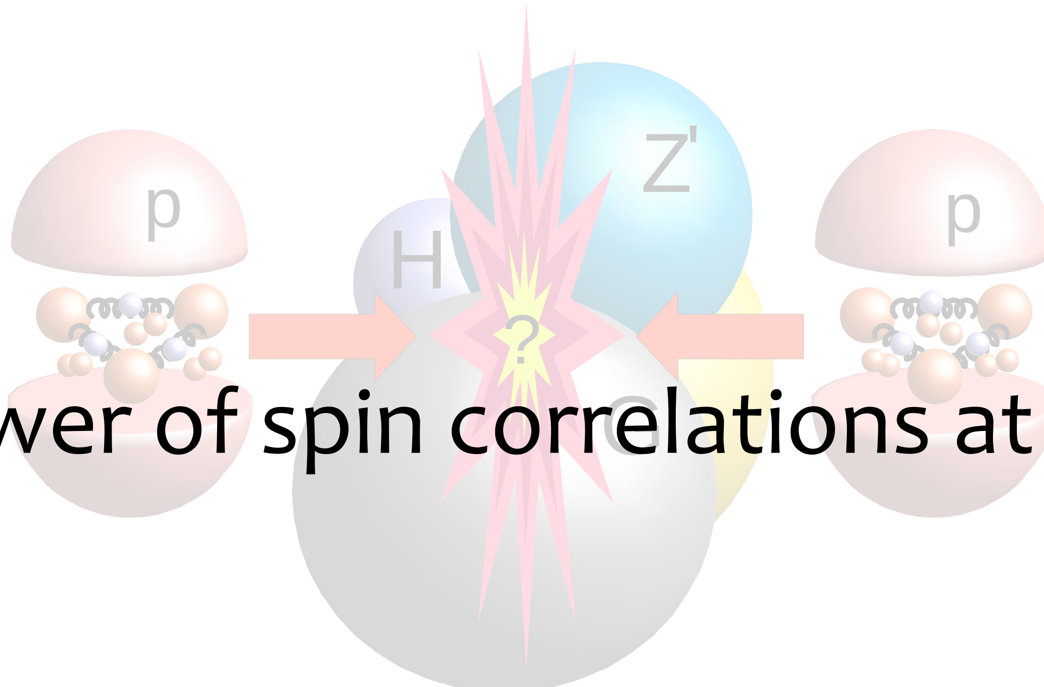


The power of spin correlations at the LHC

Nhan Tran

Johns Hopkins University

FNAL, 06.07.2011

The power of spin correlations

Such techniques have been used for nearly 50 years

PHYSICAL REVIEW VOLUME 126, NUMBER 5 JUNE 1, 1962

Parity of the Neutral Pion and the Decay $\pi^0 \to 2e^+ + 2e^-$†

N. P. Samios
Columbia University, New York, New York and Brookhaven National Laboratory, Upton, New York

AND

R. Plano,* A. Prodell,‡ M. Schwartz, and J. Steinberger
Columbia University, New York, New York

(Received January 17, 1961)

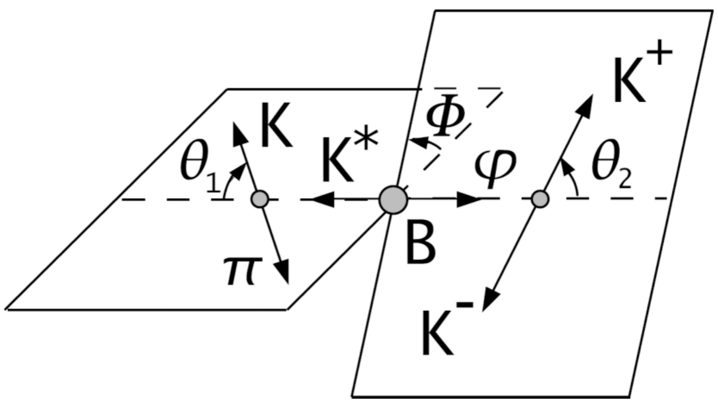
Two hundred and six electronic decays of the π^0, $\pi^0 \to e^+ + e^- + e^+ + e^-$, have been observed in a hydrogen bubble chamber. The decay distributions of the electron pairs and the total rate for this process are shown to be in good agreement with theory. An examination of correlations of the e^+e^- pair decay planes on the basis of electrodynamic predictions is in agreement with the hypothesis that the π^0 is pseudoscalar, but disagrees for scalar pions by 3.6 standard deviations.

Much of presented techniques and experience comes from *B* decays:

One of the lessons learned is that there can be surprises… in the SM or beyond.

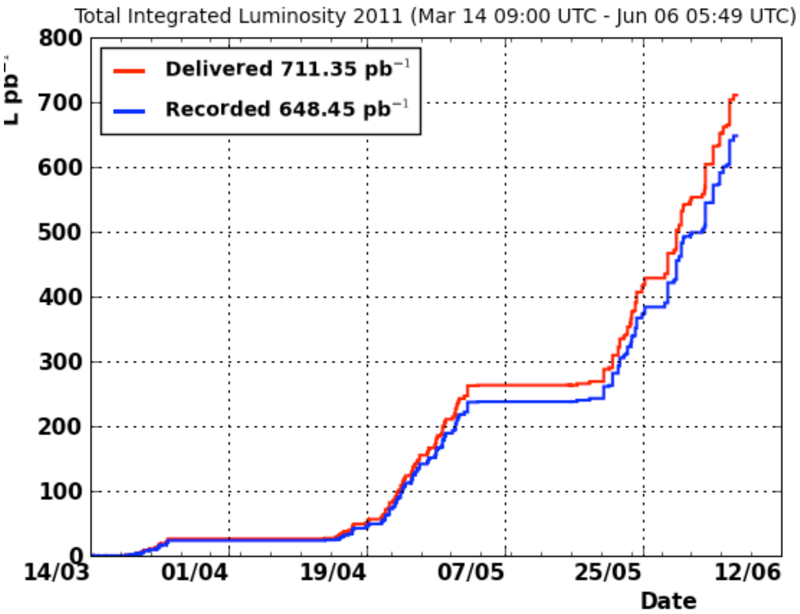
Example $B \to \phi K^*$:

BaBar Collaboration, Phys. Rev. D 78, 092008 (2008)



CMS in 2010 and beyond

- An exciting time to be a member of CMS!
- Many physics results from electroweak to supersymmetry to exotica in 2010 with ~36 pb^{-1}
- Large dataset (~fb^{-1}) on the horizon for 2011/2012!



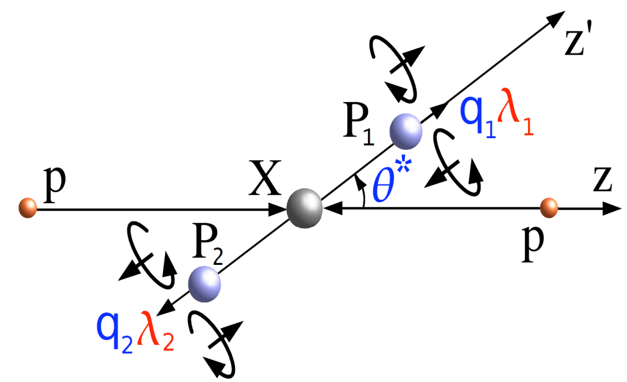
- Important to refine analysis techniques and approach to maximize our sensitivity
- Spin correlations fundamental way understand event shape and kinematics and connecting to underlying physics

Spin correlations at the LHC

- We discuss the application of spin correlations at the LHC in two cases
  - EWK - measurement of weak mixing angle
    - Test SM parameters and understand expected sensitivity
    - Analysis of the Drell-Yan angular distribution
  - EWK/Higgs/BSM - spin/CP determination of resonances
    - If we find a new resonance at the LHC, how can we distinguish between different physics scenarios?
- Introduce formalism, techniques and tools for understanding decay kinematics
  - Multivariate likelihood analysis to extract maximal information

Conservation of angular momentum

- Helicity amplitude formalism
  - Projection of spin in momentum direction
  - massive spin J particle with helicity states
    λ = -J, -J+1, … , J-1, J



- For a given 1 ⟶ 2 process with initial spin J and projection m, decaying to helicity states λ_1, λ_2:

$$\boxed{\langle \Omega, \lambda_1, \lambda_2 | S | J m \rangle = \sqrt{\frac{(2J+1)}{4\pi}} D^{J*}_{m,\lambda_1-\lambda_2}(\Omega) A_{\lambda_1 \lambda_2}}$$

  - Rotational generators for SU(2) - Wigner functions

$$D^{J*}_{m,\lambda_1-\lambda_2} = e^{-im\phi} d^{J}_{m,\lambda_1-\lambda_2}(\theta) e^{i(\lambda_1-\lambda_2)\phi}$$

  - A_{λ_1,λ_2} helicity amplitudes determined from specific couplings
  - Angles θ, Φ are polar and azimuthal directions in CM frame

Simple SM example: $Z/\gamma \to f\bar{f}$

Amplitude of Z/γ to fermions^

$$A(X_{J=1} \to q\bar{q}) = \epsilon^{\mu} \bar{u}_f \gamma_\mu (\rho_V + \rho_A \gamma_5) v_f$$

$$A_{+-} = \sqrt{2}(\rho_V + \rho_A)$$
$$A_{-+} = \sqrt{2}(\rho_V - \rho_A)$$

polarization vectors - Z: +1, (0), -1 and f: -½, ½

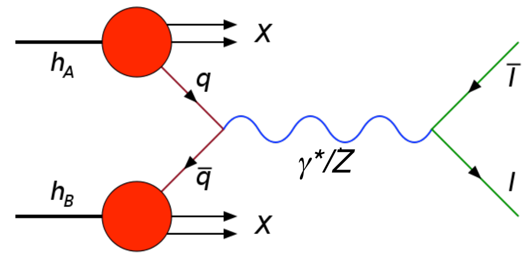
Based on fermion polarizations we can determine the helicity amplitudes to build angular distributions

We have the usual vector and axial vector couplings (ρ_V, ρ_A) which can be expressed in terms of the weak mixing angle ($\sin^2\theta_W$)

process	$\rho_1^{(1)} = c_V$	$\rho_2^{(1)} = c_A$
$\gamma \to l^+l^-$	$-e$	0
$\gamma \to u\bar{u}$	$+2e/3$	0
$\gamma \to d\bar{d}$	$-e/3$	0
$Z \to l^+l^-$	$\frac{-3+12\sin^2\theta_W}{6\sin(2\theta_W)} e \simeq -0.045e$	$\frac{+1}{2\sin(2\theta_W)} e \simeq +0.593e$
$Z \to u\bar{u}$	$\frac{+3-8\sin^2\theta_W}{6\sin(2\theta_W)} e \simeq +0.227e$	$\frac{-1}{2\sin(2\theta_W)} e \simeq -0.593e$
$Z \to d\bar{d}$	$\frac{-3+4\sin^2\theta_W}{6\sin(2\theta_W)} e \simeq -0.410e$	$\frac{+1}{2\sin(2\theta_W)} e \simeq +0.593e$

^ Assuming massless fermions

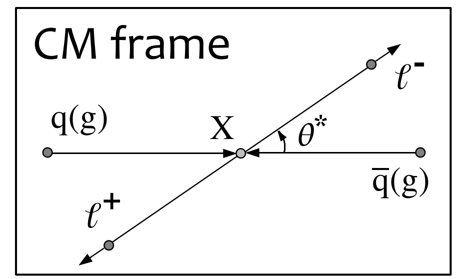
Drell-Yan process: $q\bar{q} \rightarrow \gamma^{*}/Z \rightarrow l^{+}l^{-}$



In applying the above formalism, we can extract the "textbook" differential cross-section.

This simplifies to the classic $\sim A\,(1+\cos^2\vartheta) + B\cos\vartheta$

Mass dependence of the coefficients A/B gives sensitivity of to $\sin^2\theta_W$; linear terms dominated by interference of γ^*/Z



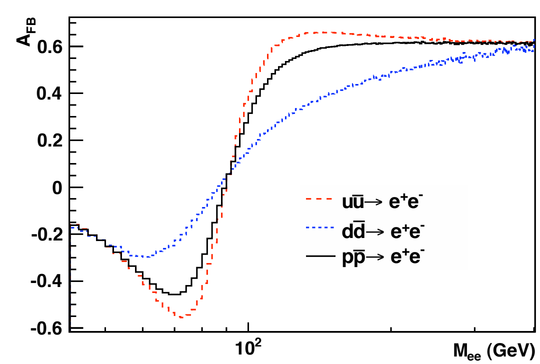
$$\frac{d\Gamma(ab \to X_J \to P_1 P_2)}{d\cos\theta^*} \propto \frac{1}{s}\left(J+\frac{1}{2}\right) \sum_{\chi_1,\chi_2,\lambda_1,\lambda_2} \left(d^J_{\chi_1-\chi_2,\lambda_1-\lambda_2}(\theta^*)\right)^2$$

$$\times \left| A^{(\gamma\to ll)}_{\lambda_1,\lambda_2} B^{(q\bar{q}\to\gamma)}_{\chi_1,\chi_2} + A^{(Z\to ll)}_{\lambda_1,\lambda_2}(\theta_W) B^{(q\bar{q}\to Z)}_{\chi_1,\chi_2}(\theta_W) \times \frac{s}{(s-m_Z^2)+im_Z\Gamma_Z} \right|^2$$

Traditionally, measurements of $\sin^2\theta_W$ are done with forward-backward asymmetry:

$$A_{FB} = \frac{N_F - N_B}{N_F + N_B} = \frac{3}{8}\frac{B}{A}$$

D0 2011, arxiv:1104.4590

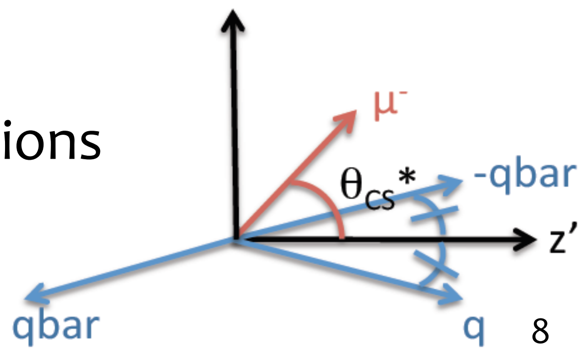


The real world

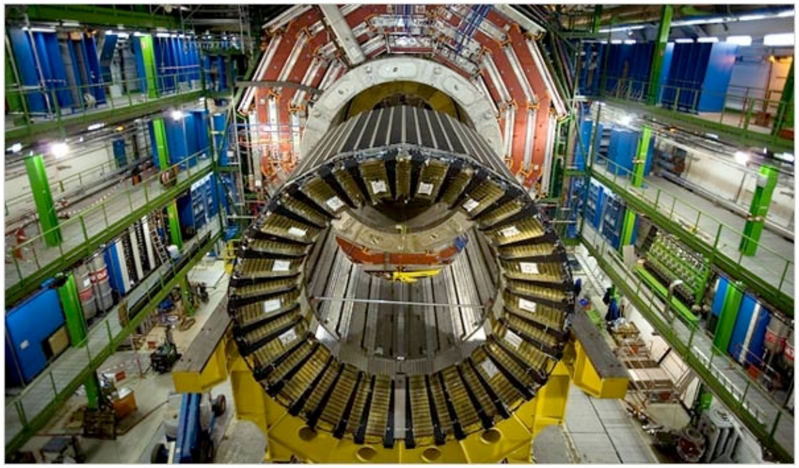
- Parton distribution functions (PDFs)
  - Partonic luminosities affect the mass and rapidity distributions

