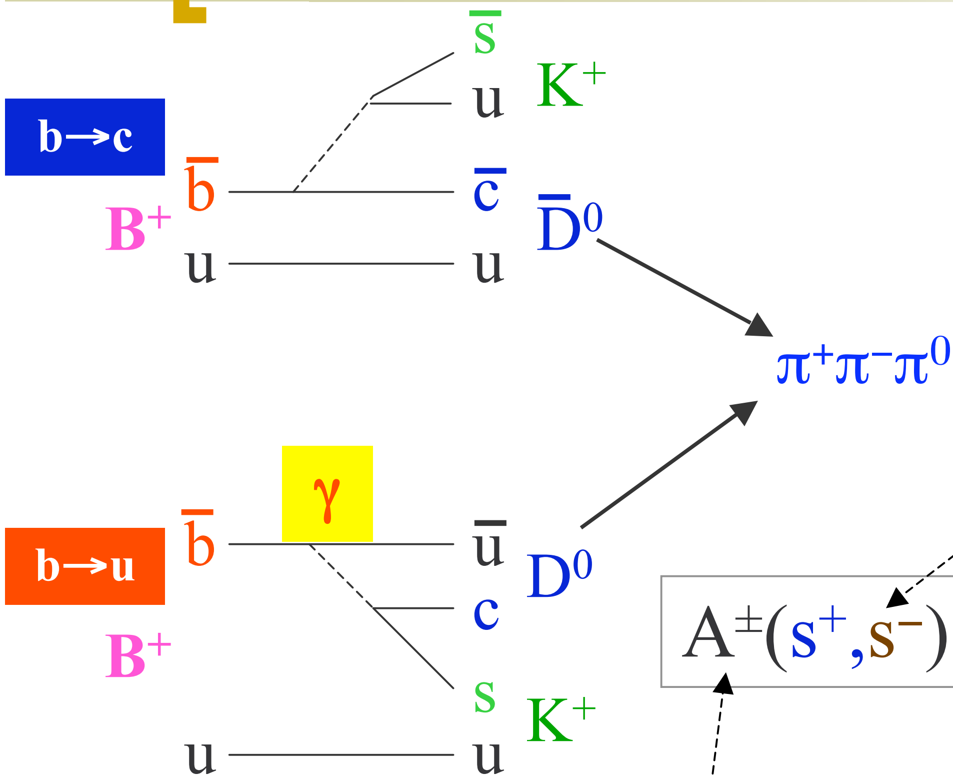
$BR(B^-\rightarrow D_{\pi\pi\pi 0}K^-) = (4.6 \pm 0.8 \pm 0.7) \times 10^{-6}$

$A(B^-\rightarrow D_{\pi\pi\pi 0}K^-) = -0.02 \pm 0.15 \pm 0.03$

Step 3

Extraction of γ : Basic Idea

Atwood et al., PRL78, 3257 (1997)
Giri et al., PRD68,054018 (2003)



- δ_B is strong phase diff. of $A(B^- \rightarrow D^0K^-)$ and $A(B^- \rightarrow \bar{D}^0K^-)$
- Unknowns: r_B, δ_B and γ

Dalitz plot variables

$$A^{\pm}(s^+,s^-) = f_D(s^+,s^-) + r_B e^{i(\delta\pm\gamma)} f_D(s^-,s^+)$$

$B^{\pm}\rightarrow D_{\pi\pi\pi^0}K^{\pm}$ amplitude $\quad$ $\bar{D}^0\rightarrow\pi\pi\pi^0$ amplitude $\quad$ $D^0\rightarrow\pi\pi\pi^0$ amplitude

- Based on GGSZ method of **PRD68, 054018**, so far used only with $D\rightarrow K_S\pi^+\pi^-$
- Goal: add modes for maximum γ precision

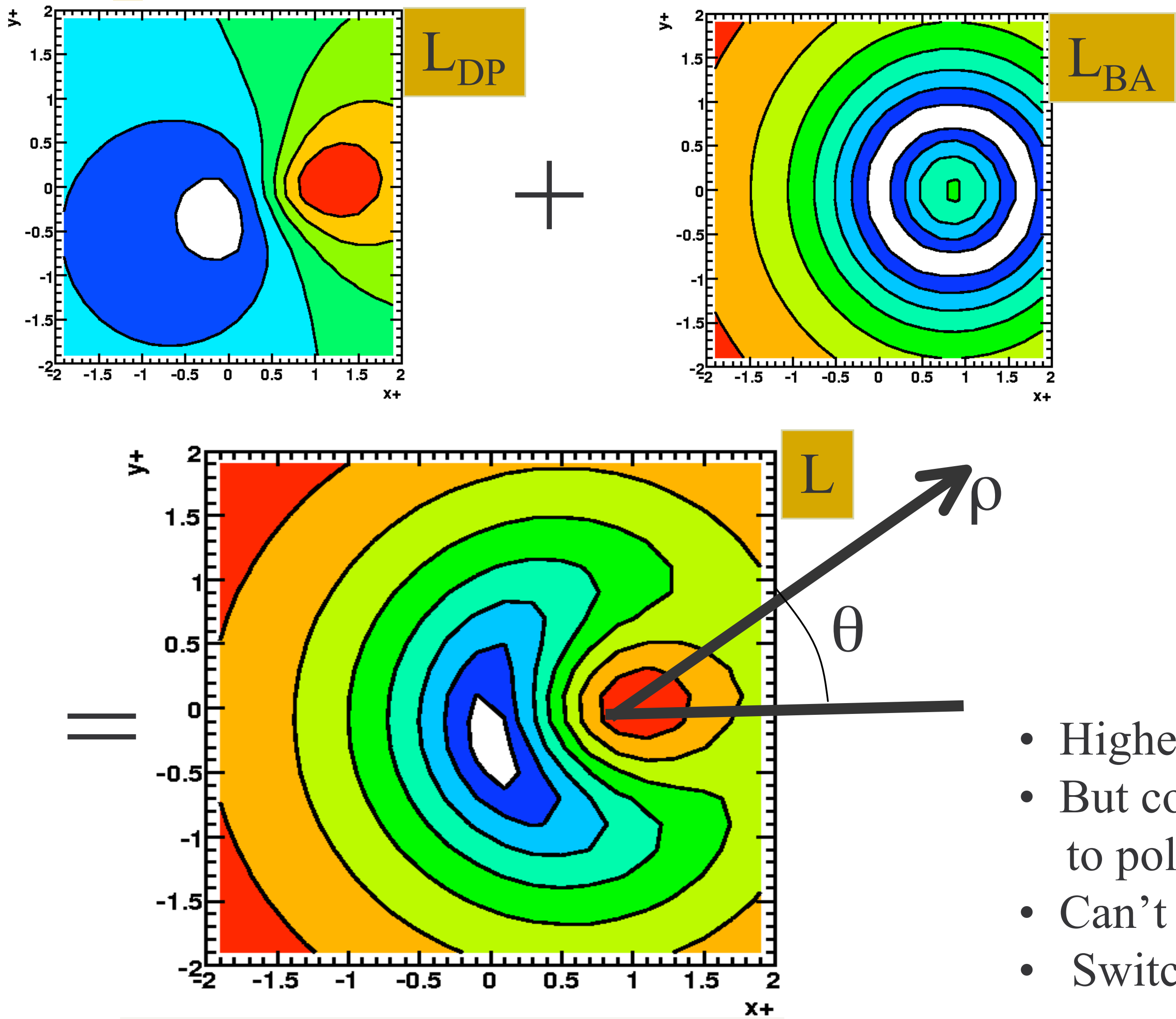
Step 3

Add more Information to the Likelihood

- The Dalitz plot shape $|A^{\pm}(s^{+},s^{-})|^2$ depends on the CP parameters $r_B e^{i(\delta\pm\gamma)}$
  - Previous Dalitz analyses, with $K_S\pi^+\pi^-$, used only this signature
- But the branching fractions = $\int|A^{\pm}(s^{+},s^{-})|^2$ are also sensitive to the CP parameters
  - Using both the shape and the absolute rates gives higher sensitivity
- It turns out that in this mode , the BRs give a much higher sensitivity
  - Don't know how it is in $K_S\pi^+\pi^-$ – need to check. If the same is true there, expect significant improvement in $K_S\pi^+\pi^-$ sensitivity to γ

Step 3

Combined behavior $L = L_{DP} + L_{BA}$



- Highest sensitivity
- But correlated contours due to polar symmetry of L_{BA}
- Can't quote sensible errors
- Switch to polar coordinates

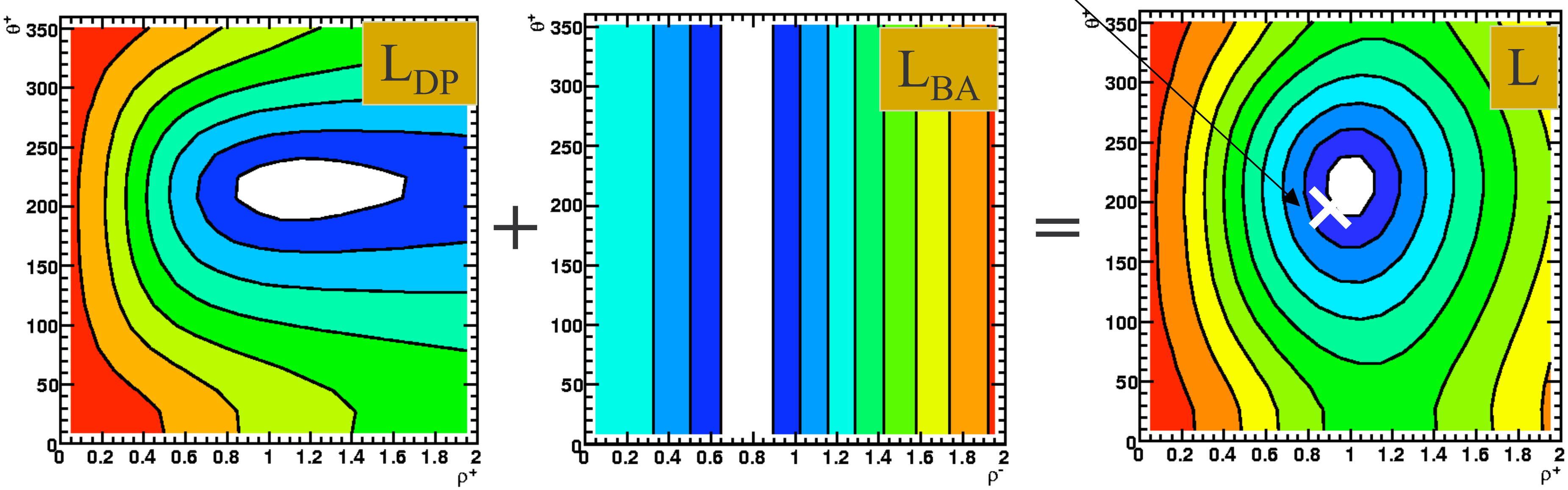
Step 3

Polar coordinates

$$\rho_\pm \equiv \sqrt{(x_\pm - x^0)^2 + y_\pm^{\ 2}} \qquad \theta_\pm \equiv \tan^{-1}\left(\frac{y_\pm}{x_\pm - x^0}\right)$$

$$x^0 = \int f_D(s^+,s^-)^* \, f_D(s^-,s^+) \, ds^- ds^+ = 0.85$$

$\rho_\pm = x^0$ and $\theta = 180°$ for $r_B = 0$ (no CP violation)