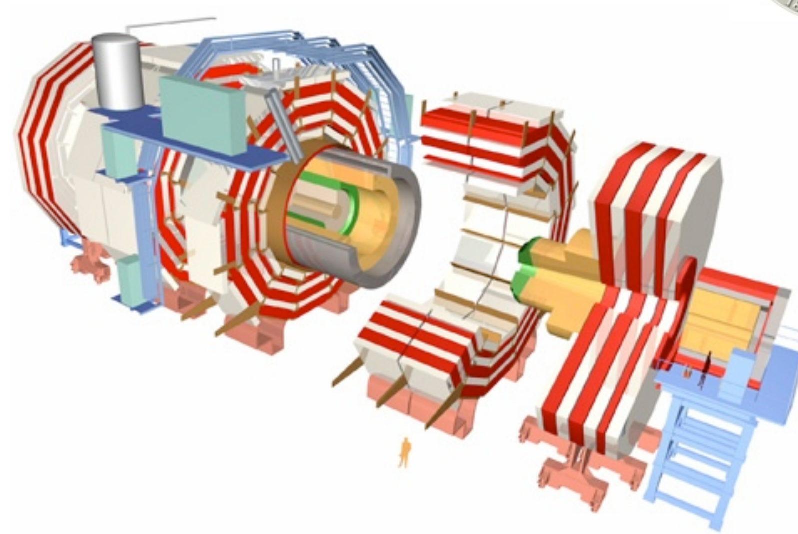
$$\frac{d\sigma_{pp}(px_1, px_2, \cos\theta^*)}{dx_1\, dx_2\, d\cos\theta^*} = \sum_{ab} \hat{\sigma}_{ab}(Q^2, \cos\theta^*)\, \tilde{f}_a(x_1, Q^2)\, \tilde{f}_b(x_2, Q^2)$$

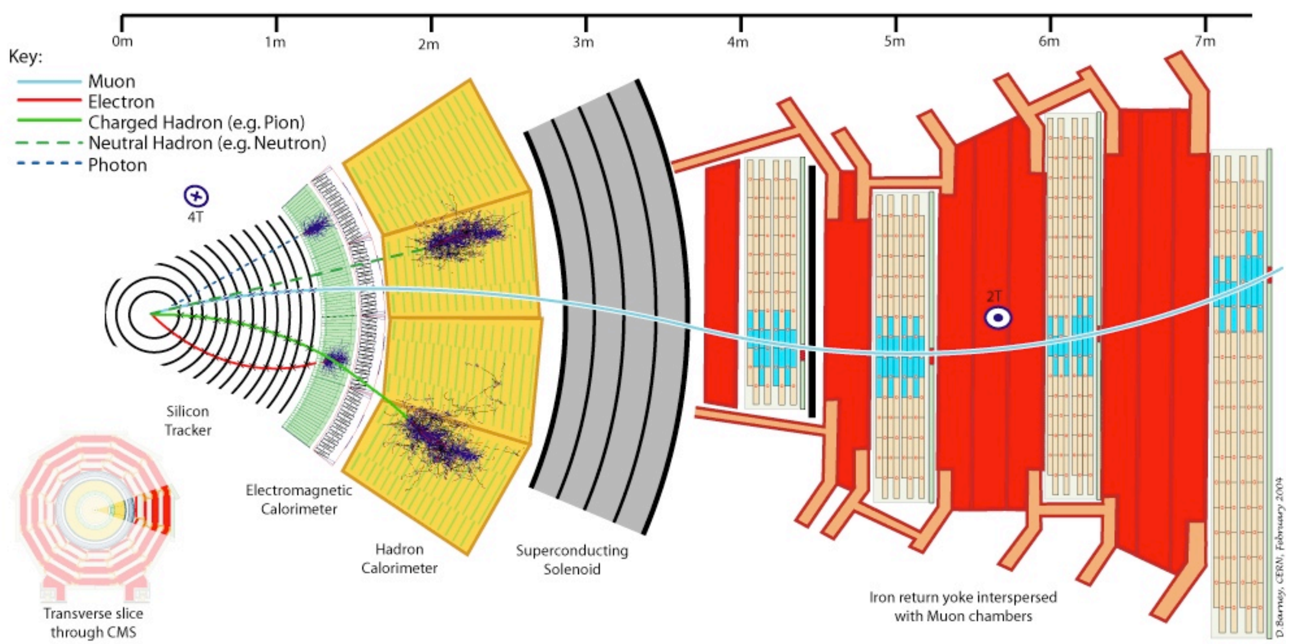
  - Dilution: LHC is a pp collider, which direction does the quark come from? Statistical solution: use the boost direction (rapidity) of the dilepton system
- Detector resolution/acceptance
  - Detector sculpts phase space and smears track parameters
  - Final State Radiation (FSR) shifts dilepton mass
- (N)NLO effects
  - Higher order effects distort angular distributions
  - Collins-Soper frame reduces affect of $p_T(l^+l^-)$



CMS



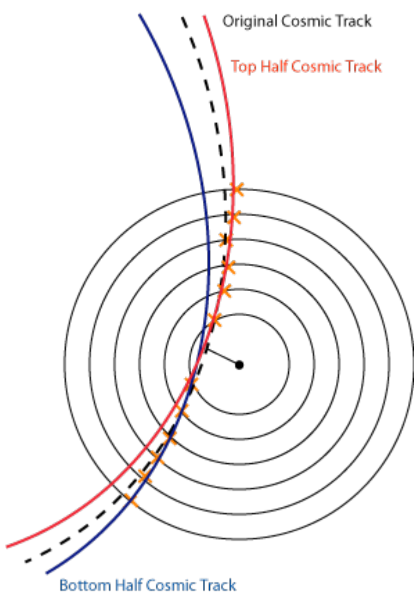




0m
1m
2m
3m
4m
5m
6m
7m
Key:
Muon
Electron
Charged Hadron (e.g. Pion)
Neutral Hadron (e.g. Neutron)
Photon
4T
2T
Silicon Tracker
Electromagnetic Calorimeter
Hadron Calorimeter
Superconducting Solenoid
Iron return yoke interspersed with Muon chambers
Transverse slice through CMS
D.Barney, CERN, February 2004

CMS commissioning

- Many great results from CMS in 2010!
- One of the main reasons is due to commissioning period with cosmic rays to understand the detector
- Highlight, one of many great results from commissioning: Tracker alignment (a biased choice ;)

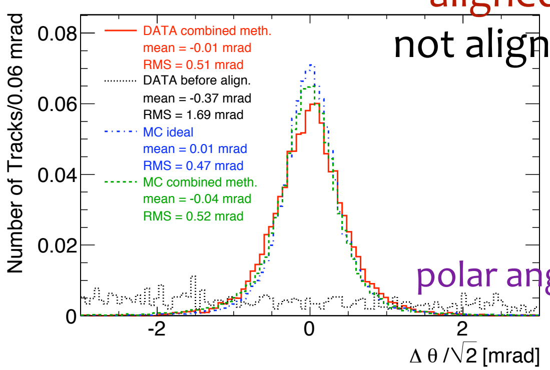


Alignment: use cosmic tracks to determine position of >15000 silicon sensors *in situ* to precision at micron level

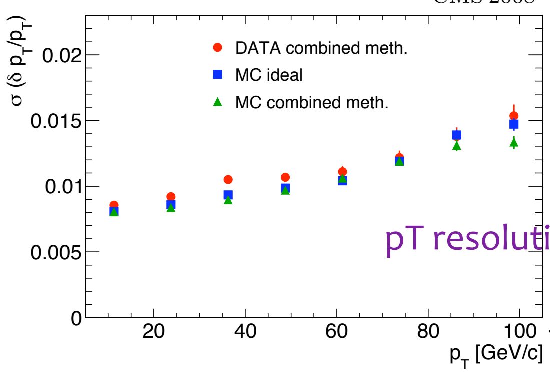
Validate by splitting cosmic tracks in two and comparing their track parameters

aligned

not aligned



polar angle



pT resolution

Analysis concept

- Idea: per event likelihood function to extract maximal information from the event
  - Sensitivity increase (×1.4) over traditional methods [see appendix]

$$\mathcal{L} = \exp\left(-n_{\mathrm{sig}} - n_{\mathrm{bkg}}\right) \prod_{i}^{N} \left(n_{\mathrm{sig}} \times \mathcal{P}_{\mathrm{sig}}(\boldsymbol{x}_i;\ \zeta;\ \boldsymbol{\xi}) + n_{\mathrm{bkg}} \times \mathcal{P}_{\mathrm{bkg}}(\boldsymbol{x}_i;\ \boldsymbol{\xi})\right)$$

- Requires probability distribution function in mass, angle, rapidity: $\mathcal{P}_{\mathrm{sig}}(m, Y, \cos\theta; \sin^2\theta_W, \mathrm{PDFs})$
- Single parameter likelihood fit of $\sin^2\theta_W$
- Building the probability distribution function:
  - Start with Drell-Yan mass-angle distribution including partonic

$$\frac{d\sigma_{pp}(Y, s, \cos\theta^*)}{dY\, ds\, d\cos\theta^*} = \frac{1}{s_0} \sum_{q=u,d,s,c,b} \left[\hat{\sigma}^{\mathrm{even}}_{q\bar{q}}(s, \cos\theta^*) + D_{q\bar{q}}(s, Y) \times \hat{\sigma}^{\mathrm{odd}}_{q\bar{q}}(s, \cos\theta^*)\right] \times F_{q\bar{q}}(s, Y)$$

Terms dependent on PDFs, requires analytical parameterization of PDFs (more later);
$D_{qq}(s,Y)$ describes dilution term, $F_{qq}(s,Y)$ describes partonic luminosity

Likelihood analysis of $\sin^2\theta_W$

- Building the probability density function (continued)
  - Include acceptance and efficiency:

    $\mathcal{P}_{ideal}(m,Y,\cos\theta) \times \mathcal{G}_{acc}(m,Y,\cos\theta)$

    - Includes detector acceptance and pT cuts modeled analytically in the Collins-Soper frame and reconstruction/trigger efficiency
  - Include detector resolution and FSR effects:

    $[\mathcal{P}_{ideal}(m,Y,\cos\theta) \otimes \mathcal{R}(m)\,] \times \mathcal{G}(m,Y,\cos\theta)$

    - Analytically convolution of $\mathcal{R}(m)$ approximated by quadruple Gaussian

$$\mathcal{P}_{sig}(m,Y,\cos\theta;\ \sin^2\theta_W,\ \text{PDFs}) = [\mathcal{P}_{ideal}(m,Y,\cos\theta) \otimes \mathcal{R}(m)\,]\ x\ \mathcal{G}_{acc}(m,Y,\cos\theta)$$

- Model is built at LO; (N)NLO effects treated as corrections to model
- Information about $\sin^2\theta_W$ contained in the shapes of the multidimensional space

The real world, redux

- Analysis in dimuon channel, though no conceptual issues dielectron channel
- Muon selection
  - Global and tracker muons, oppositely charged
  - At least one pixel hit, 10 strip hits, 1 used muon station
  - Global muon fit, $\chi^2 < 10$; $|d_{xy}| < 2$mm
  - HLT muon matching
  - Tracker isolation (No ECAL, HCAL)
  - Angle between muons: $(\alpha - \pi) < -2.5$ mrad; cosmics rejection
  - pT > 18,7 GeV, $|\eta| < 2.4$
  - pT (CS) > 18GeV, $|\eta|$ (CS) < 2.3 [for analytical acceptance]
  - pT (Z) < 25 GeV [to suppress NLO effects]
- 2010 CMS data sample for analysis: 40 pb^{-1}

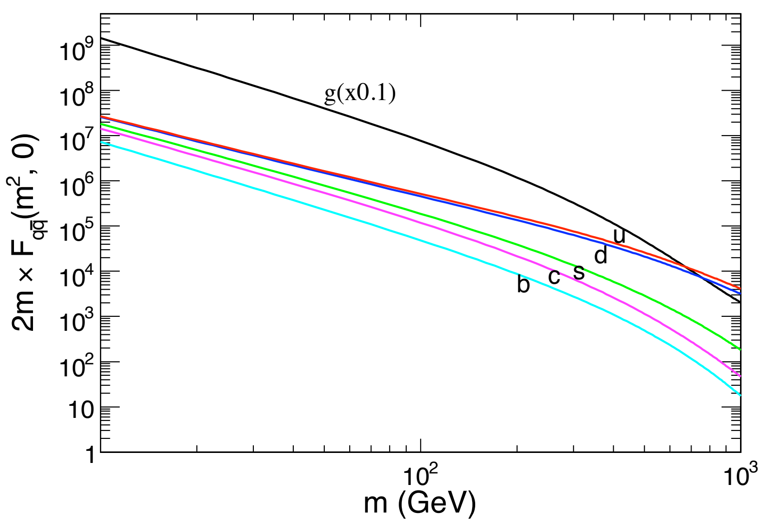
Parton Distribution Functions

We fit the data (CTEQ6QL) for *u,d,c,s,b* quarks and gluons with:

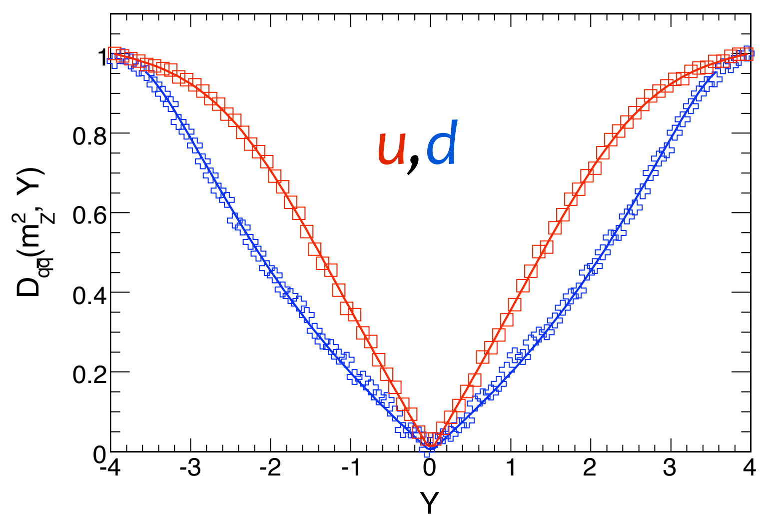
$$x\tilde{f}_a(x,Q^2) \;=\; (a_0(Q) + a_1(Q)\times x + a_2(Q)\times x^2 + a_3(Q)\times x^3)\times(1-x)^4\times x^{a_4(Q)}\times \exp(1+a_5(Q)\times x)$$