Step 3

Result with 344 M $e^+e^- \to B\bar{B}$ Events

$$r_B e^{i(\delta \pm \gamma)} = x_\pm + y_\pm$$

$$\rho_\pm \equiv \sqrt{\left(x_\pm - x^0\right)^2 + y_\pm^{\;2}}$$

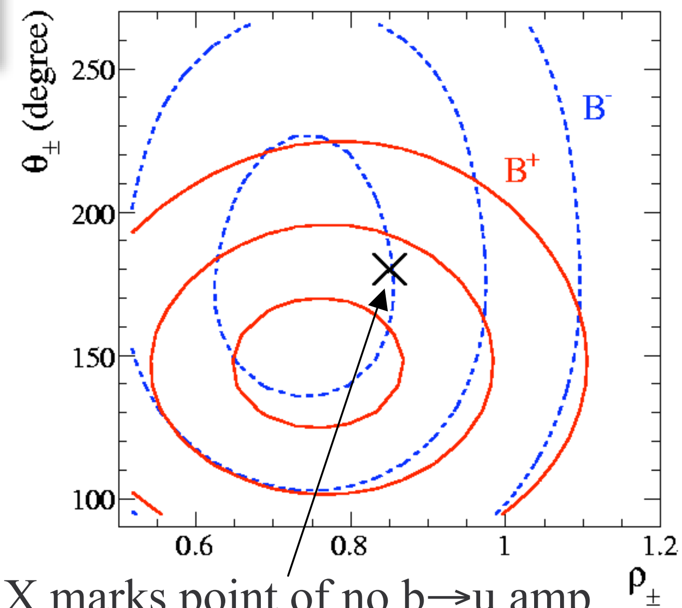
$x^0 = 0.85$

$$\theta_\pm \equiv \tan^{-1}\left(\frac{y_\pm}{x_\pm - x^0}\right)$$

$\rho^- = 0.72 \pm 0.11 \pm 0.06$;
$\theta^- = (173 \pm 42 \pm 16)°$

$\rho^+ = 0.75 \pm 0.11 \pm 0.06$;
$\theta^+ = (147 \pm 23 \pm 11)°$

However, not trivial to directly determine γ



X marks point of no $b \to u$ amp
$\rho = 0.85$, $\theta = 180°$

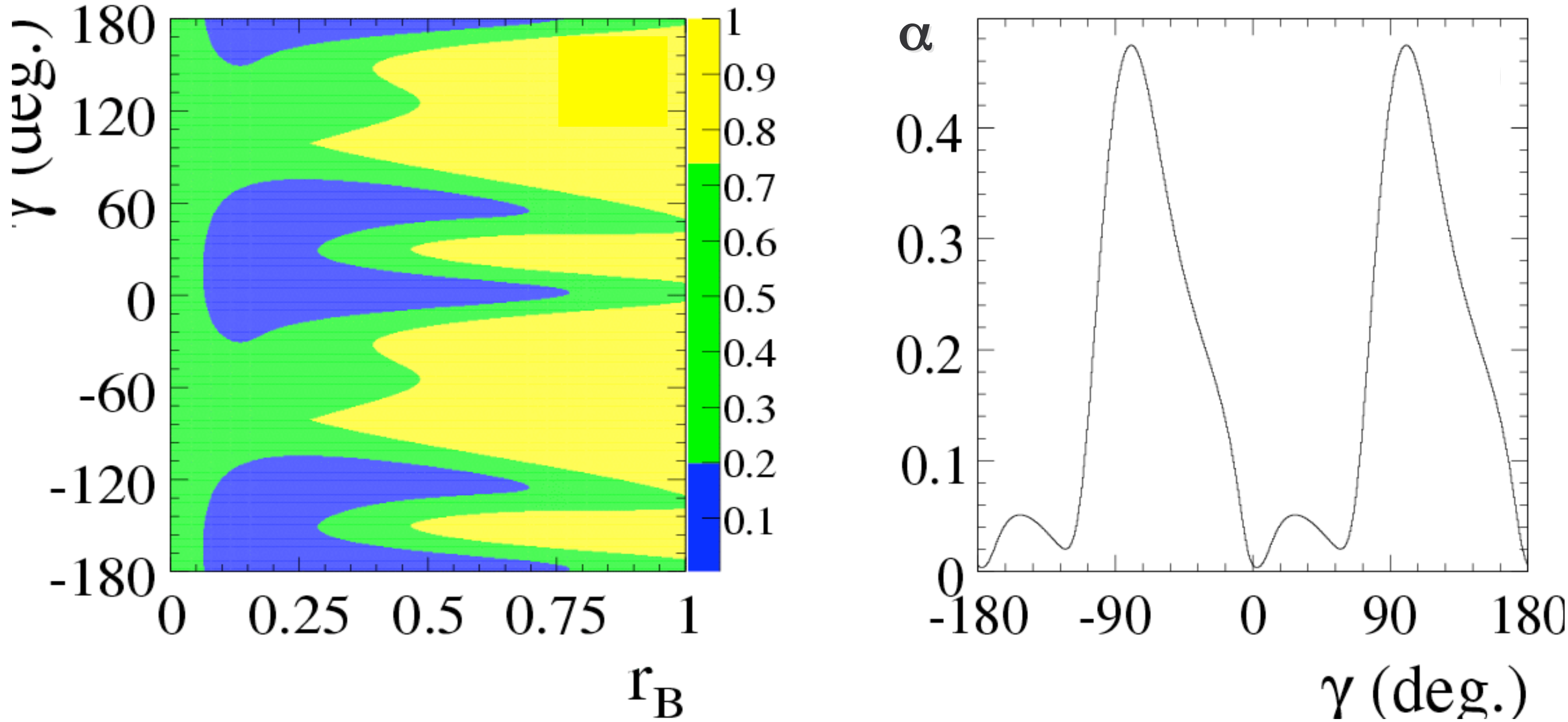
- First measurement of CP-violating quantities in $B \to D_{\pi\pi\pi^0} K$
- First combined use of DP distribution and absolute BR to extract CP parameters.
- σ_θ is too large for a meaningful extraction of γ from this analysis alone
- σ_ρ is small enough to contribute significantly to overall fits for γ

Step 3

From $(\rho_{\pm}, \theta_{\pm})$ to (r_B, δ, γ)

Use frequentist method to extract γ, r_B, δ from $(\rho_{\pm},\theta_{\pm})$

(3dim confidence intervals projections)



1σ, 2σ, and 3σ regions are defined as containing the three-dimensional significance, α, smaller than 19.9 %, 73.9 %, and 97.1 %, respectively.

Step 3

Constraints on (r_B, δ, γ)

1σ bounds on the physical parameters, including both stat. and syst. errors

First direct lower bound on r_B

$$0.06 < r_B < 0.78$$
$$-30^o < \gamma < 76^o$$
$$-27^o < \theta < 78^o$$

hep-ex / 0703037

accepted for publication in PRL

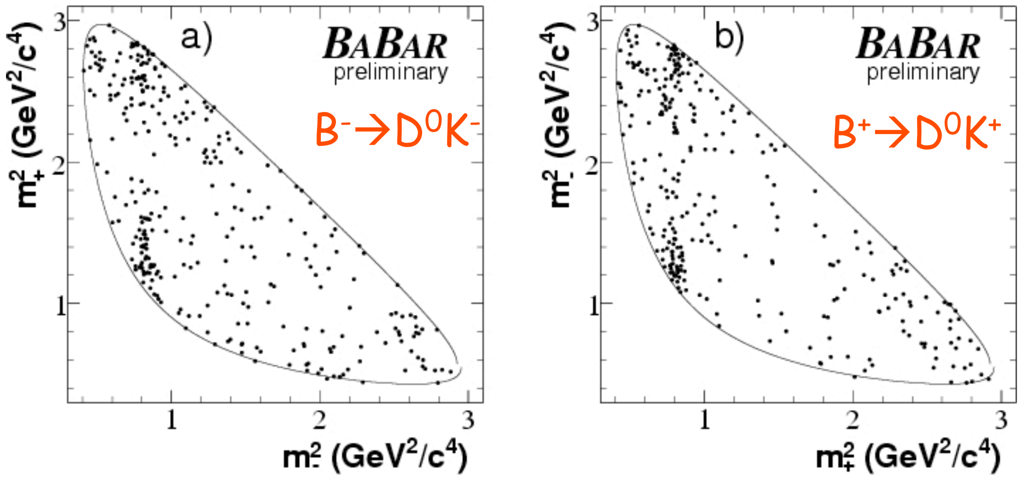
These bounds come from the results of this analysis alone.

Sensitivity to r_B, γ, and δ arises from the Dalitz plot and the BR asymmetry.

Hopefully, a more powerful bound will be obtained after combining the results of this analysis with with those from $B^{\pm} \to D[\to K_S^0 \pi^+ \pi^-] K^{\pm}$ analysis.

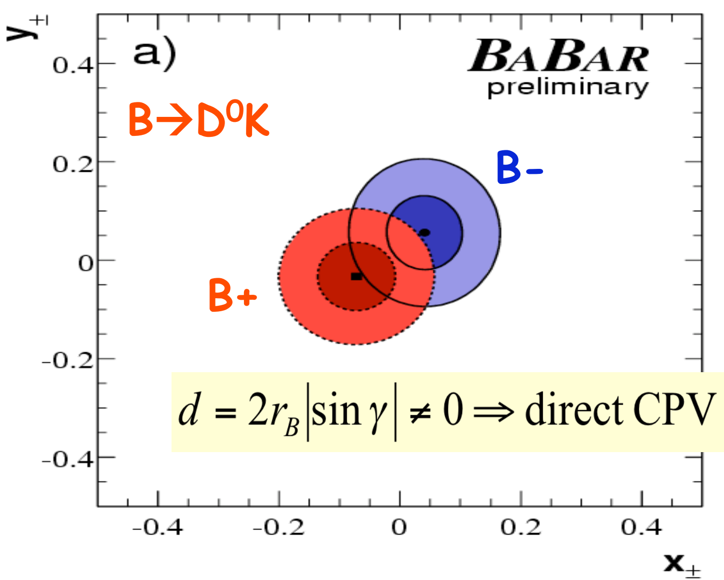
γ from $B^{\pm} \to D_{K_S^0 \pi^- \pi^+} K^{\pm}$

$D \to K_S \pi\pi$ Dalitz plot distribution in signal region

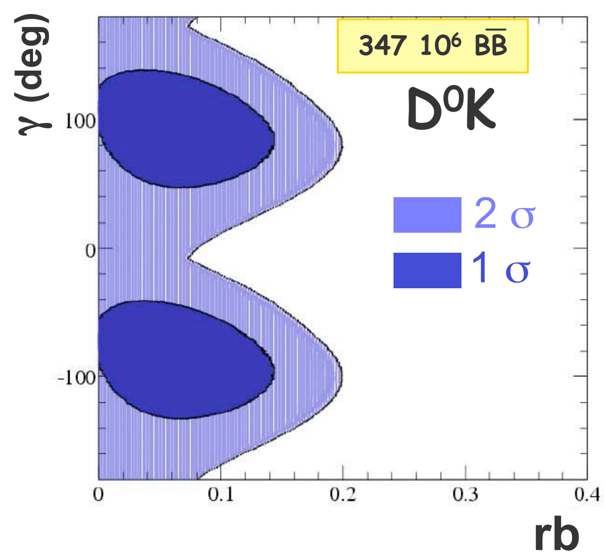


Used frequentist method to extract γ, r_B, δ_B from $(x_{\pm}, y_{\pm})$

$(x_{\pm}, y_{\pm})$ are extracted from the $D^0 \to K_S \pi\pi$ Dalitz plot



$d = 2r_B|\sin\gamma| \neq 0 \Rightarrow$ direct CPV



$r_B < 0.142$ ($r_B < 0.198$)
1σ (2σ)

$\gamma = (92 \pm 41 \pm 10 \pm 13)^{o}$
(stat) (syst) (Dalitz)

(5dim confidence intervals projections)

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Summary

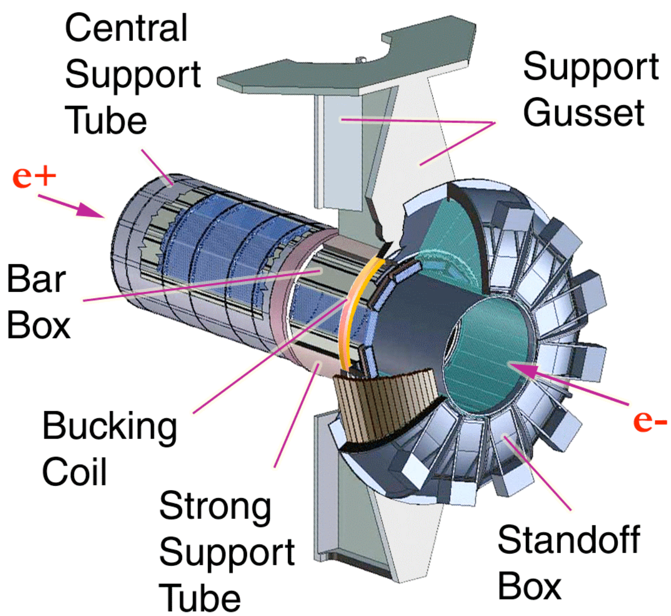
- Direct measurement of γ is crucial to constrain new physics contributions in quark sector of the Standard Model.
- Many different approaches to measure γ. Information from GLW, ADS, GGSZ, and other methods are all useful.
- The GGSZ/Dalitz method has emerged as the most powerful technique.
- Precise parameterizations of the amplitudes and phases and the inclusion of information on branching ratio and decay-rate asymmetry improve sensitivity in γ. A lot of progress made in the analysis and technique development.
- Statistics are the only thing holding us back ! Adding additional D decay modes to $B \to DK$ and combining results from them will definitely help in the future analysis.

End of Talk ! Thank You !