Analytical fit *u* quark parton distribution function, $x^*f_u(x,Q^2)$, for a given value of *Q*; then fit parameters for *Q*-dependence

Partonic luminosity vs. mass



Dilution vs. Y



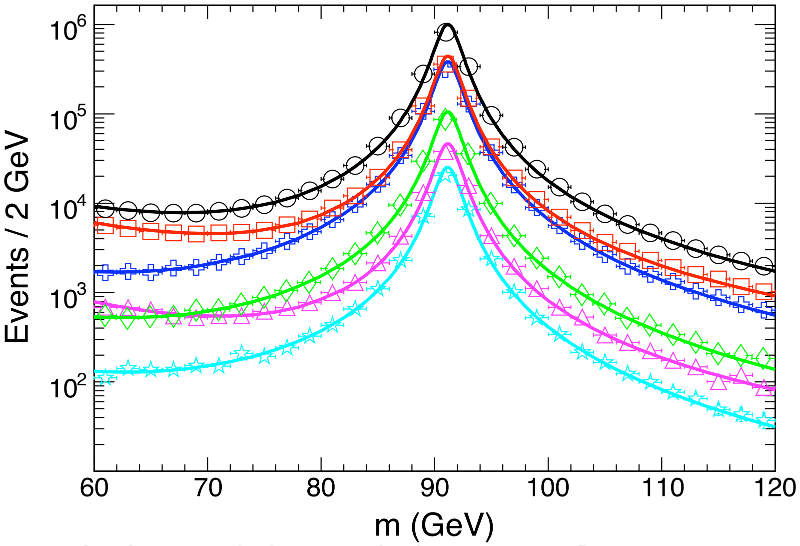
Parton Distribution Functions

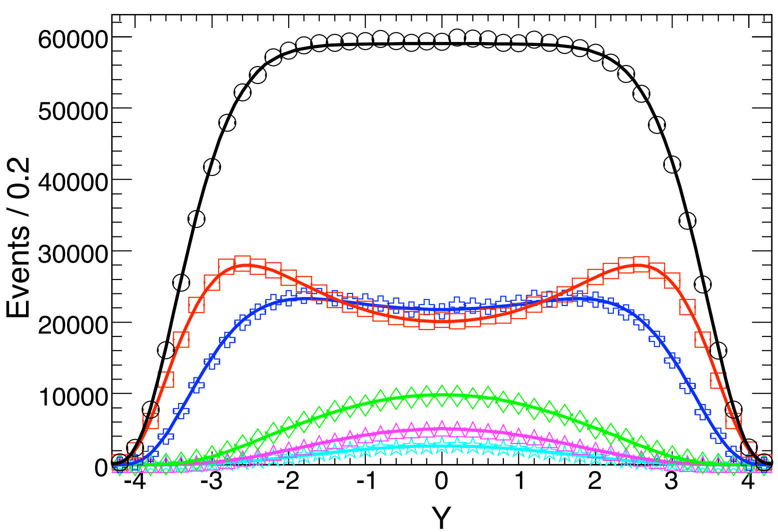
Test our model at LO:

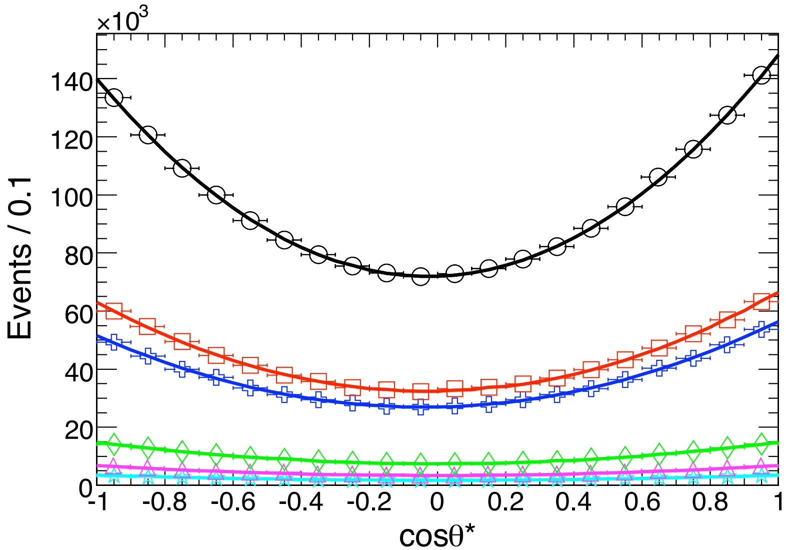
Pythia - points, Model - lines

See good agreement in our observables (m,Y,cosθ) when separated into various quark contributions.

[all, u, d, s, c, b]







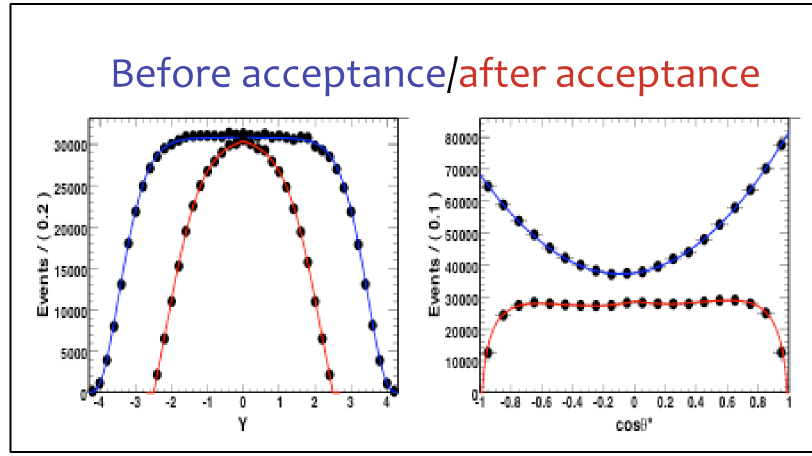
Acceptance and resolution

$\mathcal{G}_{acc}(m,Y,\cos\theta)$: Acceptance + efficiency sculpts further the Y and cosθ distributions

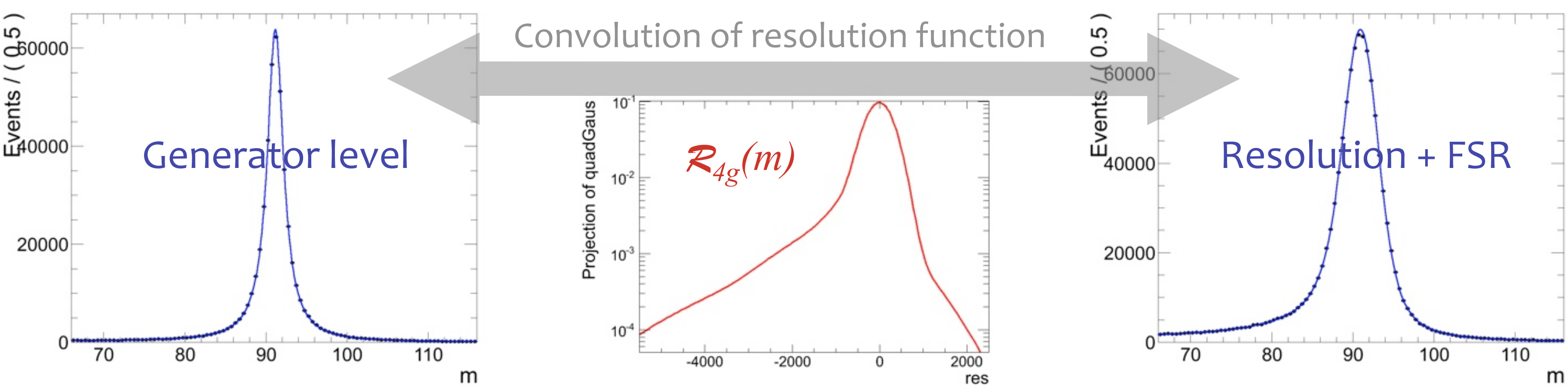
Lepton cuts: $|\eta| < Y_{max}$; $p_T > p_{T,min}$

Acceptance conditions:

$|\cos\theta| < \tanh(Y_{max} - |Y|)$; $|\cos\theta| < [1-(2p_{T,min}/m)^2]^{1/2}$

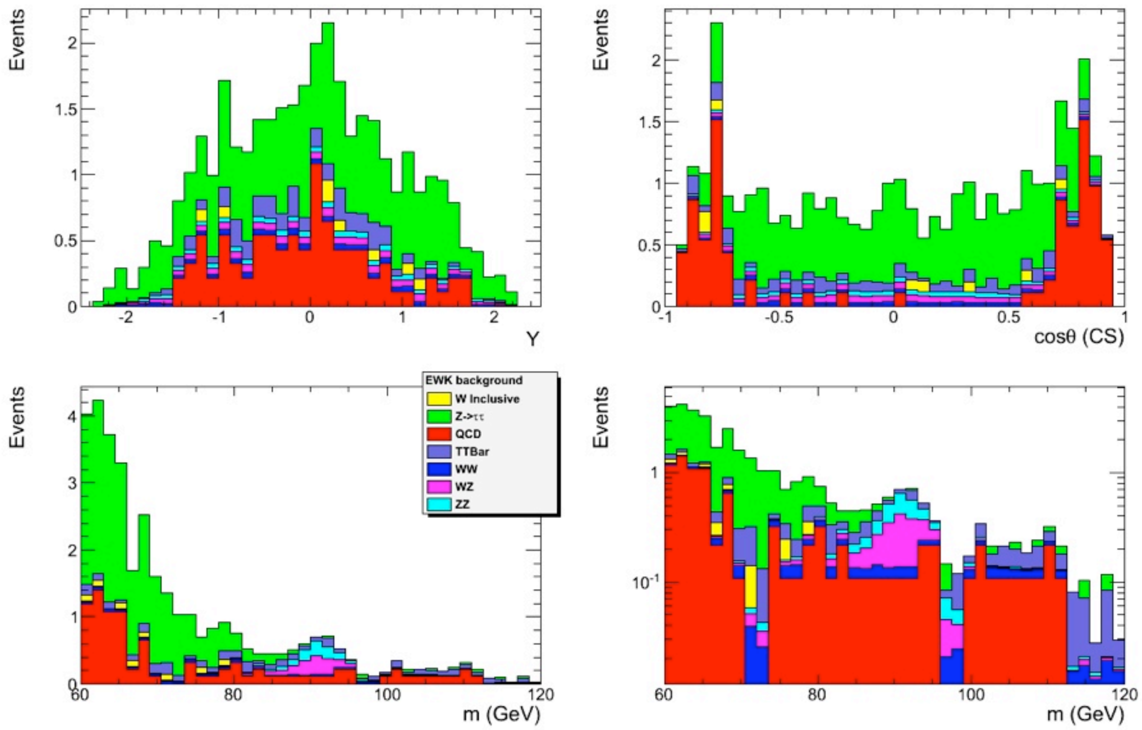


Resolution function parameters fit from MC , implemented via analytical convolution



Background: dimuon channel

Main backgrounds from $\tau^+\tau^-$ and QCD with smaller contributions from WW,WZ, W+jets, ZZ, tt; total background < 1%

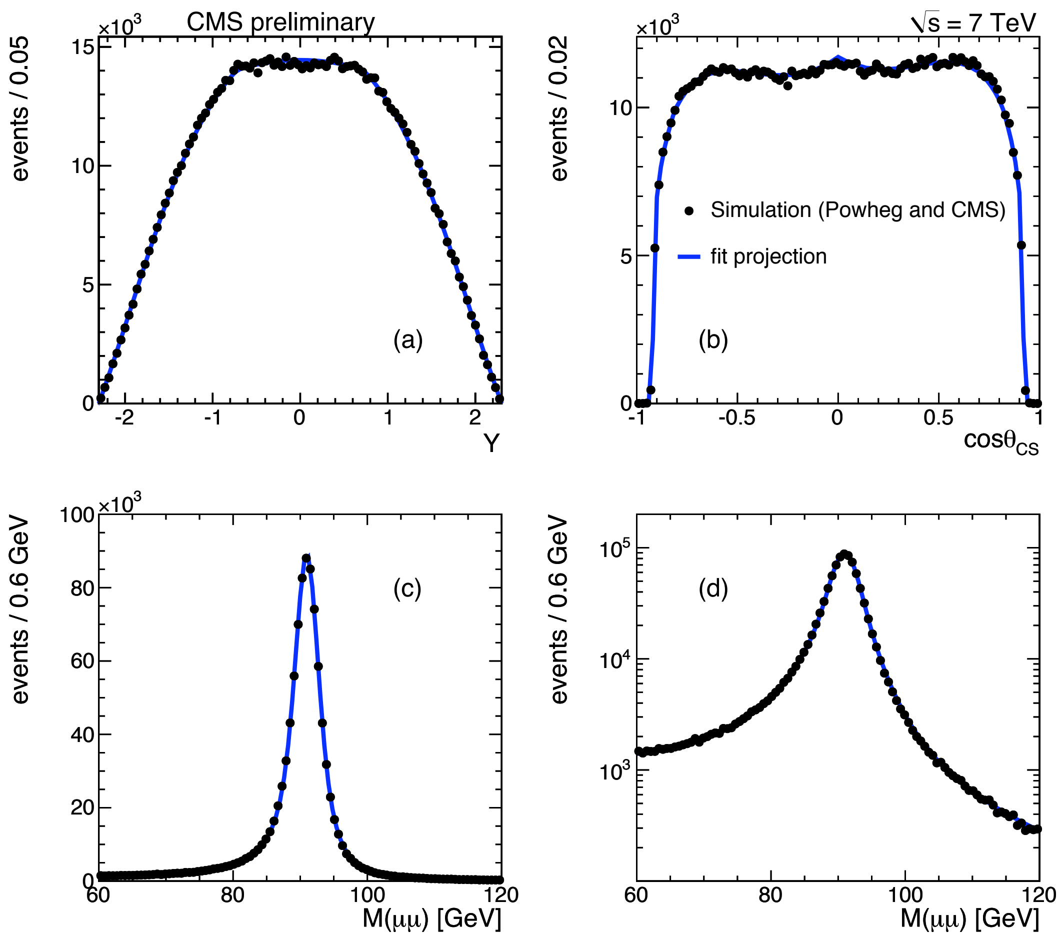


$\sin^2\theta_W$ measurement models background in likelihood function

Expect ~36 bkg events in 2010 data sample

Putting it all together: MC

Final likelihood model on Powheg+Pythia CMS simulation



Result of 400 toy experiments including sig + bkg yields:

$\sin^2\theta_W = 0.2306 \pm 0.0004$

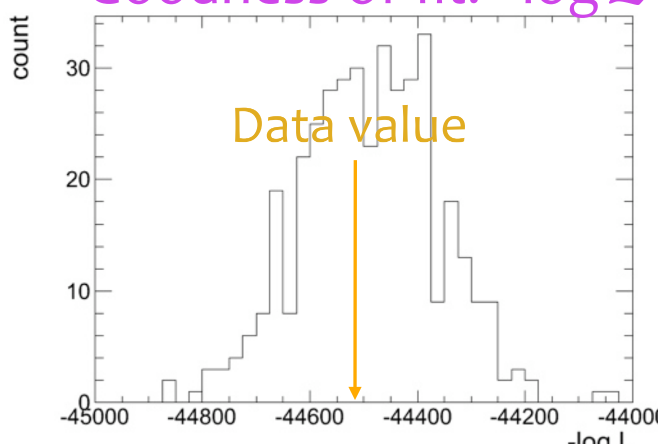
(generated value: 0.2311)

Mean expected statistical error per toy: 0.0078

Error from data: 0.0077

Pull distribution: -0.02 ± 0.96

Goodness-of-fit: - log $\mathcal{L}$



Systematic uncertainties

Dominant backgrounds from FSR and resolution/alignment

Conservative estimates, some cases statistics limited

source	uncertainty
LO model (ISR)	0.0011
PDFs	0.0015
FSR	0.0018
resolution/alignment	0.0022
fit model	0.0010
background	0.0007
total	0.0036

PDF uncertainties from CTEQ vs. MSTW

Misalignment, resolution model, momentum scale

QCD uncertainty

NLO effects

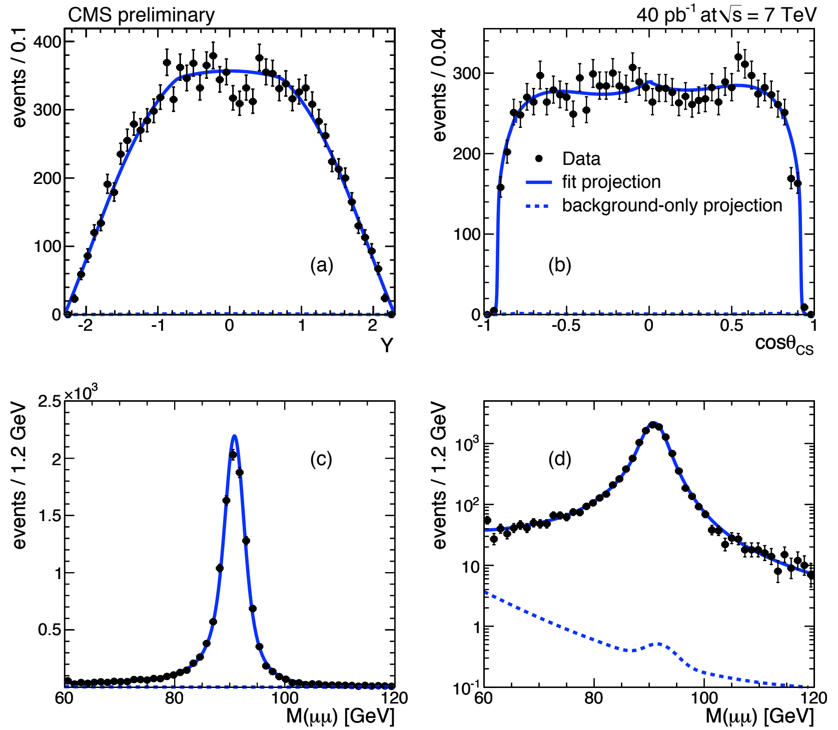
PHOTOS vs. Pythia

minimal bias from high stats MC

Total systematic error less than expected statistical error

Results with data

Data fit central value kept blind until March 1st to avoid analysis bias



CMS PAS EWK-10-011

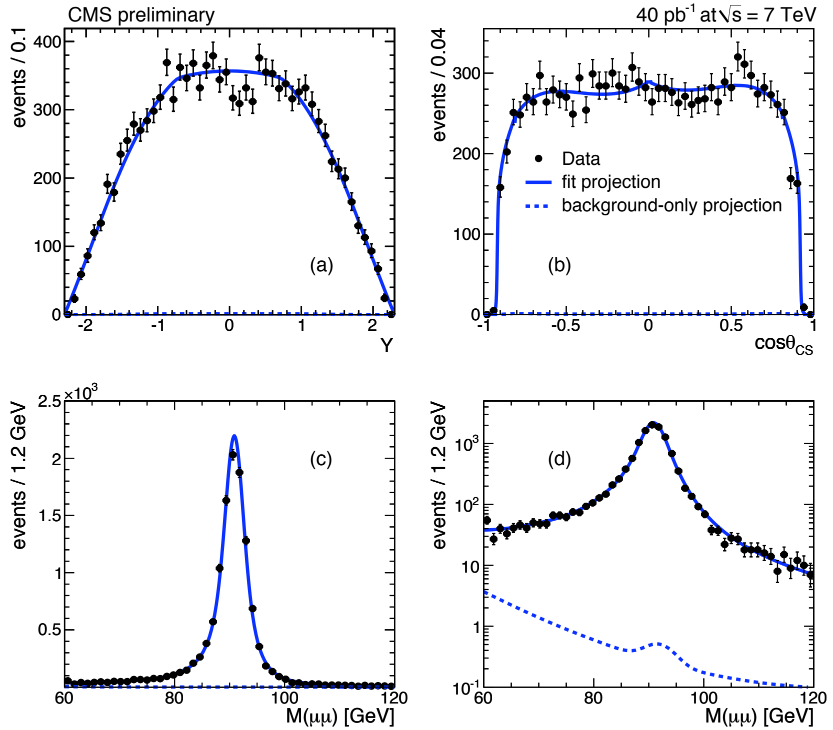
Fit result: $\sin^2\theta_W$ = X.XXXX ± 0.0077 (stat.) ± 0.0034 (sys.)

PDG value: 0.2312

Final cross-check: goodness-of-fit test yields good agreement with MC

Results with data

Data fit central value kept blind until March 1st to avoid analysis bias



CMS PAS EWK-10-011

Fit result: $\sin^2\theta_W = 0.2287 \pm 0.0077\,(\text{stat.}) \pm 0.0034\,(\text{sys.})$

PDG value: 0.2312

Final cross-check: goodness-of-fit test yields good agreement with MC

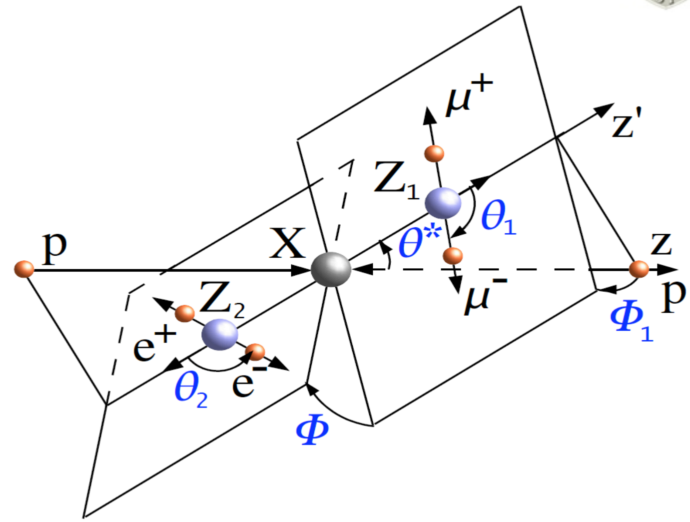
Outlook

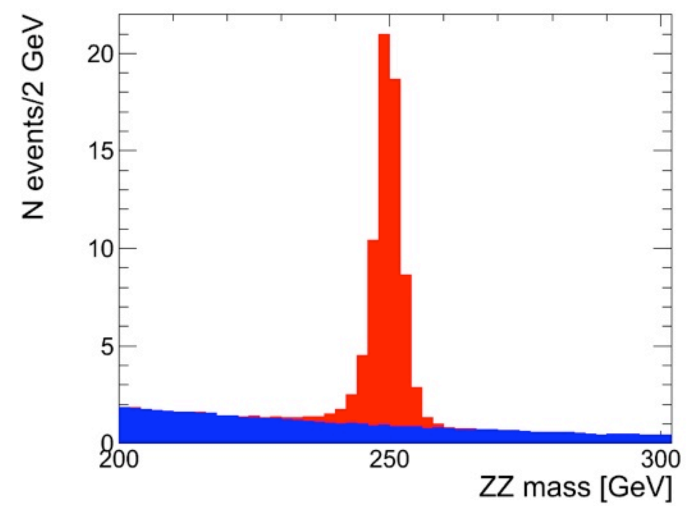
- Presentation of a novel technique for measurement of $\sin^2\theta_W$
- With the coming 2011 data, hope to approach Tevatron precision
    - Tevatron/NuTeV sensitive to $\sin^2\theta_W$ via light quark couplings at ~1%
- Focus efforts on beating down systematics (FSR, alignment) - conservative estimates made so far
- May be difficult to reach LEP precision even with infinite statistics - model flexible, can test other inputs to model

- Principles above can be applied to new physics scenarios
    - High mass dilepton resonances (e.g. Z') - can simultaneously look for broad and narrow resonances
    - Other two body final states (e.g. $H\rightarrow\gamma\gamma$)

A more complex case…

- Consider 2→4 processes, no longer assume SM-like couplings as in $\sin^2\theta_W$ analysis
- Again, let's work out the angular distribution in the helicity amplitude formalism
  - Recall, the couplings determine the helicity amplitudes
- What kind of new resonances, X, could we expect? What are its quantum numbers?
- Approach in model-independent way

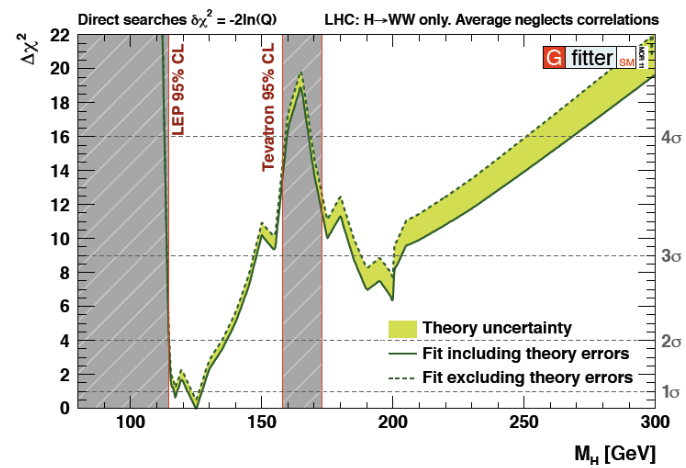


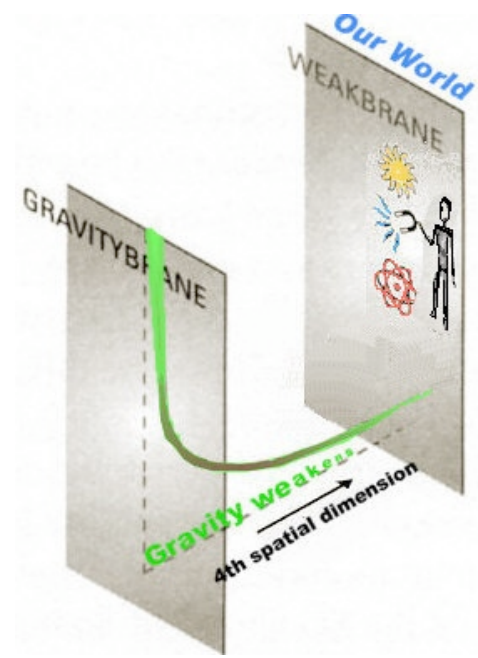


Phys. Rev. D 81, 075022(2010)

Some motivated examples

- Spin-zero
  - SM Higgs, $J^P = 0^+$, or other non-SM scalar
  - Pseudoscalar $J^P = 0^-$, multi-Higgs case
- Spin-one
  - Heavy photon
  - Kaluza-Klein gluon
- Spin-two
  - RS Graviton, $J^P = 2^+$: classic model
    - SM fields localized to TeV brane
  - Non-classic RS Graviton model
    - SM fields in the bulk
- Plus many other possibilities!

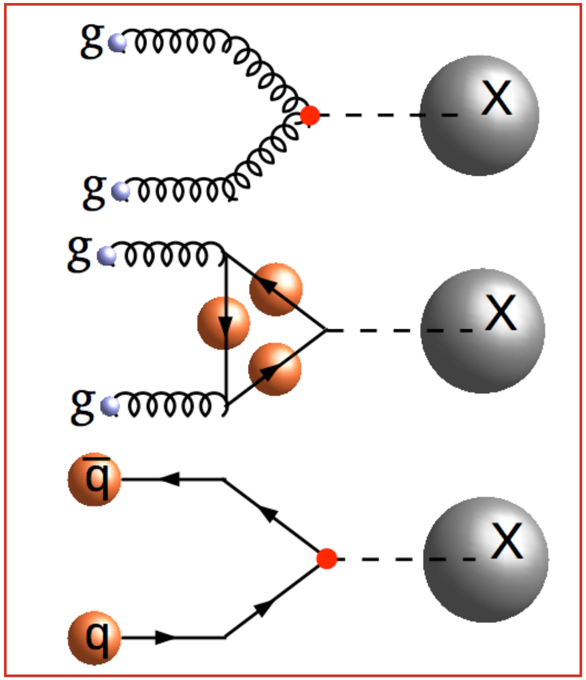


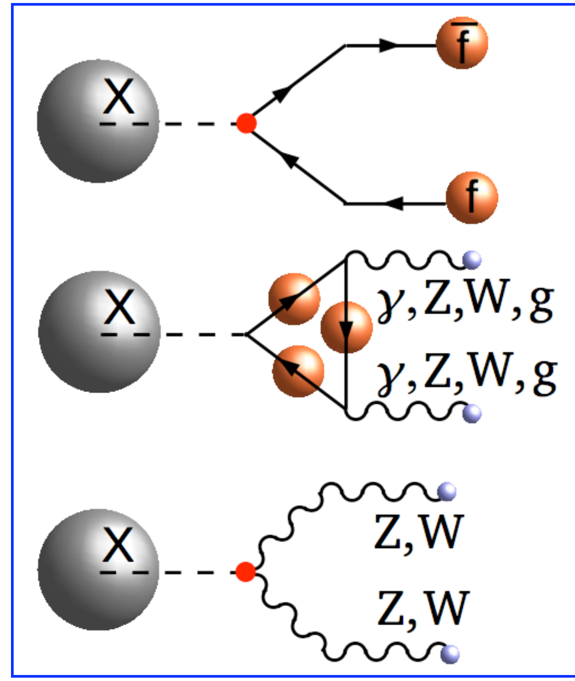


Single-produced resonances at the LHC

Consider a colorless, chargeless X with J = 0,1,2 and $J_Z = 0, \pm1, \pm2$

Production





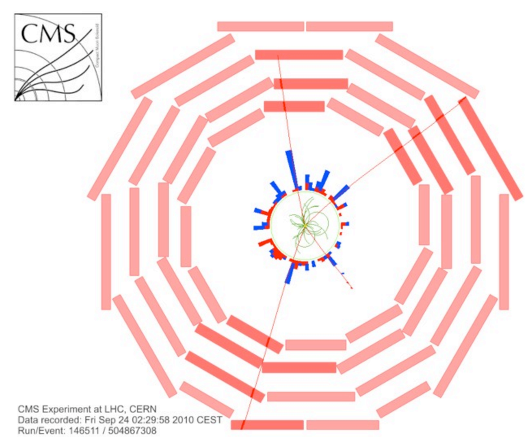
Decay

- gluon fusion: J = 0,2
  - $J_Z = 0$ or $J_Z = \pm2$
  - expect to dominate at low mass
- q-qbar: J = 1,2
  - $J_Z = \pm1$, assume chiral symmetry exact
- Decay to fermions
  - $X \to l^+l^-, q\bar{q}$
  - As $m_f \to 0$, J = 0 excluded
- Decay to gauge bosons
  - $X \to ZZ, W^+W^-, gg, \gamma\gamma$