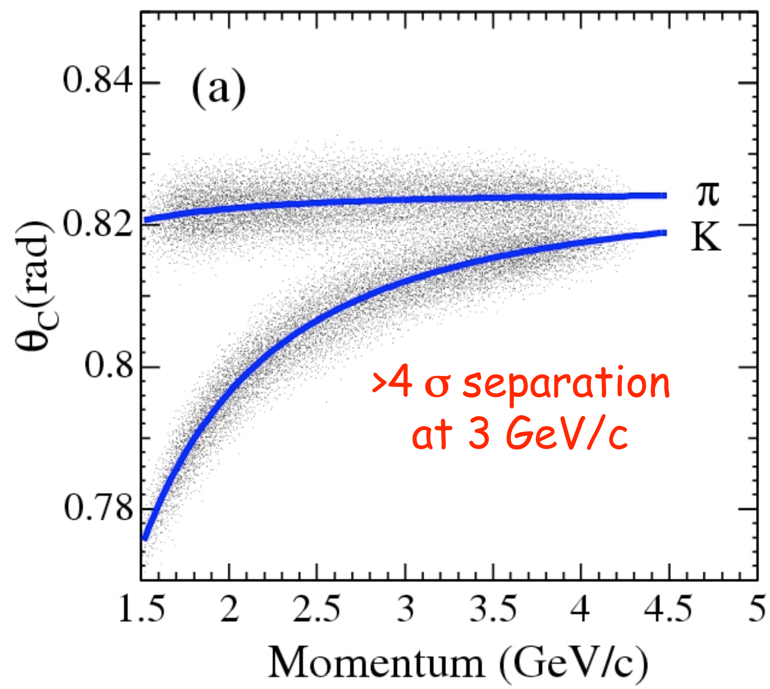
Back up slides

Kaon/Pion Discrimination: DIRC

LAYOUT

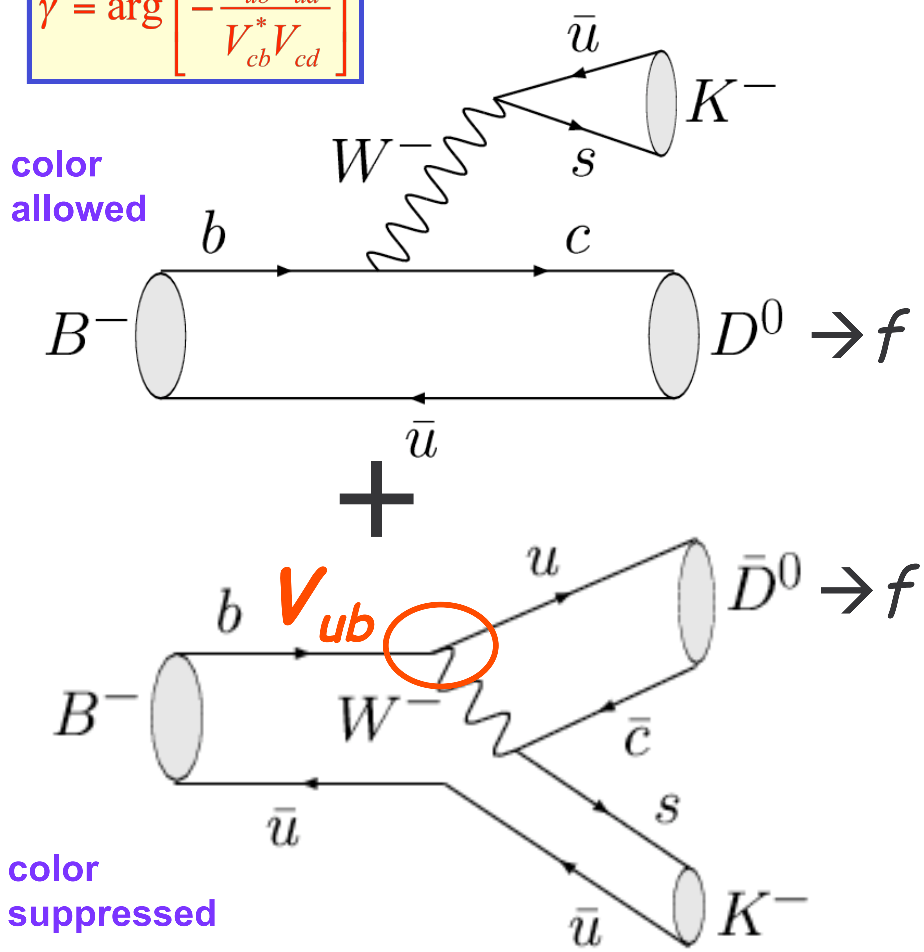


Cherenkov angle vs. momentum for pions and kaons



Methods to Extract γ

$$\gamma = \arg\left[-\frac{V_{ub}^{*}V_{ud}}{V_{cb}^{*}V_{cd}}\right]$$



- $D^0/\overline{D}^0$ decay to common final state
- The interference depends on V_{ub} and therefore on γ
- Critical parameter: ratio of amplitudes:

$$r_B \equiv \left|\frac{A(B^- \rightarrow \overline{D}^0 K^-)}{A(B^- \rightarrow D^0 K^-)}\right| \sim 0.1$$

- Select the D^0 decays that enhance the interference:
  - o 3-body (e.g. $K_S\pi\pi$): **Dalitz**
  - o CP-eigen. (e.g. $K_S\pi^0$): **GLW**
  - o DCS (e.g. $D^0 \rightarrow K^+\pi^-$): **ADS**

γ measurements are overwhelmingly dominated by statistical errors.

Dalitz Plot Method

- We saw that at least 2 D final states are needed in order to solve for all the unknowns.
- This 2-state requirement can be satisfied by a single multi-body D final states, in which each point in the final state phase space (Dalitz plot for a 3-body decay) serves effectively as a different final state.
- In terms of the γ analysis, what differentiates 2 final states is their values of r_f and/or $\delta_{f.}$. In this sense, different points in phase space can function as different D final states when they have different values of r_f or δ_f.
- Broad resonances are the most obvious cause for variation of r_f and δ_f in different points of final-state phase space.

Assessment of Some 3-body D^0 Decays

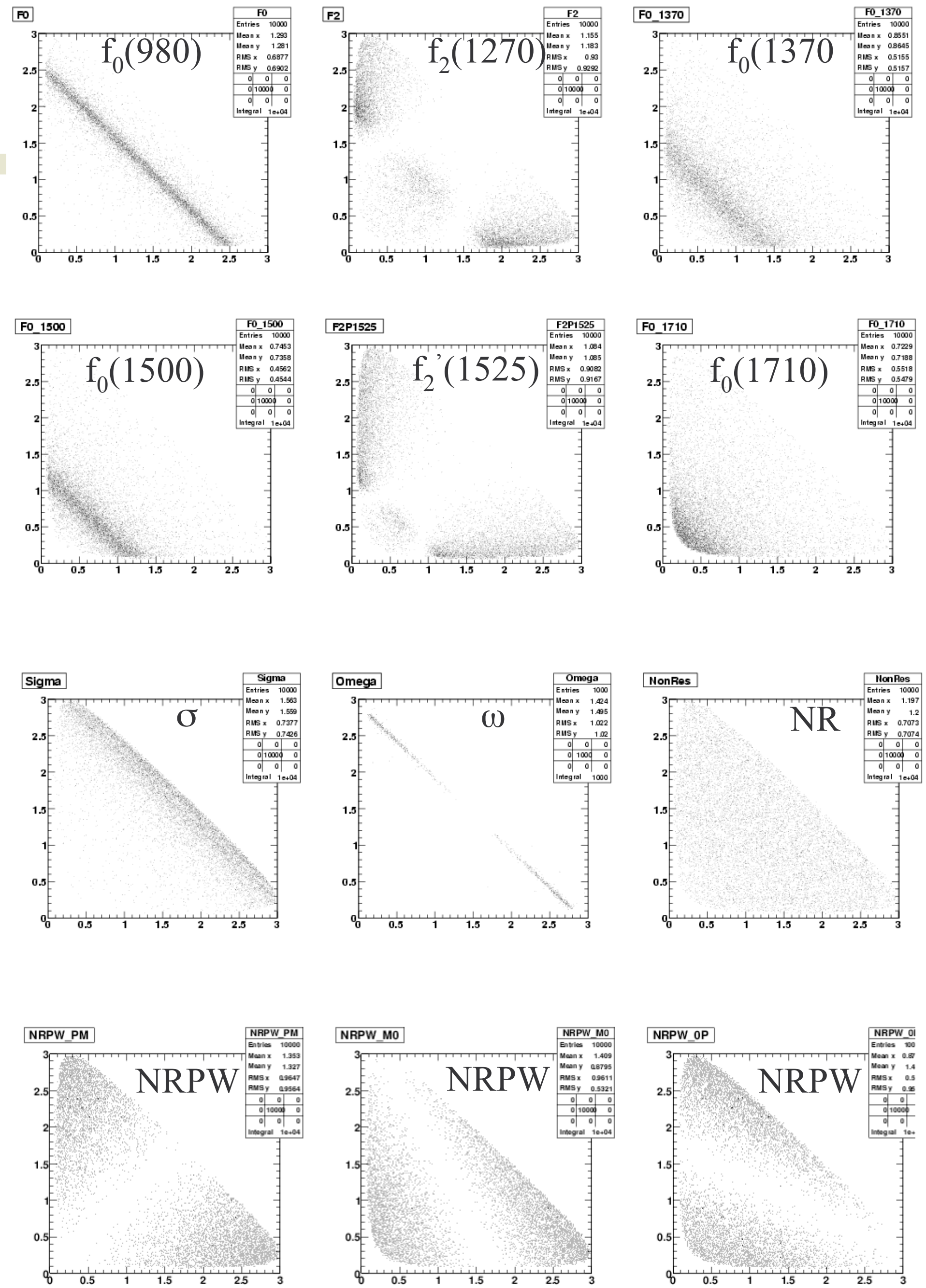
Mode	BR($D^0 \to f$)	λ^n	$\lvert A(\overline{D^0})/A(D^0)\rvert$	Bgd	Comments
$K_S\pi^+\pi^-$	**2.9%**	**n=0**	$\sim\lambda^2$ to 1	OK	Attractive due to high stat & low background
$\pi^+\pi^-\pi^0$	**1.5%**	**n=1**	~ 1	π^0	Expect similar sensitivity as $K_S\pi\pi$ if background under control
$K_SK^+\pi^-$	**(0.34 ⊕ 0.26)%**	**n=1**	~1	OK	Expect similar sensitivity as $\pi\pi\pi^0$
$K^+\pi^-\pi^0$	**~0.2%**	**n=2**	$\sim 1/\lambda^2$	π^0	S/B probably too small for now
$K^+K^-\pi^0$	**0.3%**	**n=1**	~1	π^0 bad, KK good	Low stat, but low background, so sensitivity could approach $\pi\pi\pi^0$
$K_S\pi^0\pi^0$	**~1% (+?)**	**n=0**	1	$2\pi^0$	CP eigenstate, low S/B
$K_S\pi^+\pi^-\pi^0$	**5.5%**	**n=0**	$\sim\lambda^2$	So-so	High stat, but 4-body analysis is hard. Large phase space reduces D^0-D^0bar interference

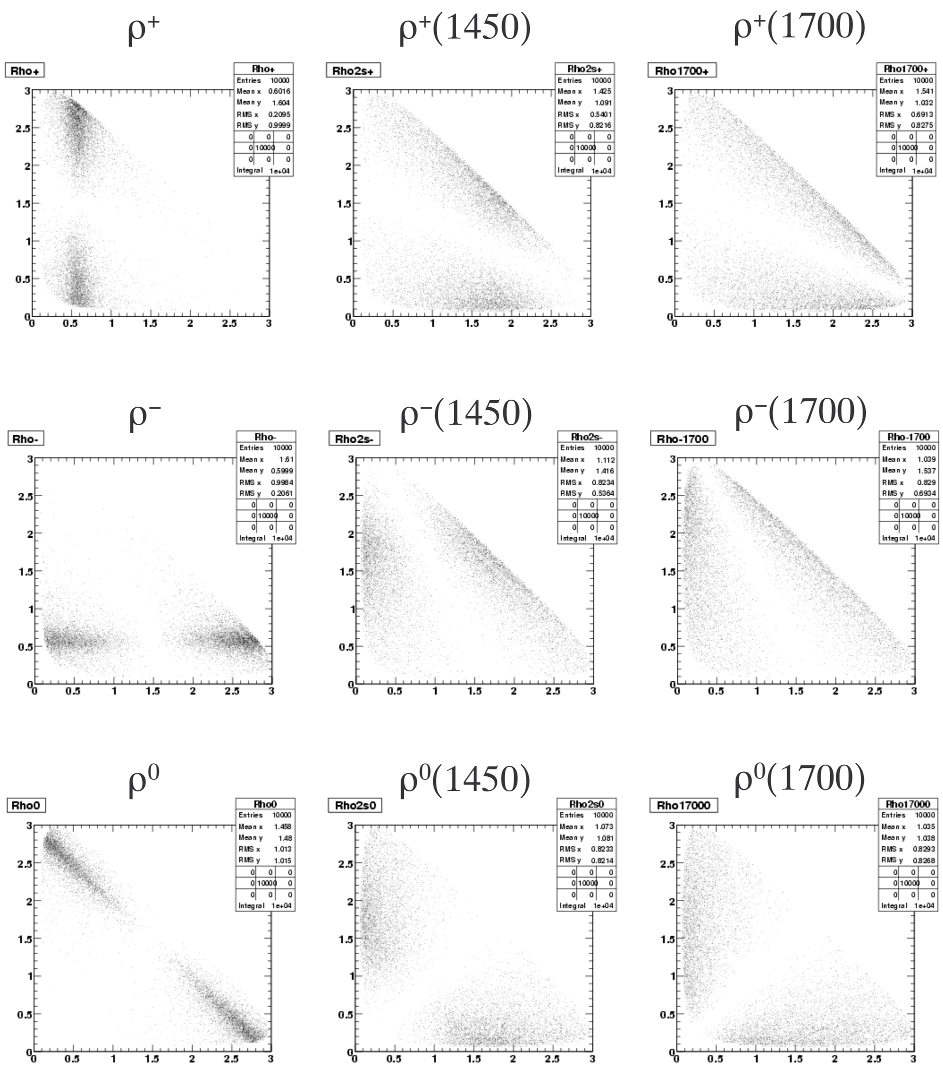
Analysis with Multi-body D^0 Final States

1. The simplest extension of the 2-body analysis.
2. Divide phase space into small bins, so that variations of r_f and δ_f within each bin can be ignored. Distant bins will have values of r_f and δ_f that are different enough so as to constitute different final states, and the analysis can be carried out, in principle, with as few as 2 bins.
3. A more accurate solution is not to ignore the variations of r_f and δ_f over the bin. But this introduces a new unknown for each bin. We now have 3 unknowns - r_f, $\sin \delta_f$, and $\cos \delta_f$. The analysis then requires a minimum of 4 bins.
4. The only approach carried out so far is to parameterize the continuous variation of r_f and δ_f over phase space by using a sum of interfering Breit-Wigner resonances.

Step 1

Signal Dalitz PDFs





Step 1

Strong-phase Diff. & Amplitude Ratio

- The strong phase difference δ_D and relative amplitude r_D between the decays of D^0 and $\bar{D}^0$ to $\rho(770)^+\pi^-$ state are defined, neglecting direct CP violation in D decays, by the equation:

$$r_D e^{i\delta_D} = \frac{a_{D^0\to\rho^-\pi^+}}{a_{D^0\to\rho^+\pi^-}} e^{i(\delta_{\rho^-\pi^+} - \delta_{\rho^+\pi^-})}$$

- We find

BaBar	Cleo
r_D = 0.714 ± 0.008 (stat) ± 0.003 (syst) **δ_D = -2.0° (stat) ± 0.6° ± 0.6° (syst)**	**r_D = 0.65 ± 0.03 (stat) ± 0.04 (syst)** **δ_D = -4° ± 3° (stat) ± 4°(syst)**
Hep-ex / 0703037 (2007)	Hep-ex / 0306048 (2003)

These measurements are consistent with each other.

Step 1

Introducing Angular Moments

Schrödinger's Equation

$$\boxed{-\frac{\hbar}{2\mu}\nabla^2\Psi(\vec{r}) + V(\vec{r})\Psi(\vec{r}) = E\Psi(\vec{r})}$$

$$\begin{cases} V(\vec{r}) = 0 \\ \vec{k} = \dfrac{\vec{p}}{\hbar} = \mu\dfrac{\vec{v}}{\hbar} \qquad \mu = \dfrac{m_1 m_2}{m_1 + m_2} \end{cases}$$

$$|i\rangle = \Psi_i = \sum_{l=0}^{\infty} U_l(r) P_l(\cos\vartheta)$$

Angular Amplitude

$$\Psi_S = \Psi_f - \Psi_i = \frac{1}{k}\sum_{l=0}^{\infty} \boxed{(2l+1)\frac{\eta_l e^{2\imath\delta_l} - 1}{2\imath}} P_l(\cos\vartheta)\frac{e^{\imath kr}}{r}$$

Dynamic Amplitude (BW, Flatte, S-wave)

In case only l = 0 (S-wave) and 1 (P-wave) amplitudes are present :

$$\begin{cases} \sqrt{4\pi}\left\langle Y_0^0 \right\rangle = S^2 + P^2 \\ \sqrt{4\pi}\left\langle Y_1^0 \right\rangle = 2|S||P|\cos\phi_{SP} \\ \sqrt{4\pi}\left\langle Y_2^0 \right\rangle = \dfrac{2}{\sqrt{5}} P^2 \end{cases}$$

For S- and P- waves only, in the absence of cross-feeds from other channels, the amplitudes and the relative phase are given by:

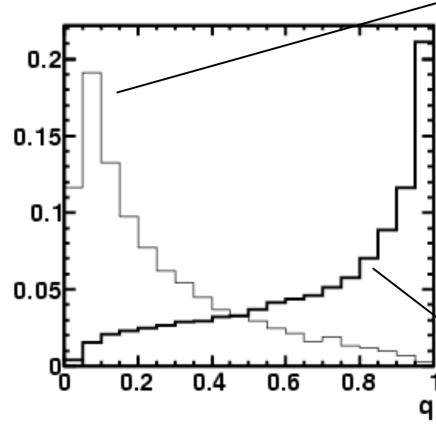
We cannot solve these Eqs for the $\pi\pi$ system (due to crossfeeds) to extract |S|, |P|, and cos ϕ_{SP} in a model independent way.

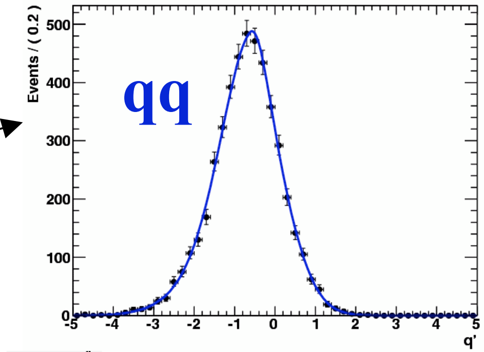
Step 2

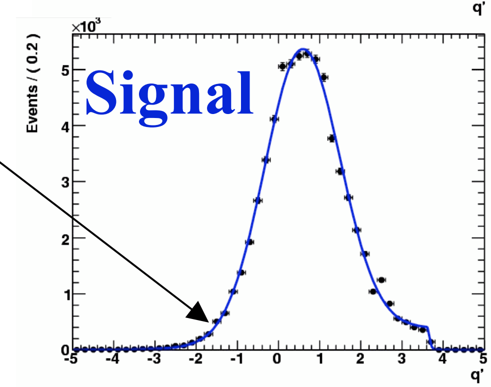
BR & Asymmetry for $B^{\pm} \to D_{\pi^-\pi^+\pi^0} K^{\pm}$

"Normalize" neural net variables q & d

$q \to q' = \tanh^{-1}[(q - ½(q_{max}+q_{min}) / ½(q_{max} - q_{min})]$





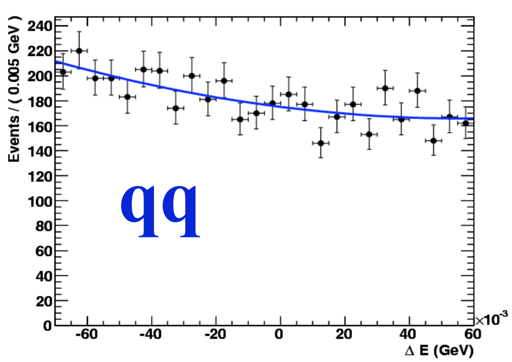


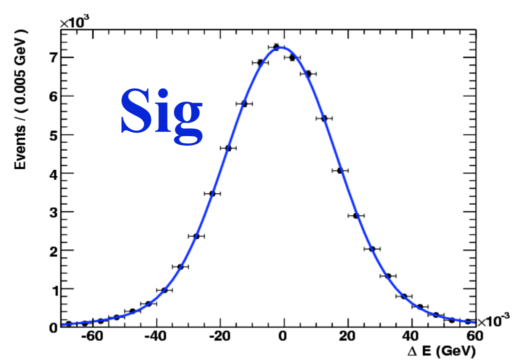
Fit $B^- \to D_{\pi\pi\pi^0} K^-$ sample with ΔE, q, d

Obtain signal yield & asymmetry

Nsig	170 ± 29
Asym	-0.02 ± 0.15

ΔE PDFs are Gaussian and 2nd-order polynomial:



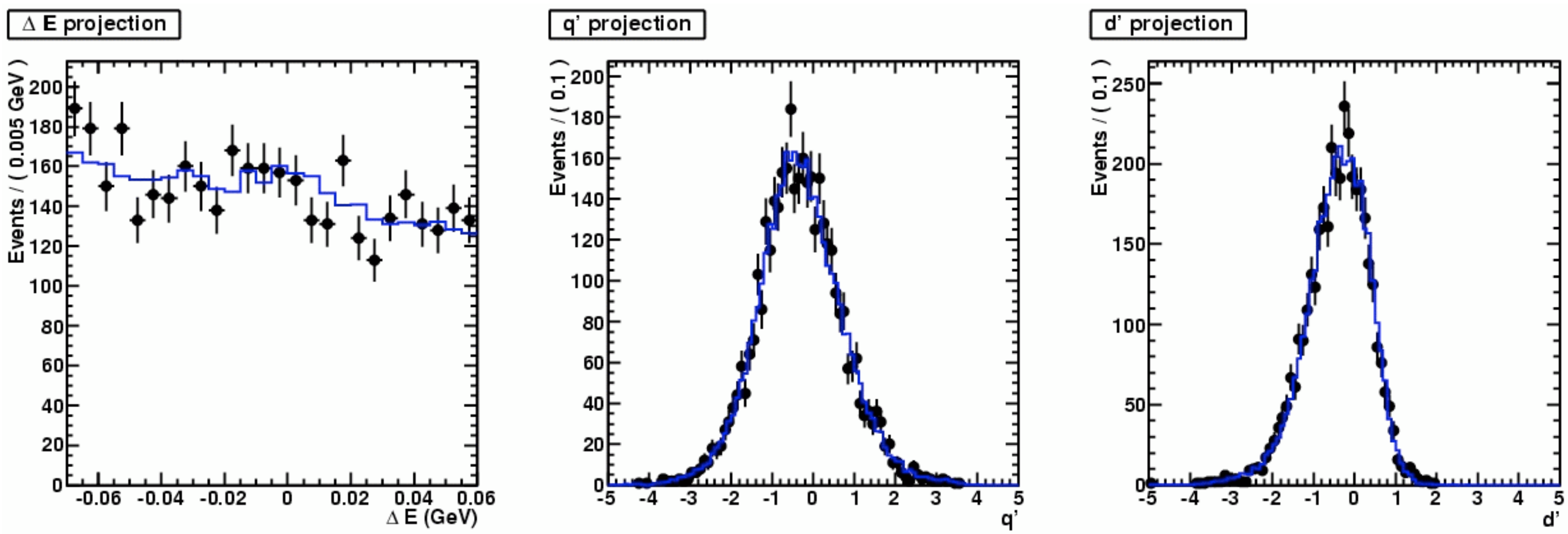


$BR(B^- \to D_{\pi\pi\pi0} K^-) =$
$(4.6 \pm 0.8 \pm 0.7) \times 10^{-6}$

$A(B^- \to D_{\pi\pi\pi0} K^-) =$
$-0.02 \pm 0.15 \pm 0.03$

Step 2

BR of $B^{\pm} \to D_{\pi^-\pi^+\pi^0} K^{\pm}$: Fit Projections



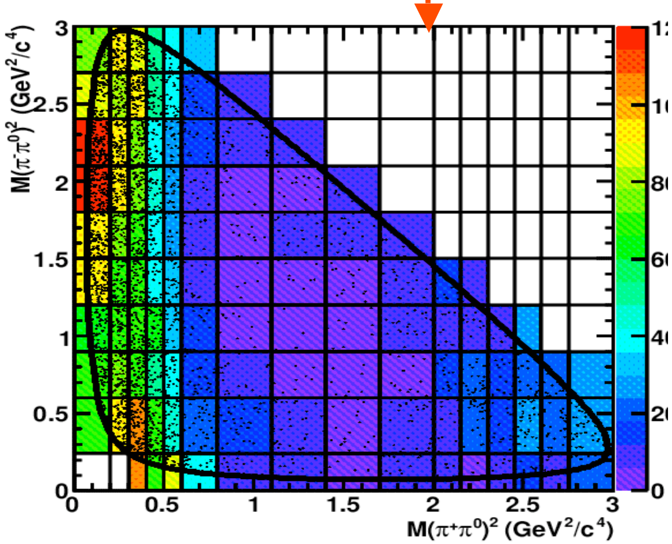
Nsig	170 ± 29
Asym	-0.02 ± 0.15
$N_{BB\ fake\ D}$	1138 ± 76
$N_{qq\ fake\ D}$	2383 ± 71
$N_{D\pi}$	57 ± 20
$N_{D\pi X}/N_{BB}$	0.53 ± 0.15

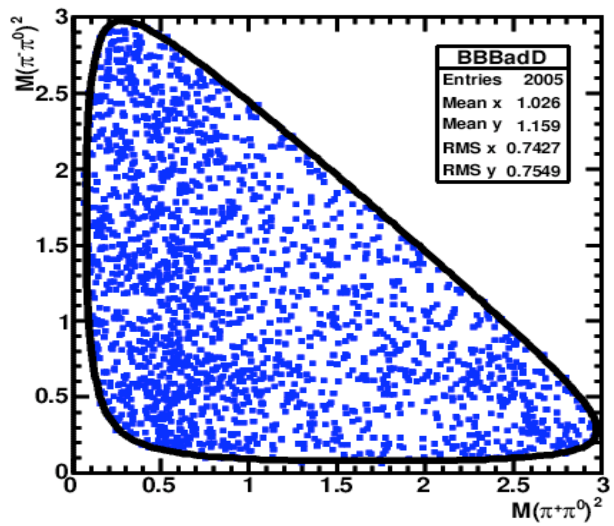
$BR(B^- \to D_{\pi\pi\pi 0} K^-) = (4.6 \pm 0.8 \pm 0.7) \times 10^{-6}$

Step 3

$B^{\pm} \to D_{\pi^-\pi^+\pi^0} K^{\pm}$: Bkg Dalitz Shapes

- Fake-D background Dalitz shapes are NR + 3 incoherent, unpolarized ρ's:
- Shape for 2 event types can't be fit to this way.
  We use an empirical shape from simulation:





For CP Fit

- Fit $D^0 \to \pi^+\pi^-\pi^0$ Dalitz plot from $B^- \to D_{\pi\pi\pi^0} K^-$ sample with ΔE, q, s^+, s^-
- NN variable d not used – highly correlated with s^+, s^-
- m_{ES} and M_D not used – correlated with other variables for the background

Step 3

CP Parameters: Max Likelihood Fit

- To make use of both the shape and the absolute decay rates, we minimize the function

$$\boxed{L = L_{DP} + L_{BA}}$$

$$\boxed{L_{DP} = -\log \prod P_{DP}}$$

$$\boxed{L_{BA} = \tfrac{1}{2}\, Y_i\, V_{ij}^{-1}\, Y_j}$$

$$Y = \begin{pmatrix} N_{meas} - N_{expected} \\ Asym_{meas} - Asym_{expected} \end{pmatrix}$$

V = error matrix from N and Asym fit

$$\underbrace{N^{\pm}_{expected} = \eta}_{1/2\, N_{BB}\, \varepsilon\, BR(D^0 \to \pi\pi\pi^0)\, BR(B^- \to D^0 K^-)} \int |A^{\pm}(s^+,s^-)|^2\, \varepsilon(s^+,s^-) \Big/ \int |f_D(s^+,s^-)|^2\, \varepsilon(s^+, s^-)$$

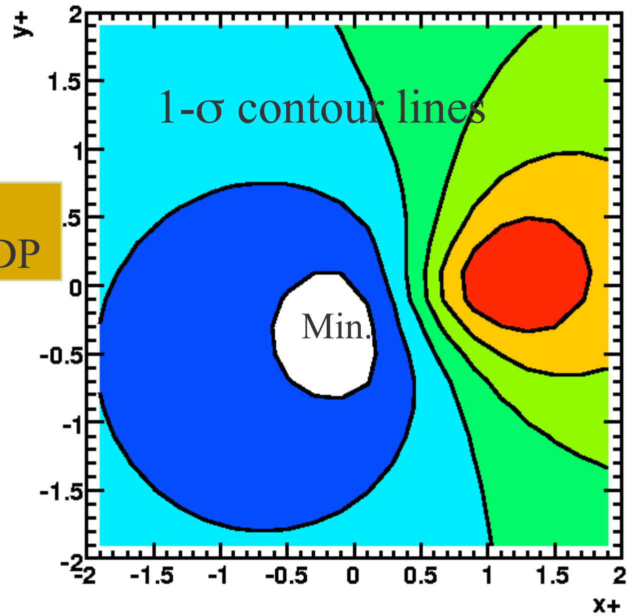
Step 3

Behavior of L_{DP} & L_{BA} for $x_{true} = y_{true} = 0$

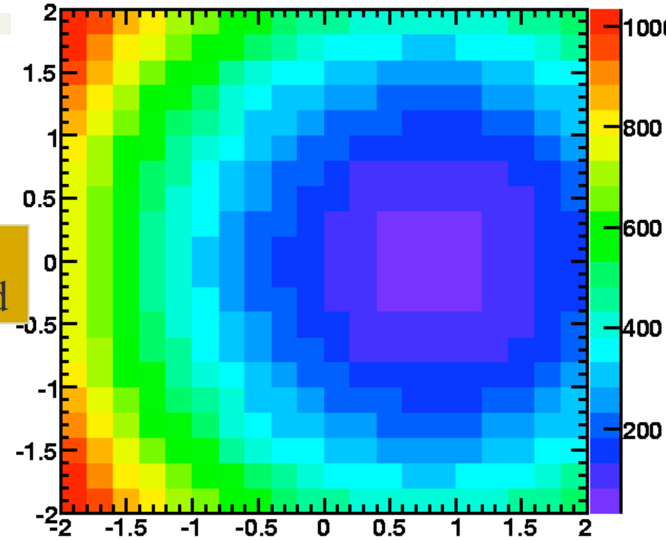
$$r_B e^{i(\delta \pm \gamma)} = x_\pm + y_\pm$$