Program

1. Consider general couplings of a spin 0/1/2 particle to SM fields
    - Lorentz invariance, Bose symmetry, and SM gauge invariance obeyed
2. Compute the helicity amplitudes for general couplings
3. Write down the generic angular distribution in terms of the helicity amplitudes
4. Use a multivariate likelihood analysis to fit angular distributions of given hypothetical new resonances^
    - Choose some specific cases for demonstrating power of multivariate likelihood

- Focus on the $X \to ZZ \to 4l$ decay channel
    - Final state fully reconstructed accurately
    - More information in four-body final state
    - *ZZ* decay can be large or even dominant
    - Identical gauge bosons give additional constraints



^ generator developed to simulate resonances, more details later

Spin-zero resonances

Interactions of spin-zero X to two gauge bosons:

$$A(X \to VV) = v^{-1} \epsilon_1^{*\mu} \epsilon_2^{*\nu} \left(a_1 g_{\mu\nu} M_X^2 + a_2 q_\mu q_\nu + a_3 \epsilon_{\mu\nu\alpha\beta} \, q_1^\alpha q_2^\beta \right)$$

Dimensionless complex coupling constants

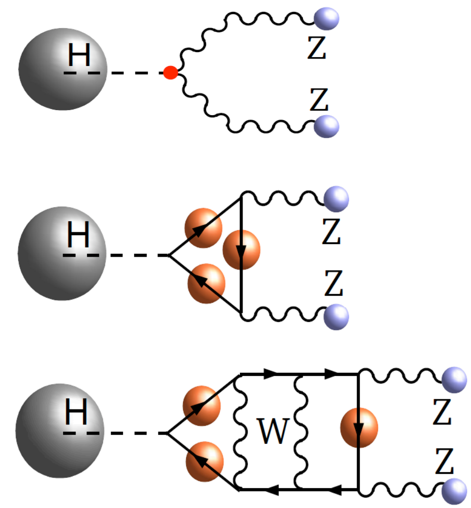
Gauge boson polarization vectors:

$$e^\mu_{1,2}(0) = m_V^{-1}(\pm\beta m_X/2, 0, 0, m_X/2)\,, \quad e^\mu_1(\pm) = e^\mu_2(\mp) = \frac{1}{\sqrt{2}}(0, \mp 1, -i, 0)$$

Plug polarization vectors into above expression to read off helicity amplitudes

---

- In the case of a SM Higgs
  - a1 = tree level
  - a2 = radiative corrections (few %)
  - a3 = 3-loop contribution O(10-11) odd-parity coupling



Helicity amplitudes

Helicity amplitudes: contribution to total amplitude from daughter helicities

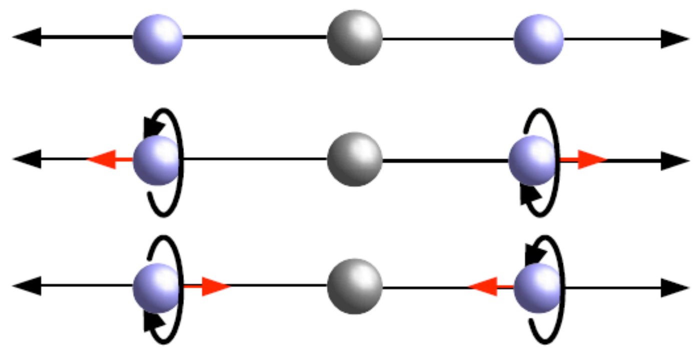
Massive gauge bosons (W,Z) have $J_z = 0, \pm 1$ possible helicity states;

9 total amplitudes, A_{kl}

Examples: A_{00}

A_{++}

A_{--}



Interactions with spin-zero particle X

$$A_{00} = -\frac{M_X^4}{4vM_V^2}\left(a_1(1+\beta^2)+a_2\beta^2\right) ,$$

$$A_{++} = \frac{M_X^2}{v}\left(a_1 + \frac{ia_3\beta}{2}\right) ,$$

$$A_{--} = \frac{M_X^2}{v}\left(a_1 - \frac{ia_3\beta}{2}\right) .$$

Longitudinal amplitude (A_{00}) dominates at large mass for SM Higgs

No longitudinal contribution for 0^- case

Spin-one resonances

Interactions of spin-one X to two gauge bosons:

$$\boxed{A(X \to ZZ) = g_1^{(1)} \left[(\epsilon_1^* q)(\epsilon_2^* \epsilon_X) + (\epsilon_2^* q)(\epsilon_1^* \epsilon_X)\right] + g_2^{(1)} \epsilon_{\alpha\mu\nu\beta} \epsilon_X^{\alpha} \epsilon_1^{*,\mu} \epsilon_2^{*,\nu} \tilde{q}^{\beta}}$$

Dimensionless complex coupling constants

g_2 corresponds to even-parity coupling ($J^P = 1^+$)

g_1 corresponds to odd-parity coupling ($J^P = 1^-$)

Interactions with spin-one particle X :

Spin-one X polarization vectors

$$e_X(0) = (0,0,0,1)$$
$$e_X(\pm) = \frac{1}{\sqrt{2}}(0, \mp 1, -i, 0)$$

$$A_{+0} = -A_{0+} = \frac{\beta m_X^2}{2m_Z}\left(g_1^{(1)} + i\beta g_2^{(1)}\right)$$
$$A_{-0} = -A_{0-} = \frac{\beta m_X^2}{2m_Z}\left(g_1^{(1)} - i\beta g_2^{(1)}\right)$$

Spin-two resonances

Interactions of spin-two X to two gauge bosons:

$$A(X \to ZZ) = \Lambda^{-1} e_1^{*\mu} e_2^{*\nu} \Big[c_1 (q_1 q_2) t_{\mu\nu} + c_2\, g_{\mu\nu} t_{\alpha\beta} \tilde{q}^\alpha \tilde{q}^\beta + c_3 \frac{q_{2\mu} q_{1\nu}}{M_X^2} t_{\alpha\beta} \tilde{q}^\alpha \tilde{q}^\beta + 2c_4 (q_{1\nu} q_2^\alpha t_{\mu\alpha} + q_{2\mu} q_1^\alpha t_{\nu\alpha}) + c_5 t_{\alpha\beta} \frac{\tilde{q}^\alpha \tilde{q}^\beta}{M_X^2} \epsilon_{\mu\nu\rho\sigma} q_1^\rho q_2^\sigma + c_6 t^{\alpha\beta} \tilde{q}_\beta \epsilon_{\mu\nu\alpha\rho} q^\rho + \frac{c_7 t^{\alpha\beta} \tilde{q}^\beta}{M_X^2} (\epsilon_{\alpha\mu\rho\sigma} q^\rho \tilde{q}^\sigma q_\nu + \epsilon_{\alpha\nu\rho\sigma} q^\rho \tilde{q}^\sigma q_\mu) \Big]$$

Dimensionless complex coupling constants

Gauge boson polarization vectors

By applying gauge boson polarization vectors to the general amplitudes, we can read off the helicity amplitudes

For identical massive gauge bosons, can have 9 A_{kl} where k,l = 0,±1

$$A_{+-} = A_{-+} = \frac{m_X^2}{4\Lambda} c_1 (1+\beta^2)\,, \qquad A_{+0} = A_{0+} = \frac{m_X^3}{m_V \sqrt{2}\Lambda} \left[\frac{c_1}{8}(1+\beta^2) + \frac{c_4}{2}\beta^2 - \frac{c_6 + c_7\beta^2}{2} i\beta \right],$$

$$A_{++} = \frac{m_X^2}{\sqrt{6}\Lambda} \left[\frac{c_1}{4}(1+\beta^2) + 2c_2\beta^2 + i\beta(c_5\beta^2 - 2c_6) \right], \qquad A_{-0} = A_{0-} = \frac{m_X^3}{m_V \sqrt{2}\Lambda} \left[\frac{c_1}{8}(1+\beta^2) + \frac{c_4}{2}\beta^2 + \frac{c_6 + c_7\beta^2}{2} i\beta \right],$$

$$A_{--} = \frac{m_X^2}{\sqrt{6}\Lambda} \left[\frac{c_1}{4}(1+\beta^2) + 2c_2\beta^2 - i\beta(c_5\beta^2 - 2c_6) \right], \qquad A_{00} = \frac{m_X^4}{m_V^2 \sqrt{6}\Lambda} \left[(1+\beta^2)\left(\frac{c_1}{8} - \frac{c_2}{2}\beta^2\right) - \beta^2 \left(\frac{c_3}{2}\beta^2 - c_4\right) \right].$$

A bit of bookkeeping...

J = 0

Production: gg^

Allowed spin projection:
0

Helicity Amplitudes:
A_{00}
A_{++}, A_{--}

J = 1

Production: $q\bar{q}$*

Allowed spin projection:
±1^

Helicity Amplitudes:
$A_{+0} = -A_{0+}$
$A_{0-} = -A_{-0}$

J = 2

Production: gg or $q\bar{q}$

Allowed spin projection:
0, ±1, ±2

Helicity Amplitudes:
A_{00},
A_{++}, A_{--}
$A_{+0} = A_{0+}$
$A_{0-} = A_{-0}$
$A_{+-} = A_{-+}$

* gg fusion forbidden due to Landau-Yang theorem

^ assume chirality a good quantum number for massless quarks

For identical vector bosons: $A_{kl} = (-1)^J A_{lk}$

For definite CP states: $A_{kl} = \eta_P (-1)^J A_{-k-l}$

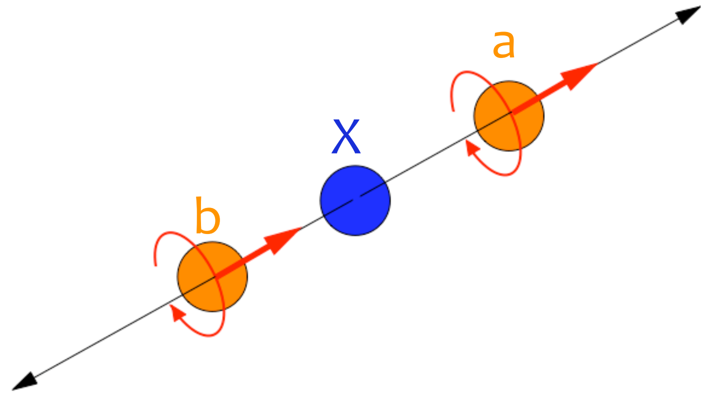
Angular Distributions

We can connect theory to experiment via angular distributions parameterized by helicity amplitudes

Recall, the process 1→2 process:

We can express the polarization of X (J,m) via the helicity amplitudes of a and b (λ_1 and λ_2)

$$\boxed{\langle \Omega, \lambda_1, \lambda_2 | S | J m \rangle = \sqrt{\frac{(2J+1)}{4\pi}} D^{J*}_{m,\lambda_1-\lambda_2}(\Omega) A_{\lambda_1\lambda_2}}$$



Now considering the sequential decay *ab→X→ZZ→4l*

$$\begin{aligned} A_{ab} \propto \quad & D^{J*}_{\chi_1-\chi_2,m}(\Omega^*) B_{\chi_1\chi_2} \times D^{J*}_{m,\lambda_1-\lambda_2}(\Omega) A_{\lambda_1\lambda_2} \\ & \times D^{s_1*}_{\lambda_1,\mu_1-\mu_2}(\Omega_1) T(\mu_1,\mu_2) \times D^{s_2*}_{\lambda_2,\tau_1-\tau_2}(\Omega_2) W(\tau_1,\tau_2) \end{aligned}$$

$B_{\chi1,\chi2}$ are helicity states of incoming partons

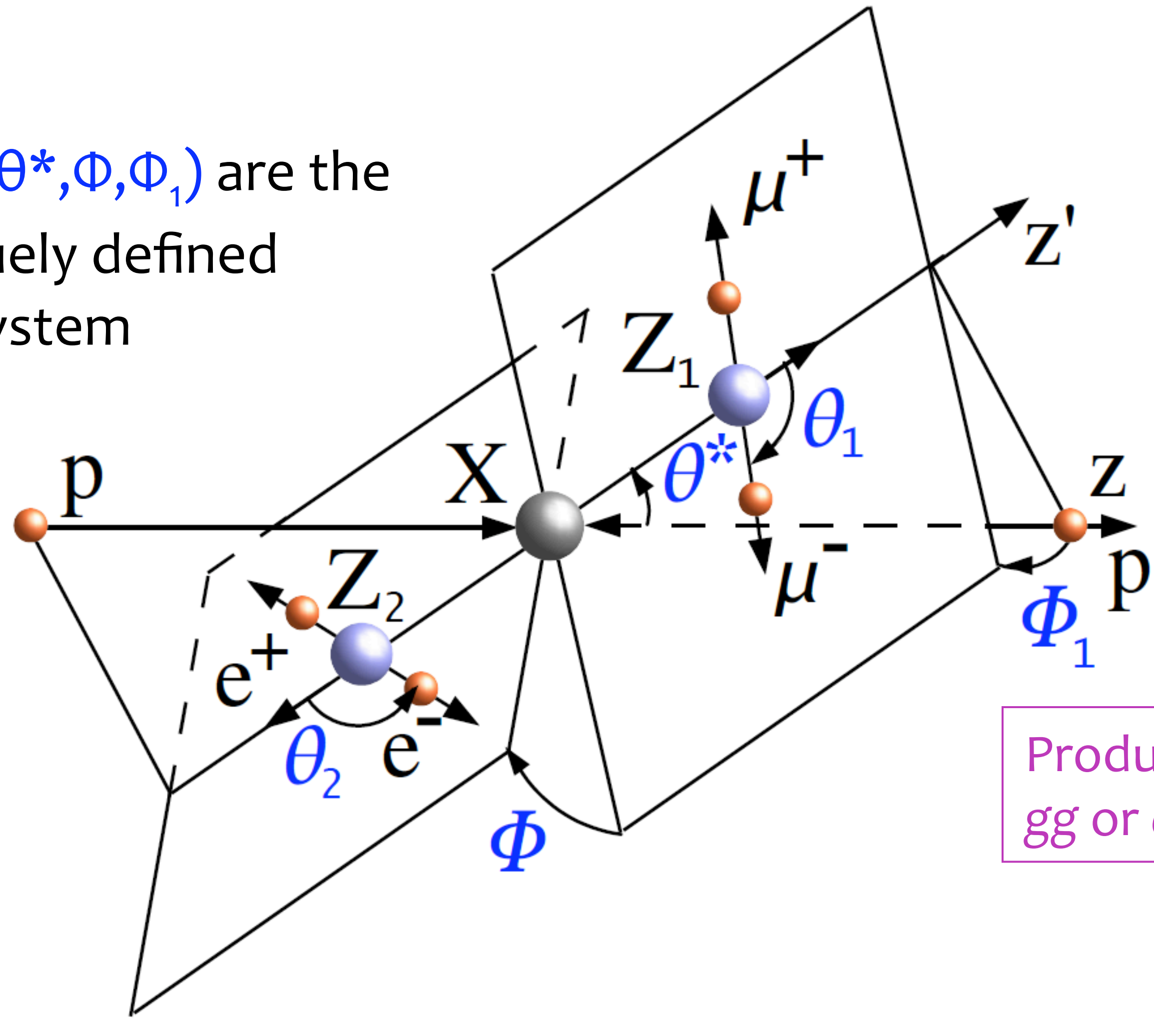
$A_{\lambda1,\lambda2}$ are helicity states of outgoing Z bosons