Toy exp., S+B

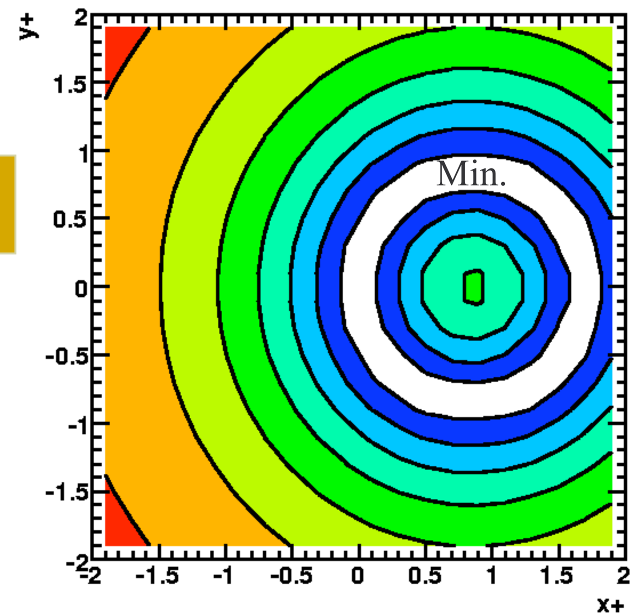
L_{DP}



$N^{+}_{expected}$



L_{BA}



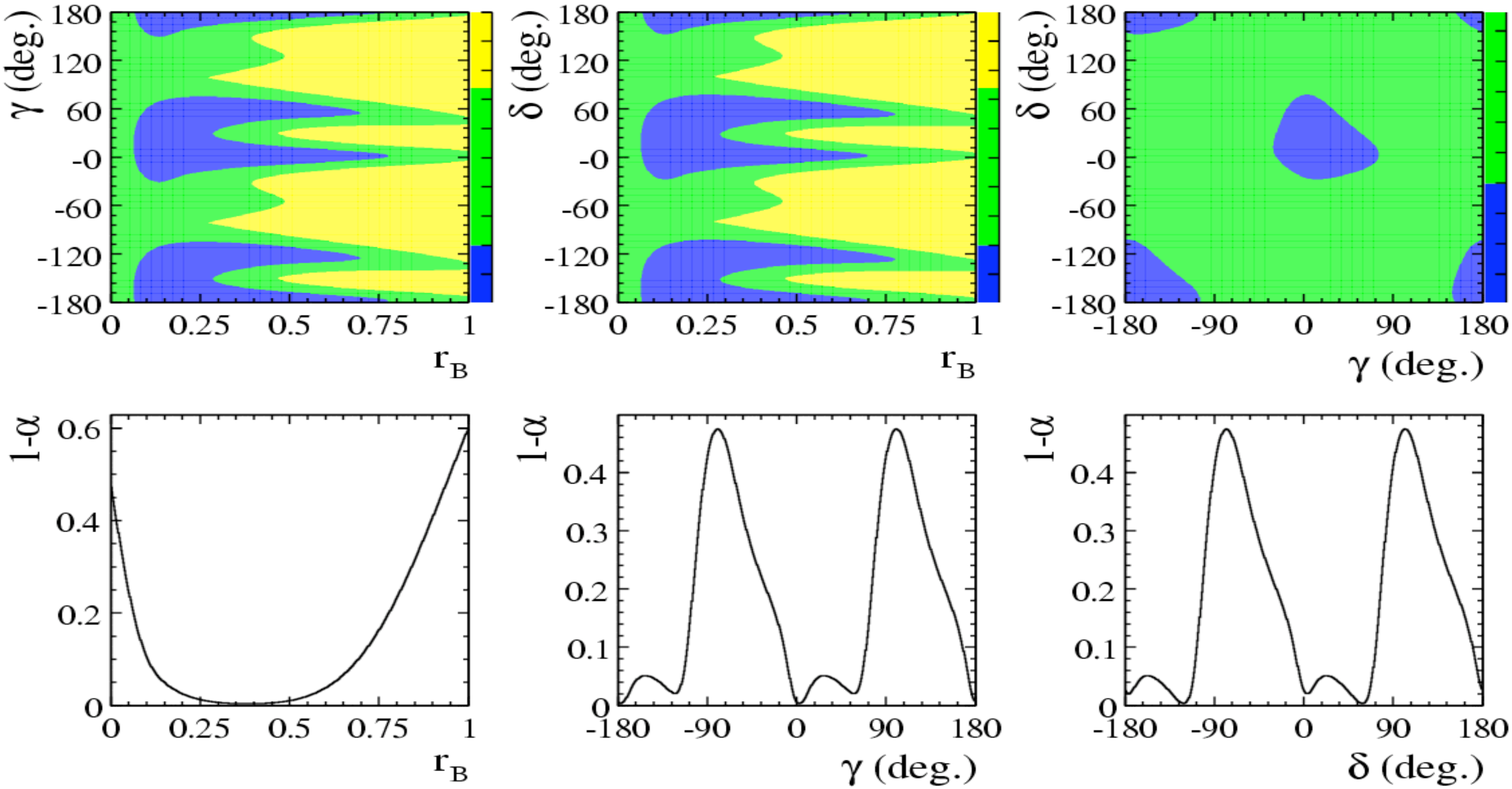
- L_{DP} (L_{BA}) has Cartesian (polar) symmetry
- L_{BA} is more sensitive (denser contour lines) in radial direction (ρ), not sensitive at all in θ

Step 3

From ($\rho_\pm$, $\theta_\pm$) to (r_B, δ, γ)

Use frequentist method to extract γ, r_B,δ_B from ($\rho_\pm$,$\theta_\pm$)

(3dim confidence intervals projections)



Step 3

Systematics details

- Dalitz Model:

Dalitz model	ρ_-	θ_-	ρ_+	θ_+
$\mathrm{NR}_S,\ \rho(770)$	0.0633	17.70	0.0359	-7.30
$+\ f_0(980)$	0.0583	22.86	0.0260	4.63
$+\ \rho(1450)$	0.0010	7.20	-0.0138	-8.50
$+\ \rho(1700)$	0.0248	4.12	0.0043	-10.46
$+\ f_0(1370,1500,1710),\ f_2(1270)$	-0.0249	-11.89	-0.0287	-1.67
$+\ \sigma$	0	0	0	0
$+\ \mathrm{NR}_P$	0.0106	-0.23	0.0086	-1.46
$+\ \omega,\ f_2'(1525)$	0.0091	2.66	0.0077	-2.07
$R=0$	0.0017	-8.56	0.0005	-0.09

- BR:

Source	BF error (%)	Section
PID efficiency	3.1	13.12
π^0 efficiency	3.0	13.16
Tracking efficiency	1.5	13.17
B counting	1.1	13.18
Total	4.70	

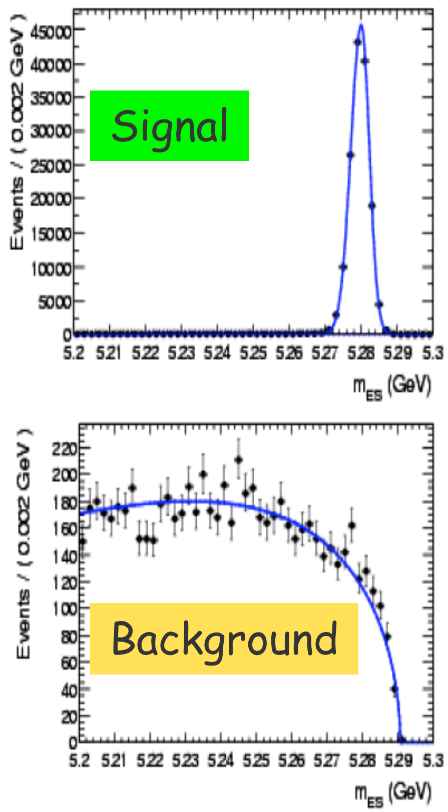
- CP systematics

Source	ρ_-	θ_-	ρ_+	θ_+	Section
$\mathcal{B}(B^- \to D^0 K^-)$	0.0288	1.56	0.0277	1.05	13.19
$\mathcal{B}(D^0 \to K^-\pi^+\pi^0)$	0.0174	0.88	0.0167	0.66	13.19
$\frac{\mathcal{B}(D^0\to\pi^+\pi^-\pi^0)}{\mathcal{B}(D^0\to K^-\pi^+\pi^0)}$	0.0058	0.01	0.0056	0.01	13.19
Signal efficiency	0.0148	0.02	0.0141	0.03	13.19
$N_{B\overline{B}}$	0.0049	0.01	0.0046	0.01	13.19
Total	0.0375	1.79	0.0360	1.24	

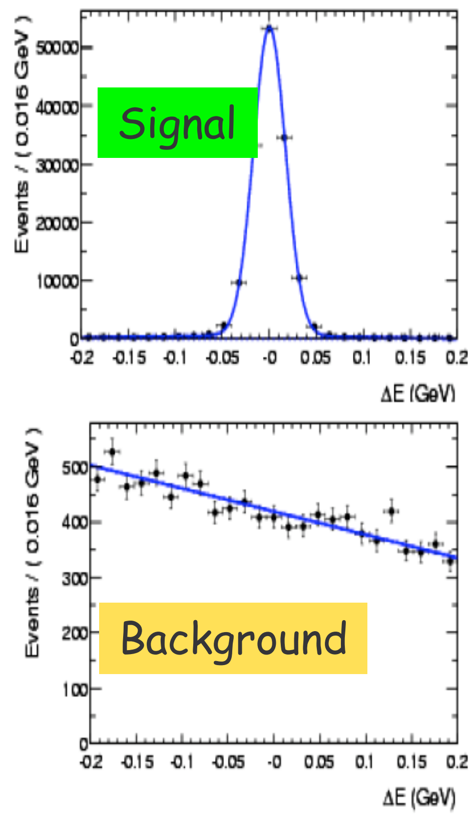
γ: Key Analysis Technique

Exploit kinematics of $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B\bar{B}$ for signal selection

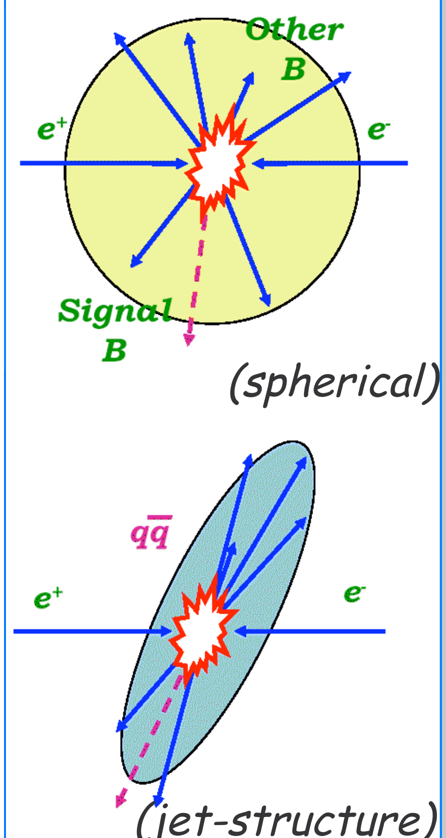
$$m_{ES} = \sqrt{E_{beam}^{*2} - p_B^{*2}}$$



$$\Delta E = E_B^* - E_{beam}^*$$



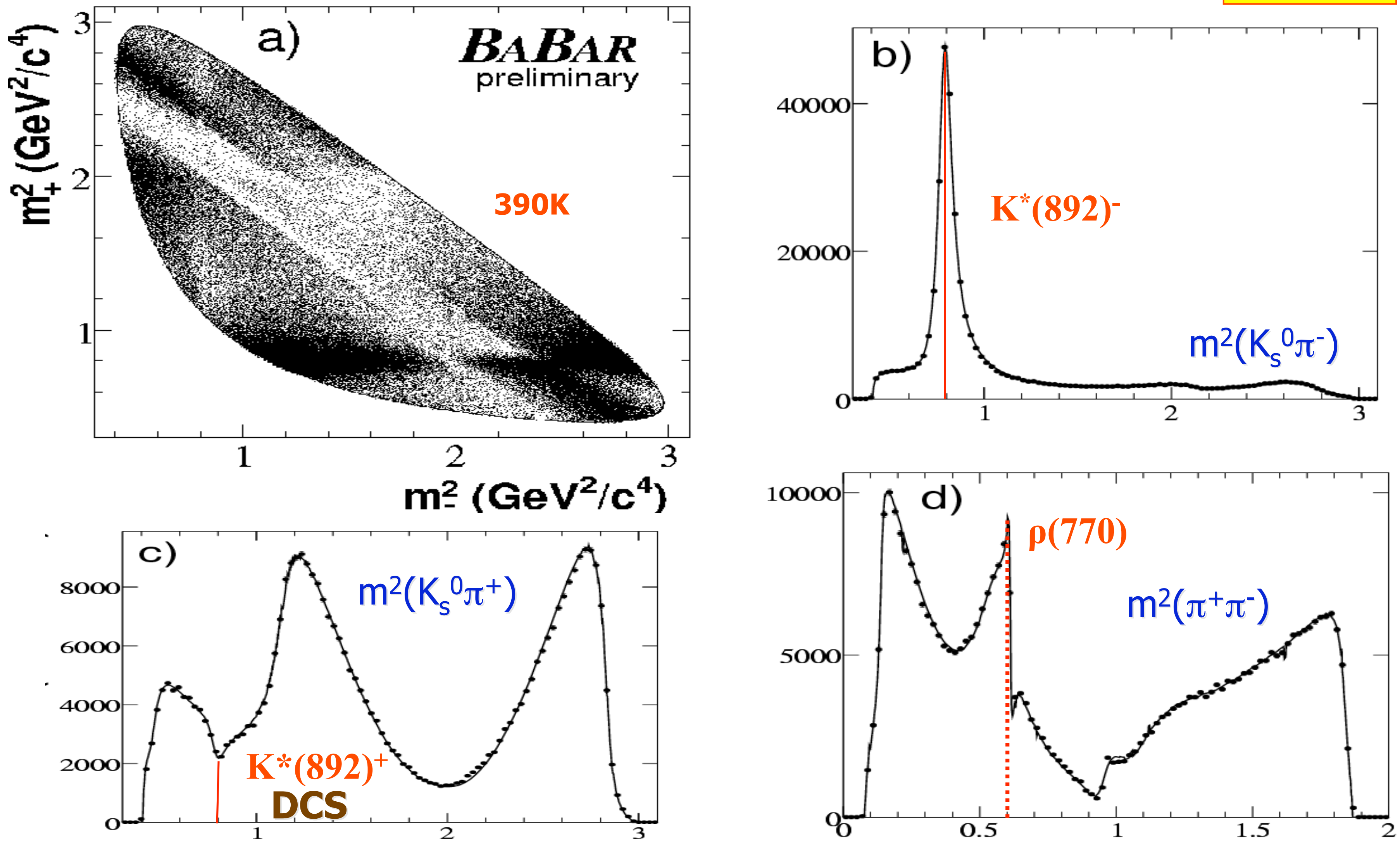
Event topology



$D^0 \to K_S^0\pi^+\pi^-$ Dalitz Plot analysis

Motivation: CKM angle γ using $B \to D[K_S^0\pi^+\pi^-]K^-$ decay

270 fb^{-1}



$D^0 \rightarrow K_s^0 \pi^+ \pi^-$ (Isobar Model)

BABAR™

Component	$Re\{a_r e^{i\phi_r}\}$	$Im\{a_r e^{i\phi_r}\}$	Fit fraction (%)
$K^*(892)^-$	-1.223 ± 0.011	1.3461 ± 0.0096	58.1
$K_0^*(1430)^-$	-1.698 ± 0.022	-0.576 ± 0.024	6.7
$K_2^*(1430)^-$	-0.834 ± 0.021	0.931 ± 0.022	3.6
$K^*(1410)^-$	-0.248 ± 0.038	-0.108 ± 0.031	0.1
$K^*(1680)^-$	-1.285 ± 0.014	0.205 ± 0.013	0.6
$K^*(892)^+$ DCS	0.0997 ± 0.0036	-0.1271 ± 0.0034	0.5
$K_0^*(1430)^+$ DCS	-0.027 ± 0.016	-0.076 ± 0.017	0.0
$K_2^*(1430)^+$ DCS	0.019 ± 0.017	0.177 ± 0.018	0.1
$\rho(770)$	1	0	21.6
$\omega(782)$	-0.02194 ± 0.00099	0.03942 ± 0.00066	0.7
$f_2(1270)$	-0.699 ± 0.018	0.387 ± 0.018	2.1
$\rho(1450)$	0.253 ± 0.038	0.036 ± 0.055	0.1
Non-resonant	-0.99 ± 0.19	3.82 ± 0.13	8.5
$f_0(980)$	0.4465 ± 0.0057	0.2572 ± 0.0081	6.4
$f_0(1370)$	0.95 ± 0.11	-1.619 ± 0.011	2.0
σ (490, 406)	1.28 ± 0.02	0.273 ± 0.024	7.6
σ'(1024, 89)	0.290 ± 0.010	-0.0655 ± 0.0098	0.9

$K^*(892)^-$: 58 %
$\rho(770)^0$: 22 %
Non-Res.: 8 %
σ (500): 8 %
$K^*(1430)^-$: 7 %
$f_0(980)$: 6 %

Important for γ and D-mixing measurements

hep-ex/0607104

The ‘Cartesian coordinates’

- Goal: Fit the Dalitz plot distributions of $D^0 \rightarrow K_S\pi\pi$ from B^- and B^+ decays to extract r_B, δ_B and γ
- Complication: The Maximum Likelihood fit overestimates r_B and underestimates the error of γ
- Solution: Write the Likelihood as a function of the cartesian coordinates $x_\pm$, $y_\pm$:

$$x_\mp = r_B \cos(\delta_B \mp \gamma)$$
$$y_\mp = r_B \sin(\delta_B \mp \gamma)$$

$$\Gamma(B^+) \propto |f_+|^2 + (x_+^2 + y_+^2)|f_-|^2 + 2x_+ \,\mathrm{Re}\left(f_+ f_-^*\right) + 2y_+ \,\mathrm{Im}\left(f_+ f_-^*\right)$$
$$\Gamma(B^-) \propto |f_-|^2 + (x_-^2 + y_-^2)|f_+|^2 + 2x_- \,\mathrm{Re}\left(f_- f_+^*\right) + 2y_- \,\mathrm{Im}\left(f_- f_+^*\right)$$

$$f_\mp \equiv A_D(m_\mp^2, m_\pm^2)$$

Likelihood is Gaussian and unbiased in $x_\pm$, $y_\pm$

- Strategy: Extract $x_\pm$, $y_\pm$ from ML fit to the $D^0 \rightarrow K_S\pi\pi$ Dalitz plot and derive r_B, δ_B and γ from $x_\pm$, $y_\pm$ with stat. procedure

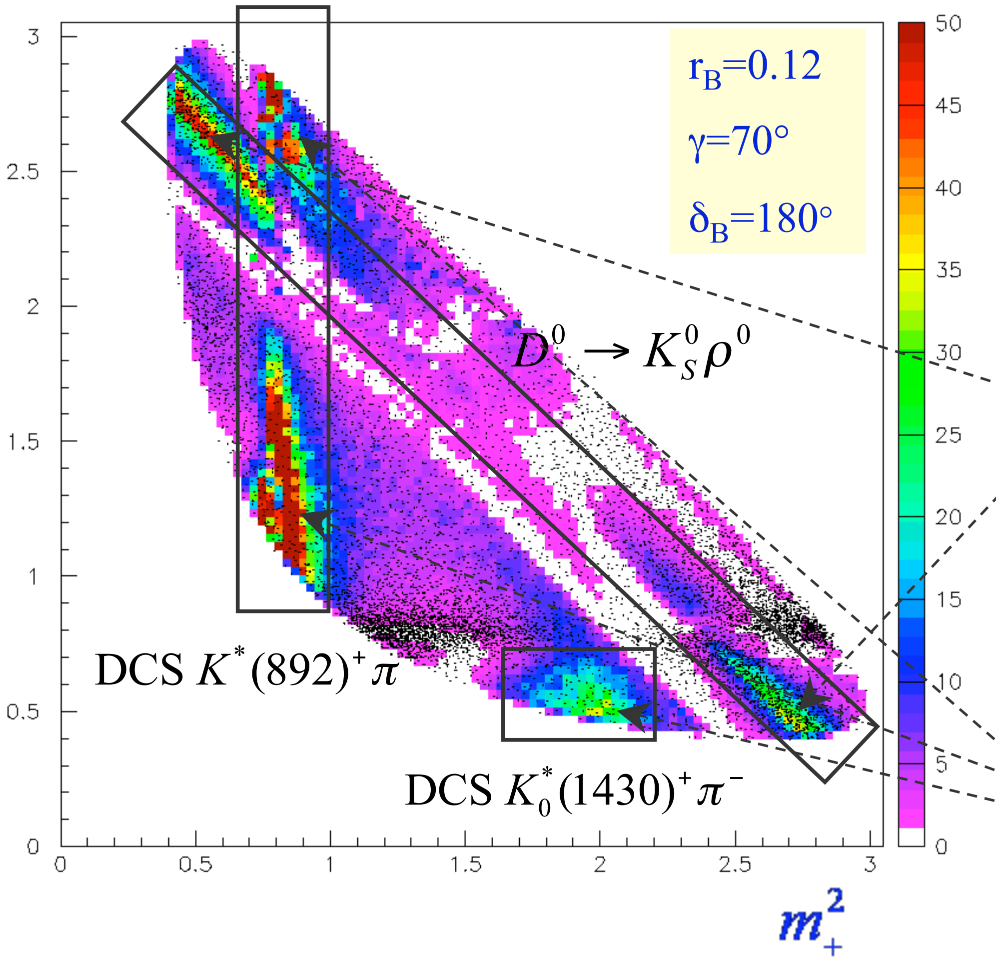
Sensitivity to γ over Dalitz plot

- Sensitivity varies strongly over Dalitz plane
- 2nd derivative of the log(L) event-by-event weighs the event

$$\sigma^2(\gamma) \sim \frac{1}{\frac{d^2 \ln(L)}{d\gamma^2}}$$

$$\text{weight} = \frac{d^2 \ln(L)}{d\gamma^2}$$

events: points (weight = 1)



Interference of $B^- \to D^0[\to K_S^0 \rho^0]K^-$

with $B^- \to \overline{D}^0[\to K_S^0 \rho^0]K^-$

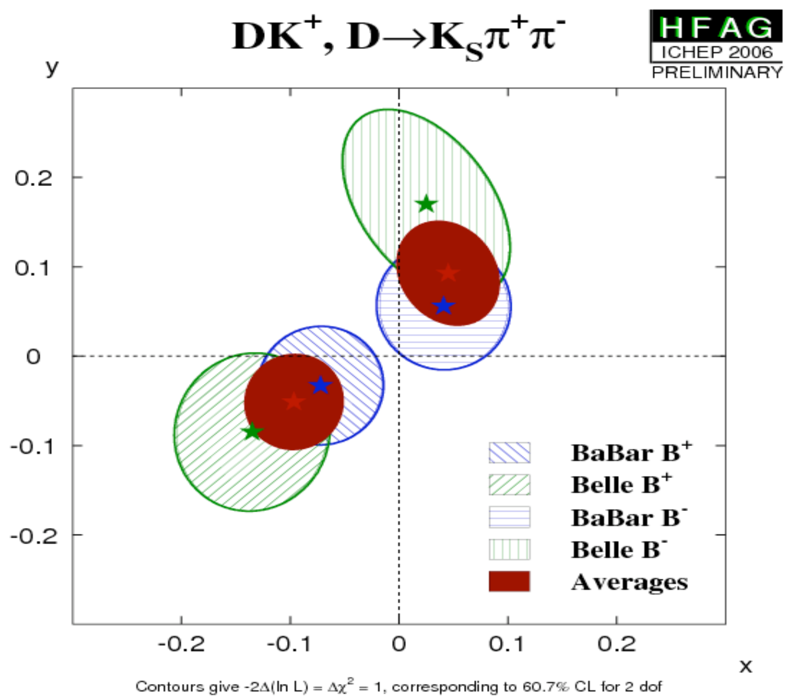
≡ GLW like

Interference of $B^- \to D^0[\to K^{*+}\pi^-]K^-$

(suppressed) with $B^- \to \overline{D}^0[\to K^{*+}\pi^-]K^-$

≡ ADS like

γ from $B^{\pm} \to D_{K_S^0 \pi^- \pi^+} K^{\pm}$, role of r_B



BaBar: $\gamma = (92 \pm 41 \pm 10 \pm 13)^{\circ}$

Belle: $\gamma = (53^{+15}_{-18} \pm 3 \pm 9)^{\circ}$

[D*K included]

better precision of BaBar (x,y) does NOT translate to a smaller error on γ. Why?

$\Delta x \approx \Delta y \approx r_B \Delta\theta$ ➔ $\Delta\gamma \sim 1/r_B$

the error of γ is ~ proportional to the uncertainty in (x,y) and inversely proportianal to the distance from (0,0).

Belle measurement is consistent with larger r_B.

Development of New Identification Selectors for K, π, P, and e

1. *"BDT Kaon"* **Selectors:**

- to replace *kaon neural net*, used in B-tagging
- use *Bagger Decision Tree* algorithm to separate kaon signal from pion background
- will continue to provide kaon id at 4 levels of strictness: Very Loose, Loose, Tight, Very Tight

2. *"KM"* **Selectors:**

- separate **K, π, p, e** from one another
- use multi-class learning
- will provide particle identification at 6 levels of strictness: Extra Loose, …, Extra Tight

Why New Selectors ?

- For B-tagging, need new Kaon selector to replace the old selectors.

- The kaon neural net hasn't been trained since circa 2001; there have been many changes in detector performance since then (e.g., **new dE/dx calibration**).
- Trained on MC, but are used to evaluate performance in real data.
- Give degraded performance for high-momentum tracks.

- For kaons, protons and pions, there is only one selector of choice for analysis: Likelihood-based. There is room for improvement.