Ω are angles defined on next slide

Definition of the system

$X \rightarrow ZZ \rightarrow 4l$:

5 angles ($\theta_1, \theta_2, \theta^*, \Phi, \Phi_1$) are the maximal, uniquely defined angles in the system



Production via gg or $q\bar{q}$

θ^*, Φ_1: production angles

θ_1, θ_2, Φ: helicity angles, independent of production

Angular distributions

General spin-J angular distribution

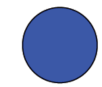
$$
\begin{aligned}
& F^J_{00}(\theta^*) \times \Big\{ 4\, f_{00} \sin^2\theta_1 \sin^2\theta_2 + (f_{++} + f_{--}) \left((1+\cos^2\theta_1)(1+\cos^2\theta_2) + 4 R_1 R_2 \cos\theta_1 \cos\theta_2 \right) \\
& \qquad - 2\,(f_{++} - f_{--}) \left(R_1 \cos\theta_1 (1+\cos^2\theta_2) + R_2 (1+\cos^2\theta_1)\cos\theta_2 \right) \\
& \qquad + 4\sqrt{f_{++} f_{00}}\,(R_1 - \cos\theta_1)\sin\theta_1 (R_2 - \cos\theta_2)\sin\theta_2 \cos(\Phi + \phi_{++}) \\
& \qquad + 4\sqrt{f_{--} f_{00}}\,(R_1 + \cos\theta_1)\sin\theta_1 (R_2 + \cos\theta_2)\sin\theta_2 \cos(\Phi - \phi_{--}) \\
& \qquad + 2\sqrt{f_{++} f_{--}}\,\sin^2\theta_1 \sin^2\theta_2 \cos(2\Phi + \phi_{++} - \phi_{--}) \Big\}
\end{aligned}
$$

$$
\begin{aligned}
& +4 F^J_{11}(\theta^*) \times \Big\{ (f_{+0} + f_{0-})(1 - \cos^2\theta_1 \cos^2\theta_2) - (f_{+0} - f_{0-})(R_1 \cos\theta_1 \sin^2\theta_2 + R_2 \sin^2\theta_1 \cos\theta_2) \\
& \qquad + 2\sqrt{f_{+0} f_{0-}}\,\sin\theta_1 \sin\theta_2 (R_1 R_2 - \cos\theta_1 \cos\theta_2)\cos(\Phi + \phi_{+0} - \phi_{0-}) \Big\} \\
& +(-1)^J \times 4 F^J_{-11}(\theta^*) \times \Big\{ (f_{+0} + f_{0-})(R_1 R_2 + \cos\theta_1 \cos\theta_2) - (f_{+0} - f_{0-})(R_1 \cos\theta_2 + R_2 \cos\theta_1) \\
& \qquad + 2\sqrt{f_{+0} f_{0-}}\,\sin\theta_1 \sin\theta_2 \cos(\Phi + \phi_{+0} - \phi_{0-}) \Big\} \sin\theta_1 \sin\theta_2 \cos(2\Psi)
\end{aligned}
$$

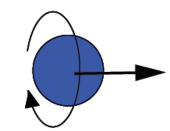
$$
\begin{aligned}
& +2 F^J_{22}(\theta^*) \times f_{+-} \Big\{ (1+\cos^2\theta_1)(1+\cos^2\theta_2) - 4 R_1 R_2 \cos\theta_1 \cos\theta_2 \Big\} \\
& +(-1)^J \times 2 F^J_{-22}(\theta^*) \times f_{+-} \sin^2\theta_1 \sin^2\theta_2 \cos(4\Psi)
\end{aligned}
$$

+ interference terms

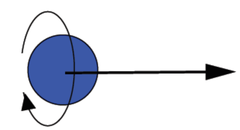
$J_Z = 0$



$J_Z = \pm 1$



$J_Z = \pm 2$



Spin-zero X: only $J_Z = 0$ part contributes

Spin-one X: only $J_Z = \pm 1$ part contributes

Spin-two X: all contributions exist $J_Z = 0, \pm 1, \pm 2$

MC Simulation

- A MC program developed to simulate production and decay of X with spin-zero, -one, or -two
  - Includes all spin correlations and all general couplings
  - Inputs are general dimensionless couplings, calculates matrix elements
  - Both gg and qqbar production
  - Contains both final states for $ZZ \to 4l$ and $ZZ \to 2l2j$
  - Output can be interfaced for hadronization/showering [LHE format]
  - All code publicly available: **www.pha.jhu.edu/spin**

Using simulation and likelihood analysis outlined in dilepton studies, we perform feasibility studies with specific examples

Case studies

Due to large amount of parameters, we choose seven signal scenarios to test a range of parameters

$J^P = 0^+$	SM Higgs-like (tree-level)
$J^P = 0^-$	pseudo-scalar
$J^P = 1^+$	exotic pseudo-vector
$J^P = 1^-$	exotic vector
$J^P = 2^+$ (min.)	Graviton-like tensor (minimal couplings)
$J^P = 2^+$ (long.)	Graviton-like tensor (longitudinally polarized)
$J^P = 2^-$	pseudo-tensor

How well can we identify these scenarios?

How well can we distinguish them from one another?

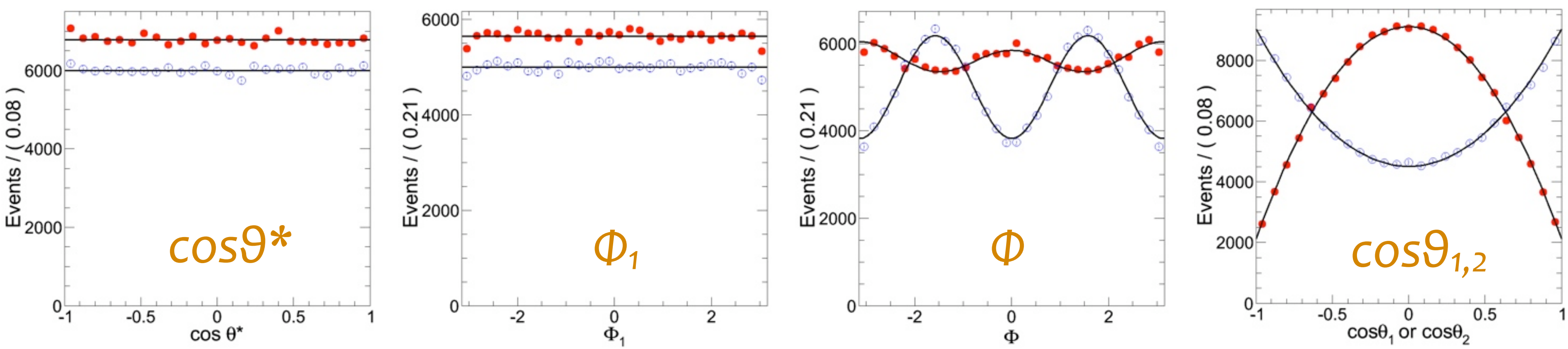
Data and angular distributions

Points are generated using MC program. Lines are derived from general spin-J angular distributions (also for following slides).

*generator level, 250 GeV

Spin Zero

0^+, 0^-



*1D projections of 5D angular distribution

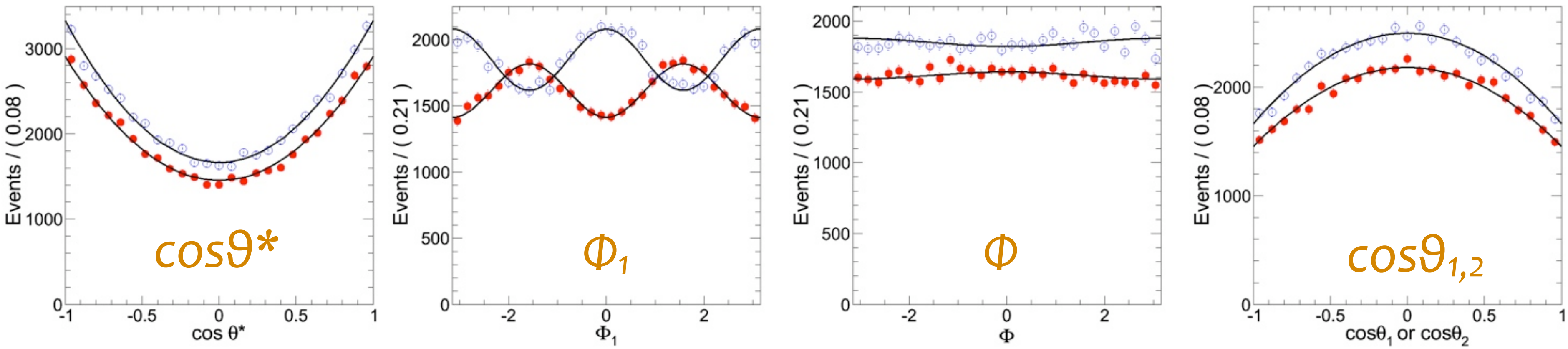
- Spin-zero particle holds no angular information, production angles (θ^*,Φ_1) expected to be flat

Data and angular distributions

*generator level, 250 GeV

Spin One

1^+, 1^-



*1D projections of 5D angular distribution

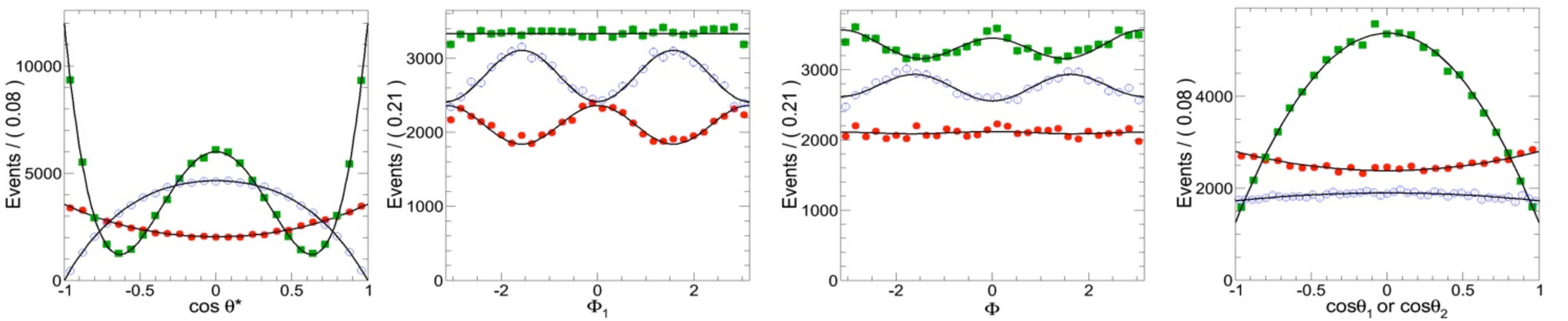
- Production angle, Φ_1, provides a powerful variable to distinguish between even- and odd-parity states
- Helicity angles good example of separation in multidimensional space - 1D projections do not seem useful

Data and angular distributions

*generator level, 250 GeV

Spin Two

2^+ (min.), 2^+ (long.), 2^-



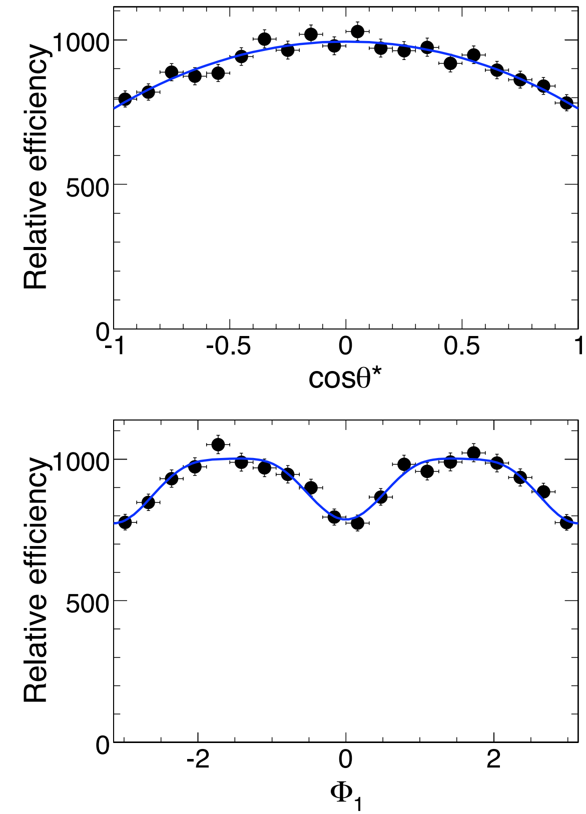
*1D projections of 5D angular distribution

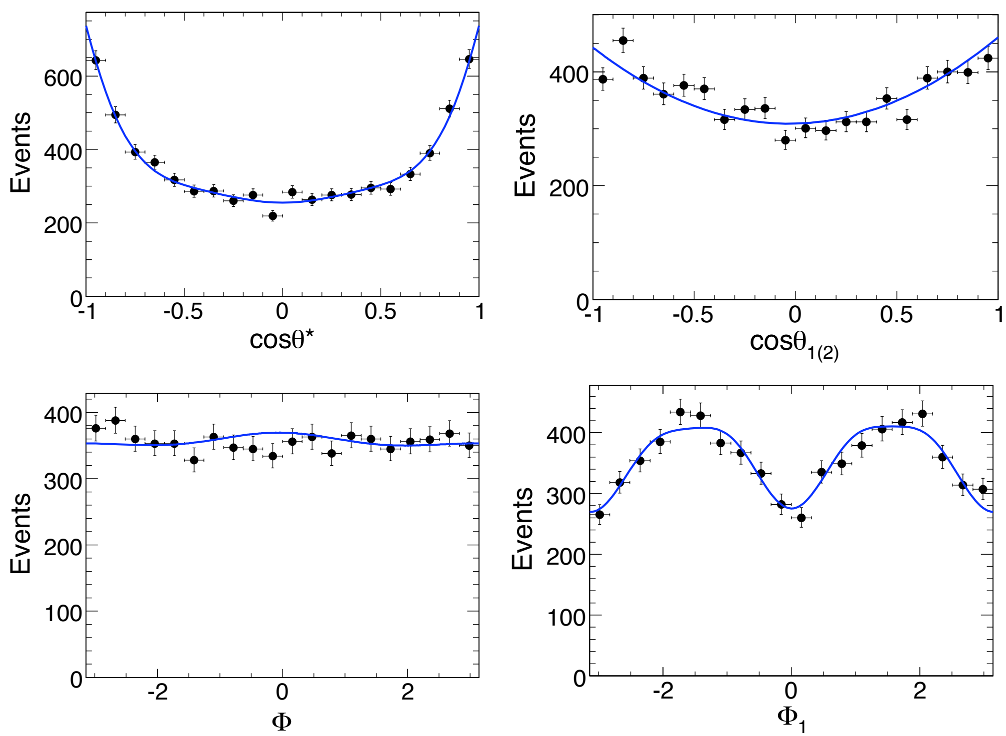
- Consider two $J^P = 2^+$ states, minimal and longitudinal (dominated by A_{00} helicity state)
- 2^+ (long.) case has the same helicity angular distribution as the 0^+ case
  - An unpolarized 2^+ (long.) is indistinguishable from the SM Higgs!