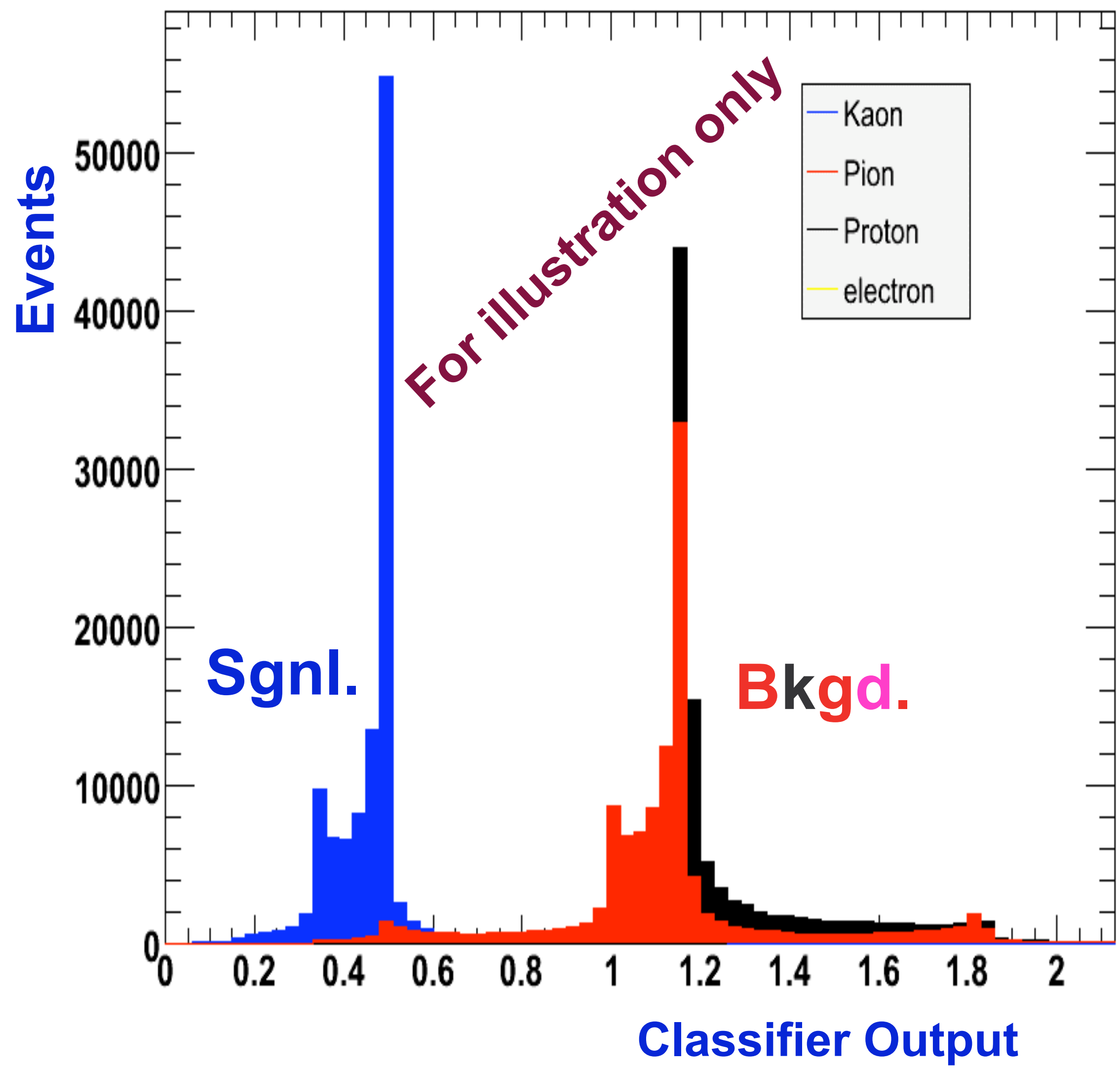
- For electron, the only available selector is likelihood-based.

- Some analyses (notably Leptonic) will benefit enormously from high-performance selectors for both low and high momentum tracks.
- Improvement in performance needed for crucial BaBar analyses looking for New Physics, rare decays, CP violation ….

What is New in the New Selectors ?

- Training on "real data".
- Include new corrections for dE/dx.
- Employ powerful statistical tools to separate signal and background, use bagging on weak classifier and multi-class training.
- For each class of particle hypothesis: "kaon", "pion", "proton", and "electron", the other three classes are treated as background for classifier training. Apart from "muon", no additional vetoes.
- Include many additional useful input variables, including P and θ after flattening the two-dimensional P: θ distribution. No need for separate trainings in P,θ bins.

Software Implementation: StatPatternRecognition



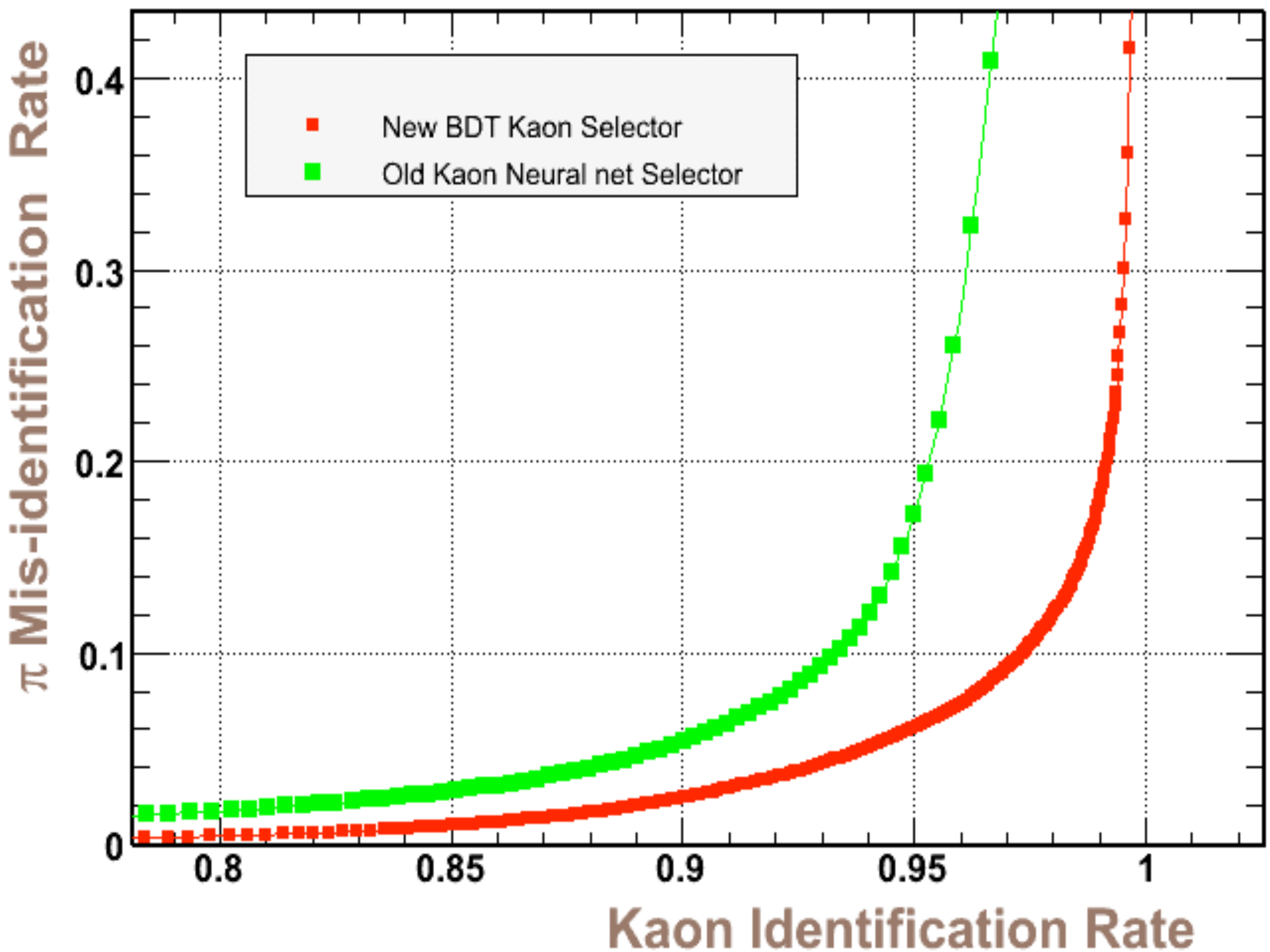
For details on the algorithms:

arXiv:physics/0507143

(by Ilya Narsky, CalTech)

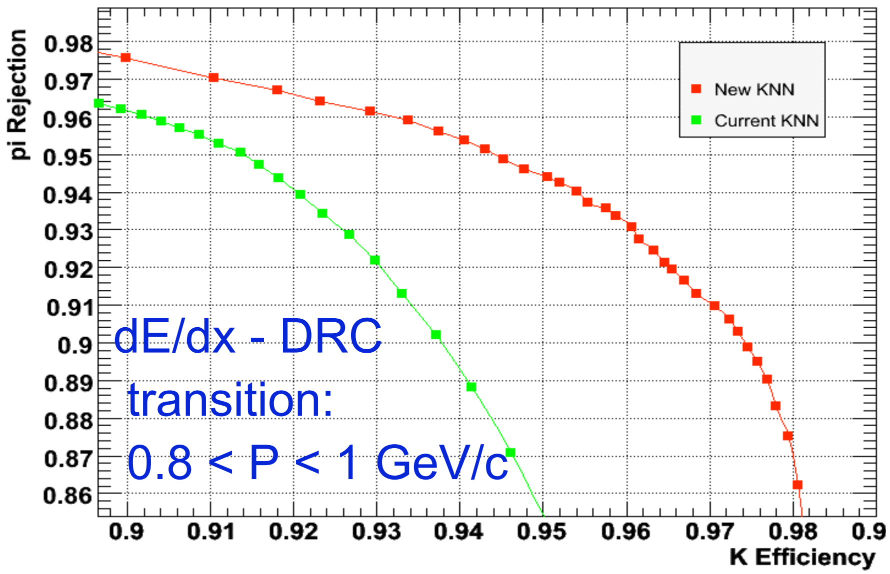
- Decision Tree splits nodes recursively until a stopping criteria is satisfied.
- Bagger decision tree divides the training data sample into a number of bootstrap replicas, and trains on each one of them separately.
- The final classification is done by majority vote.

Performance of BDT Kaon Selectors



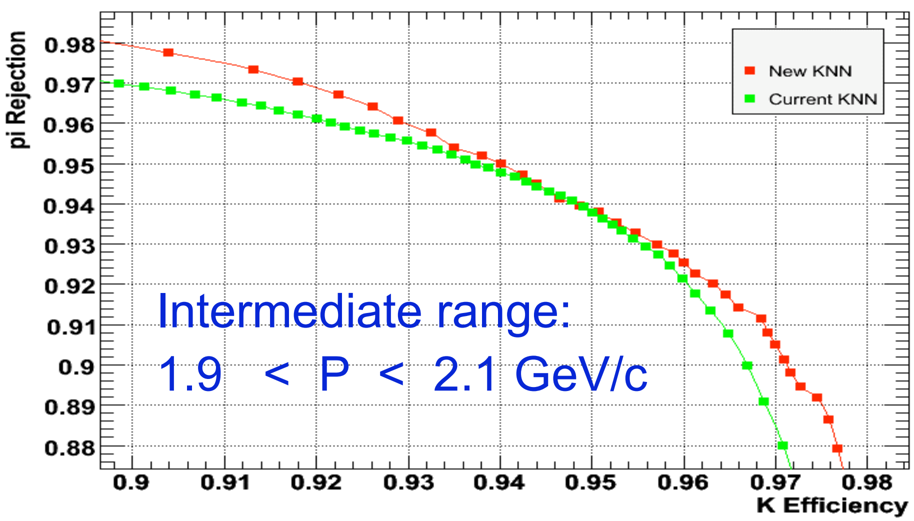
Includes all momentum and θ ranges and all tracks.

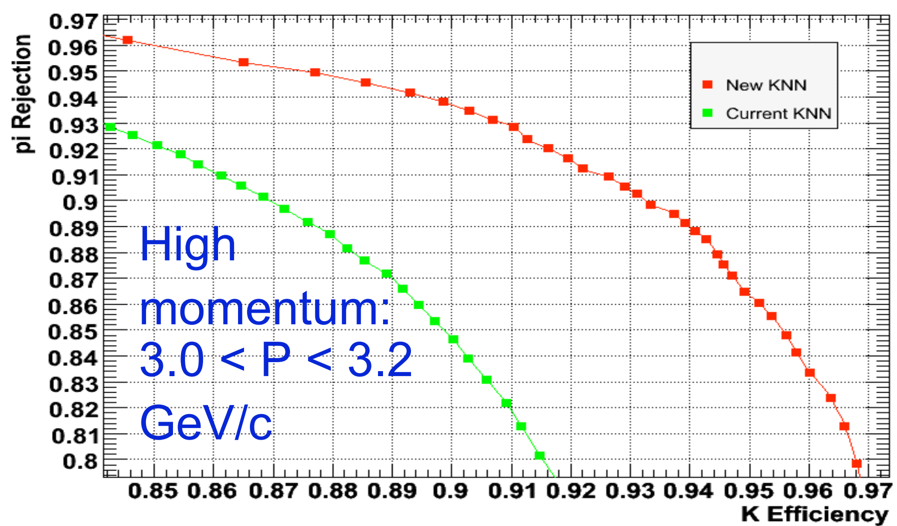
BDT Kaon Performance in Mom. Bins



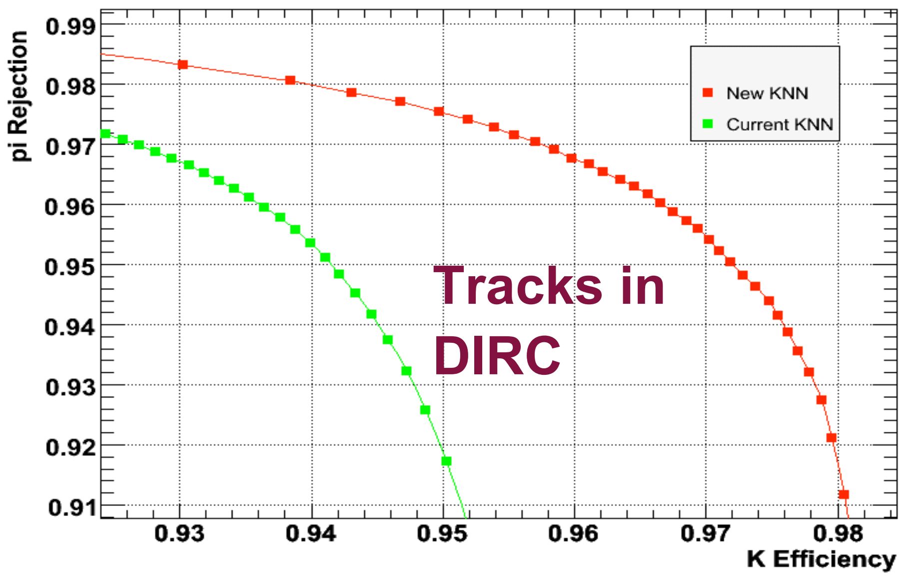
Caveat: I have changed the definition of the Y-axis variable.

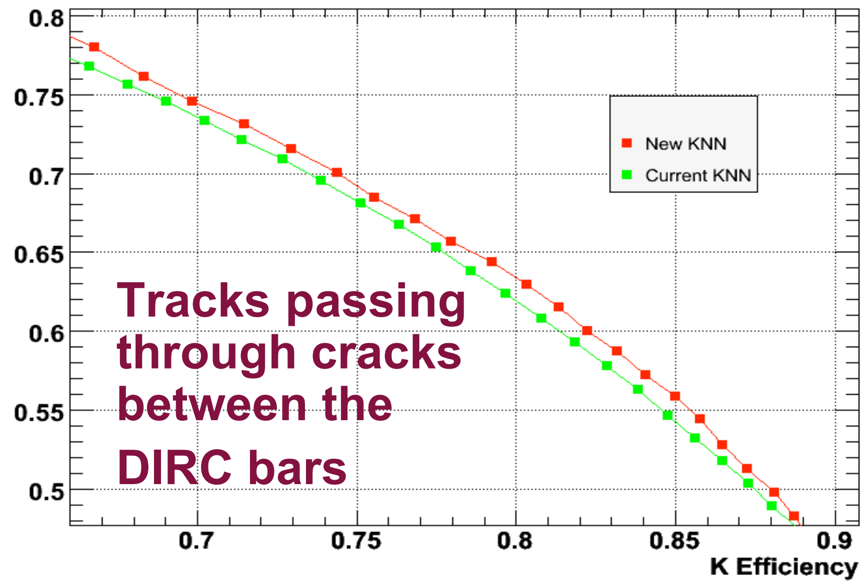
The higher curve/ point represents better performance

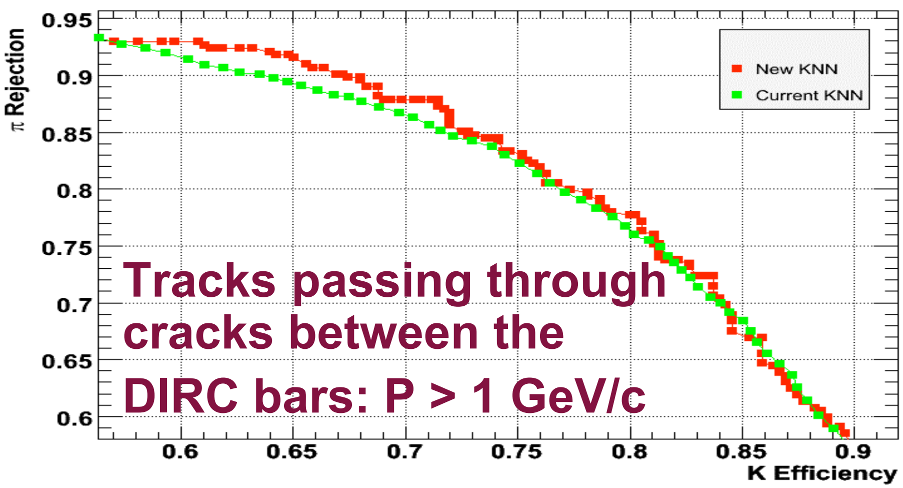




BDT Kaon Performance by Track Quality

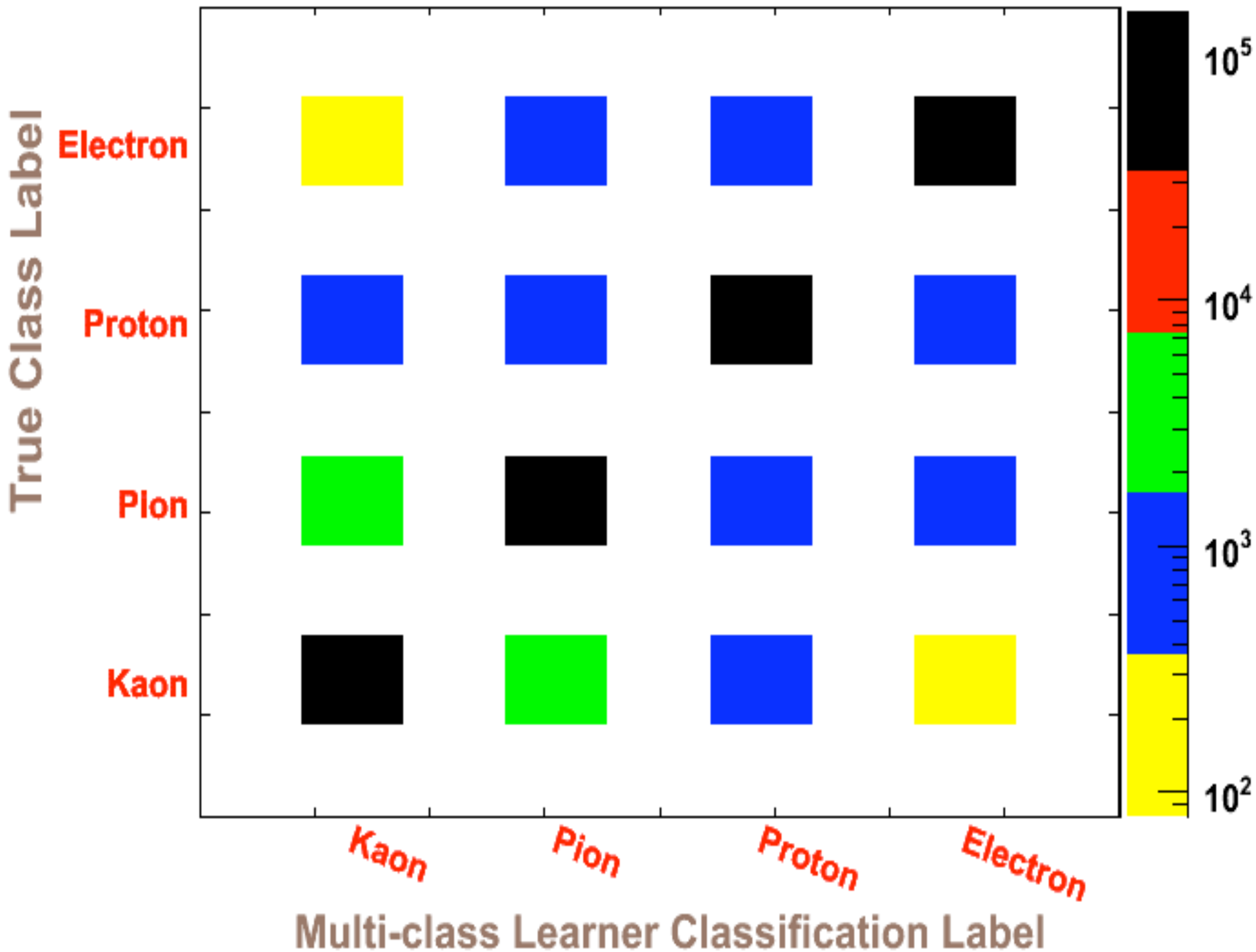






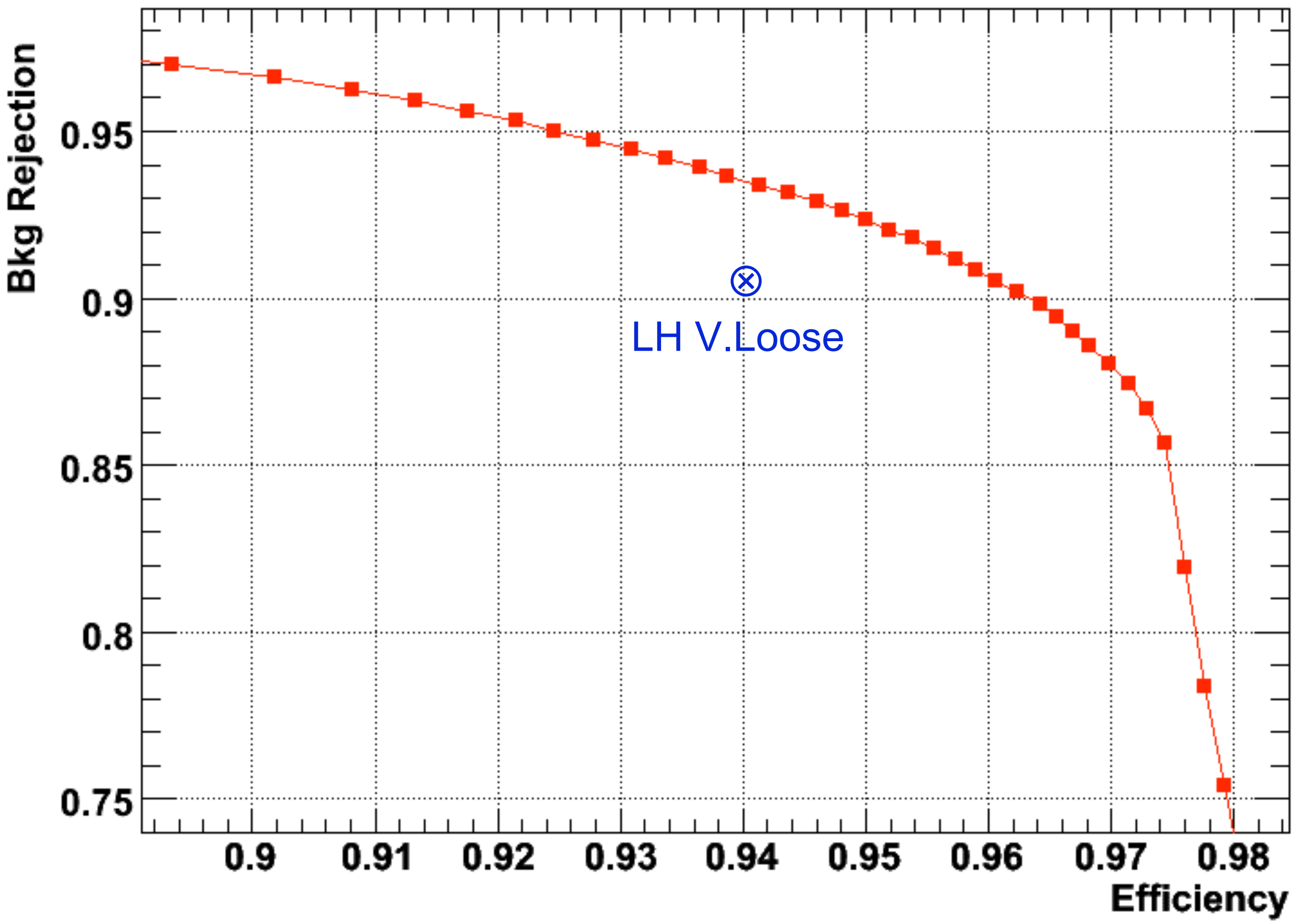
Conclusion: Improvement in performance everywhere.

Performance of KM Selectors



Number of data events used for each category = 1.22 x 10^5

Performance of Kaon Selector

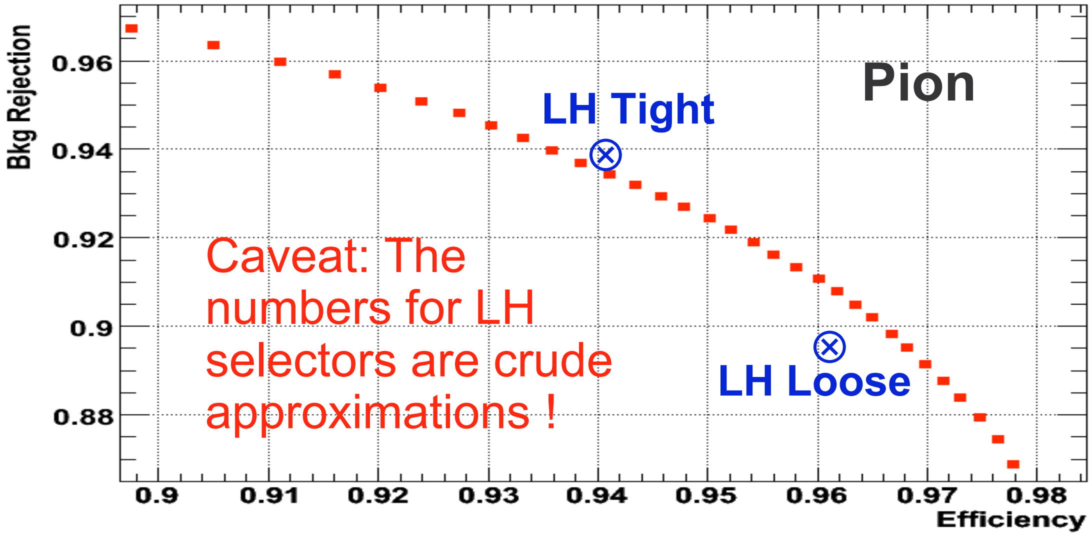


LH Loose:
Efficiency = 0.87
Pi Rej. = 0.96

LH VeryTight:
Efficiency = 0.82
Pi Rej. = 0.98

Caveat: These numbers are approximations !

Performance for Pion, Proton & Electron



Looks great ! …. But need to see performance in entire P, θ spectrum before declaring victory !