Implementation in CMS

- Detector simulation via CMSSW fast simulation
- Similar to dilepton case, detector acceptance sculpts angular distributions
  - Production angles affected, $\cos\theta^*$ and Φ_1
- Background: dominant process is SM EWK ZZ

Acceptance function





Background combination of:
$q\bar{q} \rightarrow ZZ$ (MadGraph, 85%)
$gg \rightarrow ZZ$ (gg2ZZ, 15%)

Experimental Setup

- Consider m_H = 250 GeV mass point - see analysis note[^] for 500 GeV/1 TeV
- Analysis strategy and software package from HZZ group[*]
- "Hypothesis separation" study (30 signal events, Poisson distributed)
  - 24 background events for lower mass point
  - Scenario considers statistics just enough for discovery (5σ)
- "Parameter fitting" study ("Hypothesis separation" study x5 statistics)
  - More general analysis; allows us to determine all parameters at once (spin, parity, couplings to SM particles)
  - Multiple free parameters in fit requires larger statistics
- Run 1000 toy experiments for each scenario
  - Lower statistics: how much separation between different signal hypotheses achieved?
  - Higher statistics: how well can we determine the parameters of a certain hypothesis?

^ CMS AN-2010/351, * CMS AN-2008/050

Hypothesis Separation

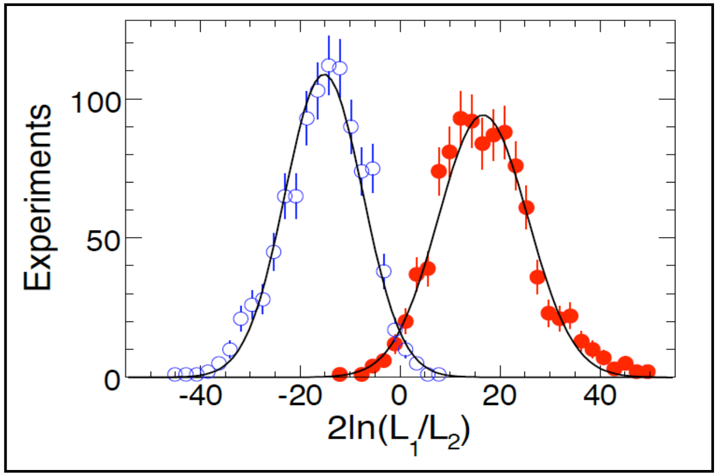
Neyman-Pearson hypothesis testing:

Run 1000 toy experiments…

Determine likelihood ratio test statistic [$S = 2*\ln(L_A/L_B)$] for data samples "A" and "B". Quote effective separation of Gaussian peaks.

Probability Density Function constructed of m_{ZZ} + angular distributions

Example case of 0^+ vs 0^- at 250 GeV



Separation of:

- Signal scenarios

	0^-	1^+	1^-	2^+_m	2^+_L	2^-
0^+	3.3	1.8	2.0	2.2	1.5	2.7
0^-		2.6	2.4	1.7	3.9	2.2
1^+			1.7	1.7	2.4	2.1
1^-				1.1	2.5	2.4
2^+_m					2.8	2.1
2^+_L						3.3

- Signal vs. Background

  L_A (S+B) and L_B (B only)

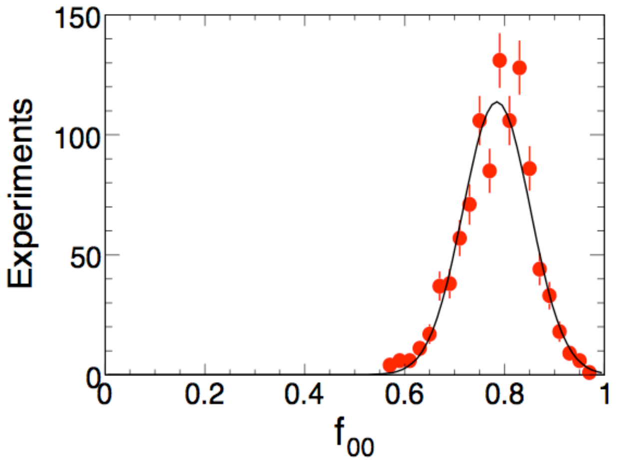
At time of discovery, can already make a statement about quantum numbers

Parameter fitting

With more statistics (x5), can say something about specific couplings to SM fields
Fit for the couplings in given hypothesis

Example fit, longitudinal contribution in case of a spin-0 resonance

	$m_X = 250$ GeV			$m_X = 1$ TeV		
	generated	fitted without detector	fitted with detector	generated	fitted without detector	fitted with detector
n_{sig}	150	150 ± 13	153 ± 15	150	150 ± 12	152 ± 12
$(f_{++} + f_{--})$	0.208	0.21 ± 0.07	0.23 ± 0.08	0.000	0.00 ± 0.03	0.00 ± 0.03
$(f_{++} - f_{--})$	0.000	0.01 ± 0.13	0.01 ± 0.14	0.000	0.00 ± 0.02	0.00 ± 0.02
$(\phi_{++} + \phi_{--})$	2π	6.30 ± 1.46	6.39 ± 1.54	2π	free	free
$(\phi_{++} - \phi_{--})$	0	0.00 ± 1.06	0.01 ± 1.09	0	free	free

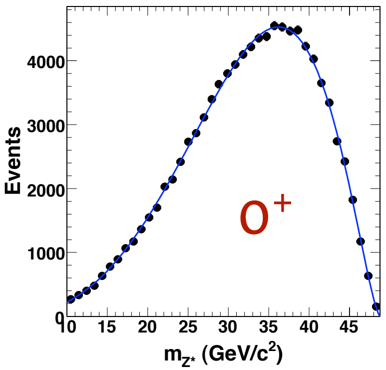


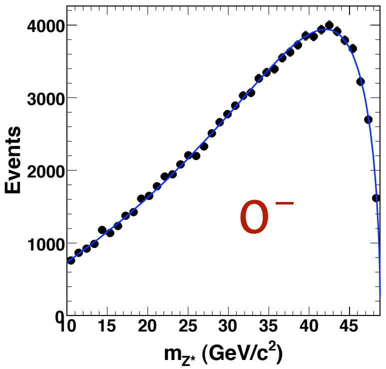
** Generator level fit

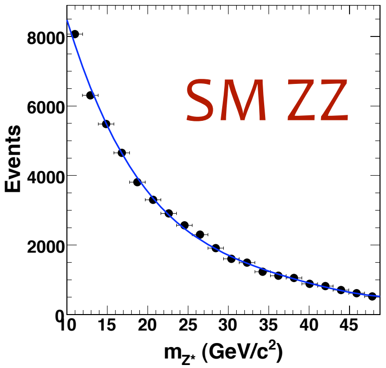
Situation analogous to the $\sin^2\theta_W$ measurement

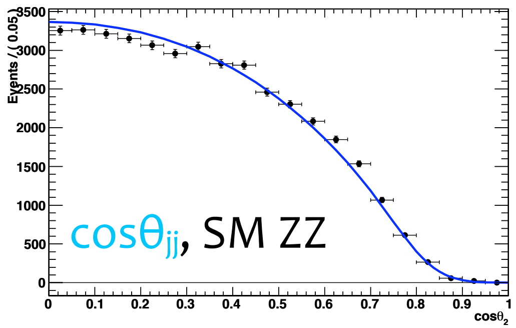
Extensions

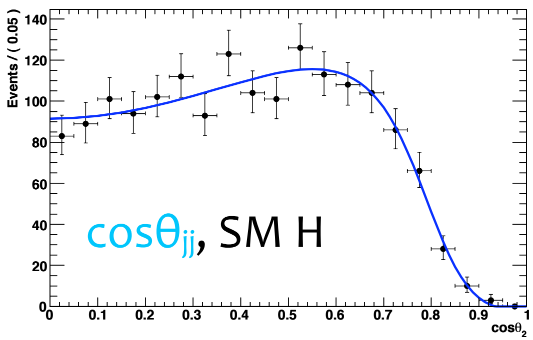
- Separation of background and signal
  - Angular information can enhance discovery potential or exclusion limits, O(10%) improvement
- Below ZZ threshold, more information in m_{Z^*} distribution (right)
  - Included into multivariate analysis and simulation for spin 0 resonances
- Angular analysis of the 2l2j final state (below)
  - $\cos\theta_{jj}$: q and $\bar{q}$ indistinguishable, strong acceptance effect from jet pT cuts











Summary

- Presentation of two analysis at CMS showcasing the power of spin correlations
- EWK analysis of the Drell-Yan process
  - Differential measurement (m,Y,cosθ) of the dimuon final state allows the measure weak mixing angle
  - With 40 pb^{-1}:
    $\sin^2\theta_W = 0.2287 \pm 0.0077$ (stat.) ± 0.0034 (sys.)
- Model-independent angular analysis of ZZ resonances
  - Be ready for anything!
  - At time of discovery, can make statements about spin/CP
- Full multidimensional analysis to extract maximal information from data

Appendix A: comparison of likelihood method with traditional template method

Comparison of methods

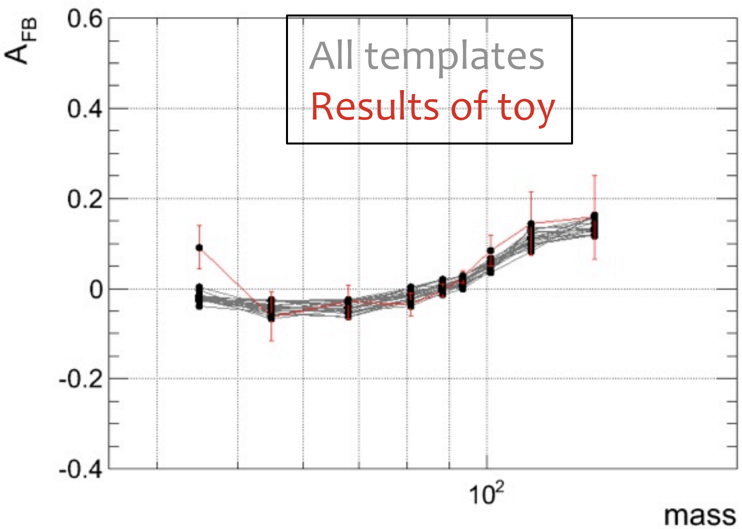
- What is the (statistical) improvement of the likelihood method over traditional methods for determining $\sin^2\theta_W$?
  - "template" method: D0 extracts $\sin^2\theta_W$ from A_{FB} by generating templates of A_{FB} for many values of Weinberg angle, finds the most probable value of $\sin^2\theta_W$ [PRL 101, 191801 (2008)]
- Problem: we don't have tons of MC at different values of $\sin^2\theta_W$ to generate templates
  - Solution: we make a generator level study adding in FSR and realistic "fast smearing"
- Not trivial to extract $\sin^2\theta_W$ from unfolded A_{FB}, requires proper statistical treatment and full correlation matrices; extract most probable $\sin^2\theta_W$ value from raw A_{FB} for convenient error estimation
- More details of this on hypernews

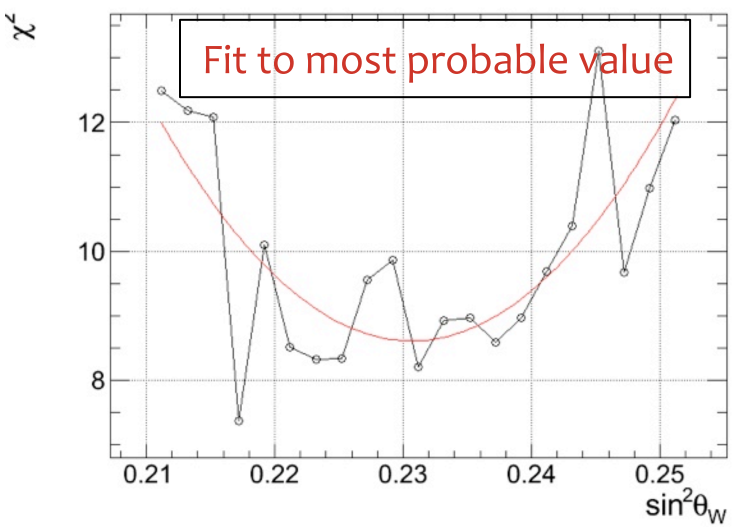
https://hypernews.cern.ch/HyperNews/CMS/get/EWK-10-011/11.html

Comparison of methods

- The setup: generate 1M events Powheg+Pythia+fast smear for different values of $\sin^2\theta_W$ (0.2112 to 0.2512 in increments of 0.002)
- Extract the raw AFB for each sample to use as "templates"
- Run toys (~15k events per) with the $\sin^2\theta_W = 0.2312$ sample and find the most probable value and error on $\sin^2\theta_W$
- Compare this with the likelihood fit method using the same toy experiments

Result of one toy exp.





Mean error on $\sin^2\theta_W \rightarrow$ 0.0014 (templates), 0.0080 (likelihood)

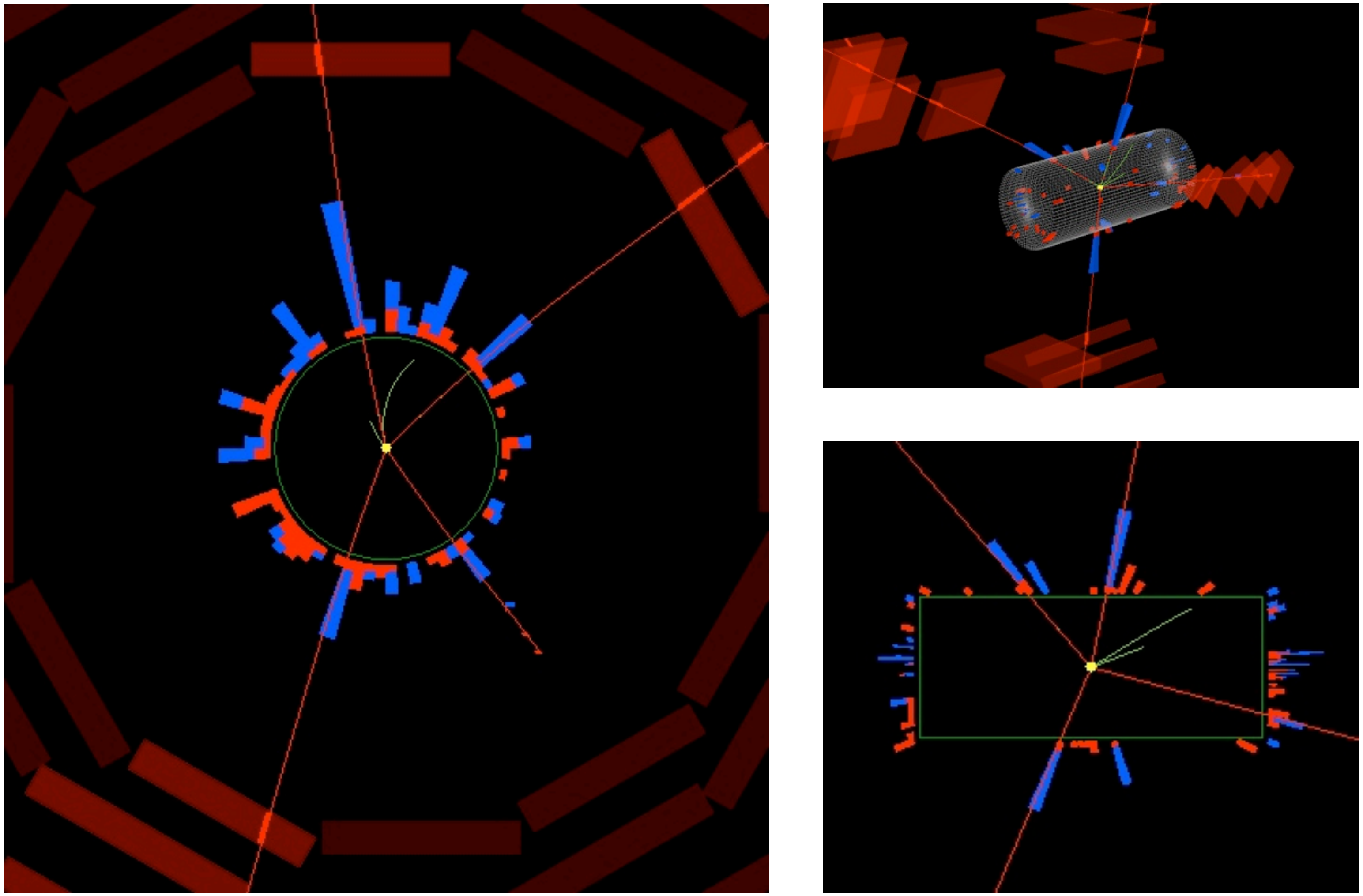
Template method has errors factor of 1.4 larger than likelihood method

Likelihood method equivalent to doubling the statistics!

Appendix B: first 4μ CMS event

4μ event

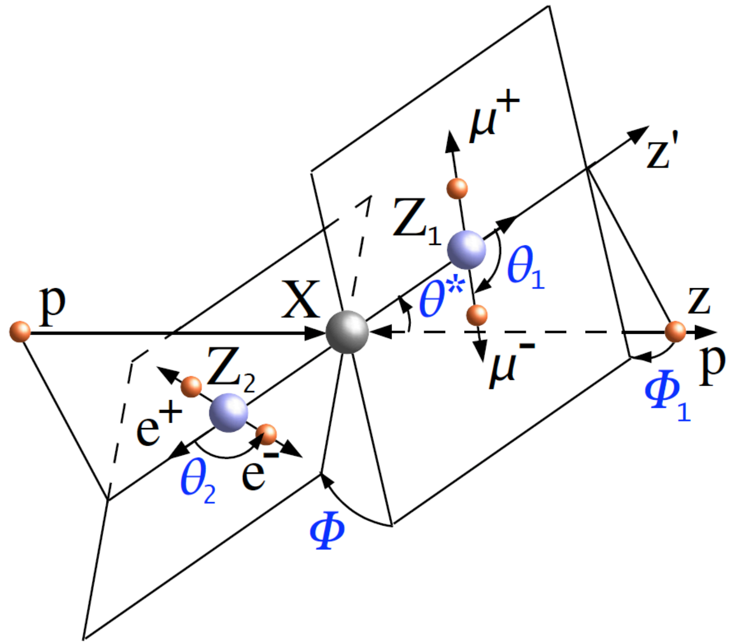
Run: 146511, Lumi: 724, Event: 504867308



Z_1 mass = 92.15; Z_2 mass = 92.24, ZZ mass = 201.75

4μ event: angles

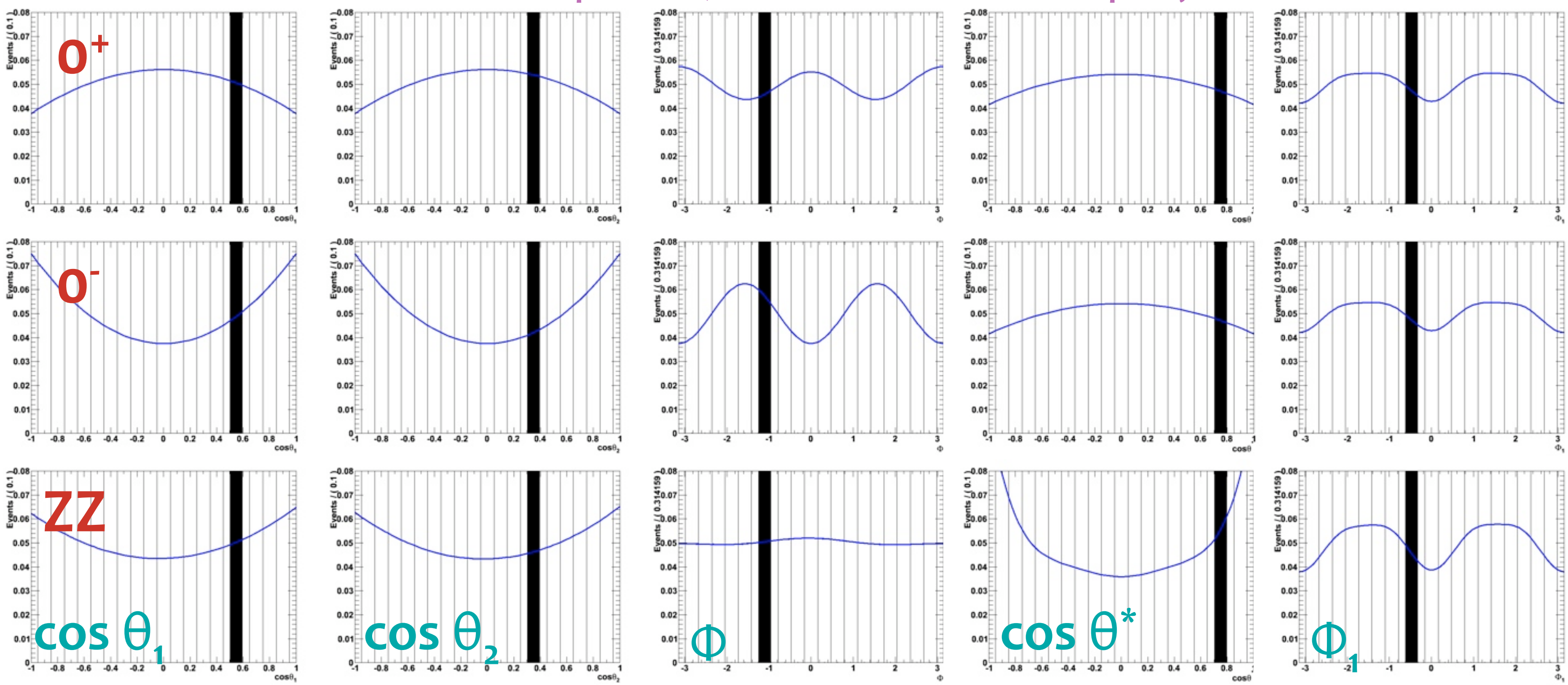
- Look at angular information
  - $\cos\theta_1 = 0.566$
  - $\cos\theta_2 = 0.325$
  - $\Phi = -1.240$
  - $\cos\theta^* = 0.731$
  - $\Phi_1 = -0.460$



- A quick and dirty angular analysis
  - Likelihood fit over 5 angles
  - Try 3 hypotheses: $J^P = 0^+$, $J^P = 0^-$, SM ZZ
  - Acceptance taken using 250 GeV simulation
  - SM ZZ is 85% qqbar→ZZ and 15% gg→ZZ

4μ event

Black lines are data points, blue lines are PDF projections



$-\ln L_{0+} = 6.998$; $-\ln L_{0-} = 6.773$; $-\ln L_{ZZ} = 6.818$

More likely scenario: L_{0-}… but within SM ZZ

$\chi^2 = -2\ln(L_{0+}/L_{0-}) = 0.45$; $\chi^2 = -2\ln(L_{0+}/L_{ZZ}) = 0.36$

Backup

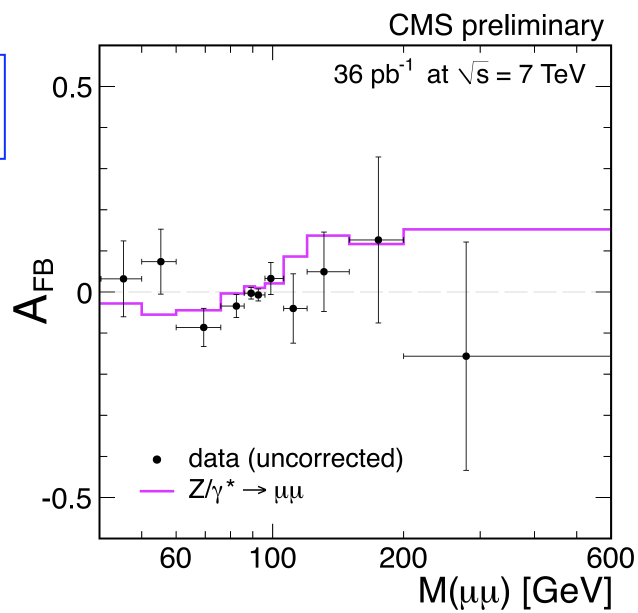
Forward-backward asymmetry

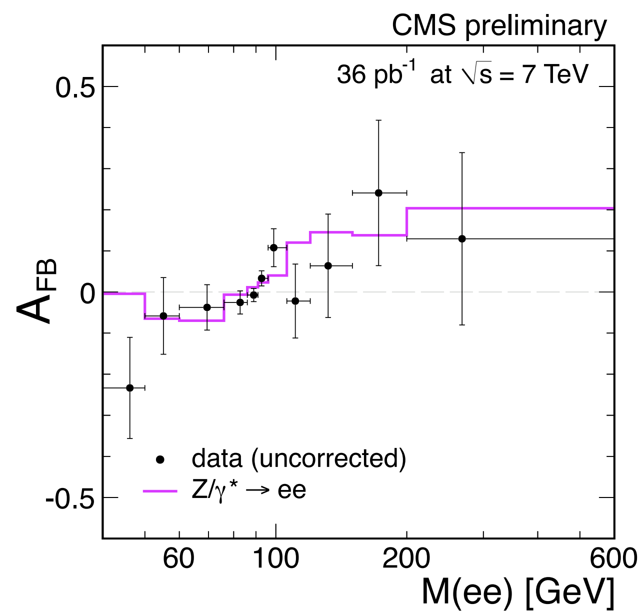
- Forward-backward asymmetry, A_{FB}: simple analysis of Drell-Yan angular distribution with 36 pb^{-1}
  - Sensitive to broad high-mass resonance; slope sensitive to couplings
- Idea: measure $\cos\theta$ asymmetry in bins of mass:

$$A_{FB} = (N_F - N_B)/(N_F + N_B)$$

- A_{FB} in good agreement with the Powheg and CMS simulation

$\mu^+\mu^-$





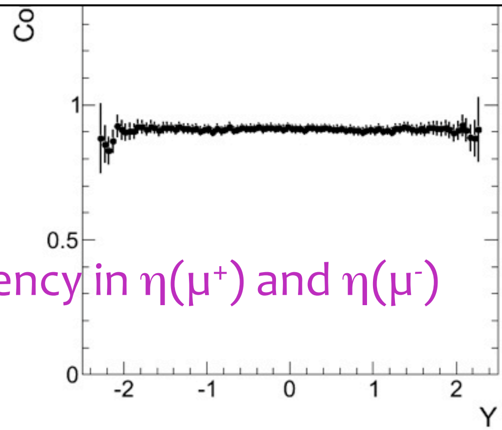
e^+e^-

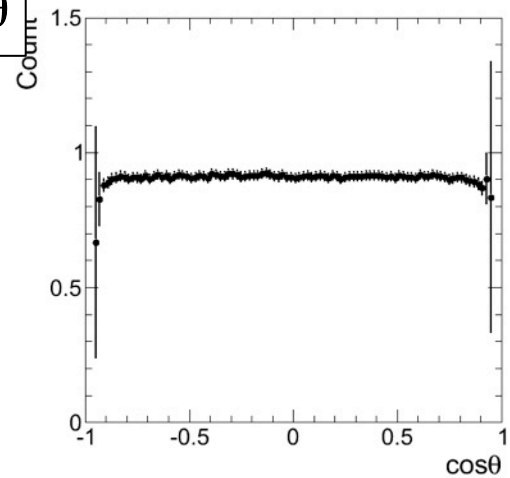
Acceptance and efficiency

Further acceptance function to include efficiency effects and NLO corrections

Probability Density Function ~ $\mathcal{P}(m,\cos\theta,Y) \times \mathcal{G}_a(m,\cos\theta,Y) \times \mathcal{G}_b(\cos\theta,Y)$

Efficiency vs. Y and cosθ





Reflection of efficiency in $\eta(\mu^+)$ and $\eta(\mu^-)$

Add into the fit model a 2D description of the efficiency in Y and cosθ.

Polynomial fit of efficiency in cosθ in bins of Y to construct 2D interpolation function.