Top Quark Mass Measurement using a Matrix Element Method in the Lepton+Jets Channel

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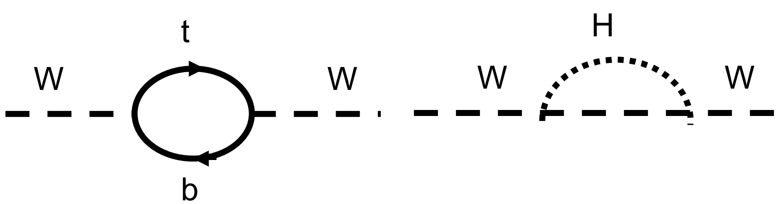
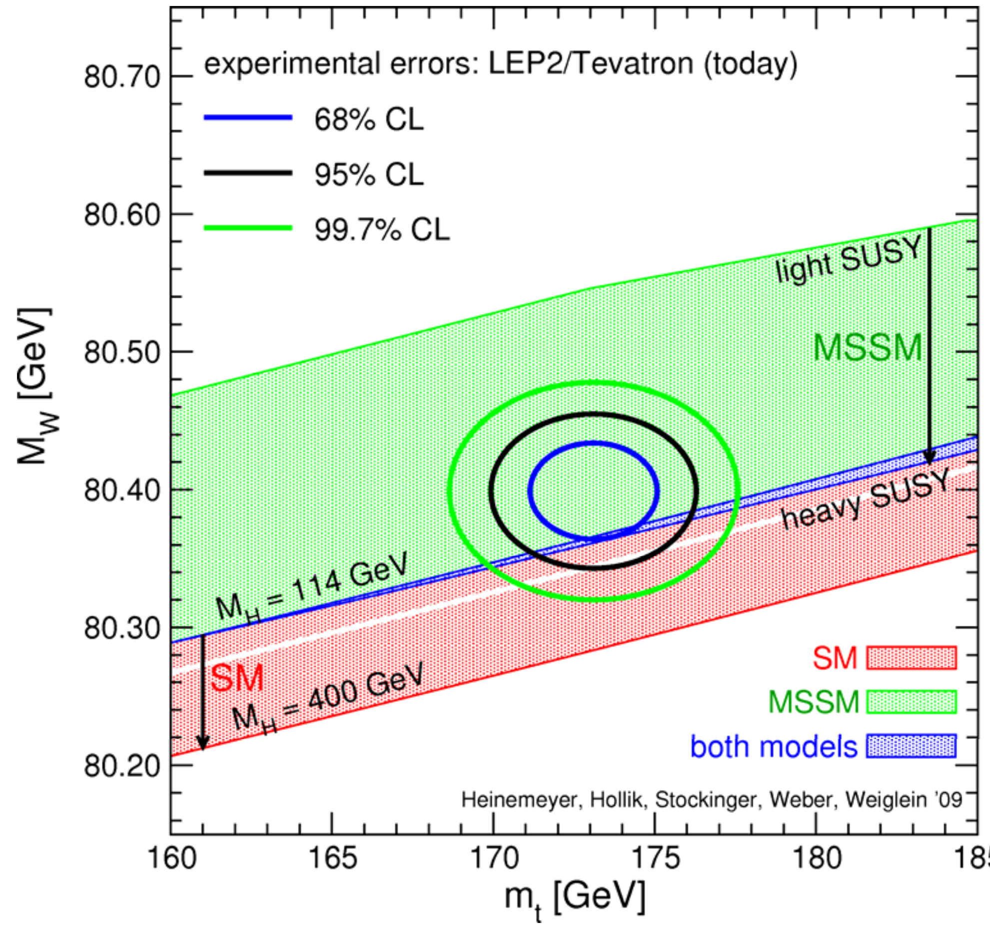
Why measure m_t?

- Top mass, m_t, is fundamental parameter of Standard Model
- Strikingly large value
- Yukawa coupling ~1
- Electroweak mass corrections $\propto m_t^2$ and $\ln(m_H)$



$\delta m_W \propto m_t^2$ $\delta m_W \propto \ln(m_H)$

- Precision measurements of m_W and m_t give constraints on SM m_H
  - consistency test of the SM
  - new physics?



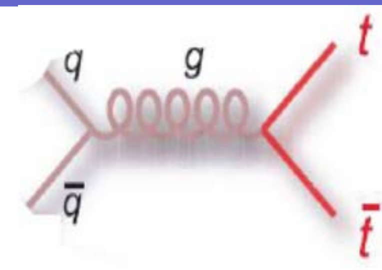
Top quark

- Dominantly produced in pairs (strong interaction)
- t decays before it can hadronize
  - Mass can be directly measured from daughter particles
- Dominant decay t → Wb with W → qq’ or W → ℓν
- Measurement uses $t\bar{t}$ pair events so three possible scenarios:

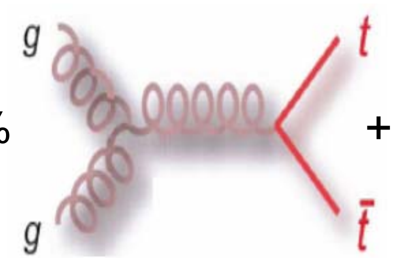
Channel	Branching Ratio
dilepton	11%
lepton+jets	44%
all hadronic	44%

- $t\bar{t} \rightarrow W^+ b\ W^- \bar{b} \rightarrow \ell^+ \nu_\ell b\ \bar{q} q' \bar{b}$
  - Charged lepton, neutrino and 4 jets

~85%



~15%



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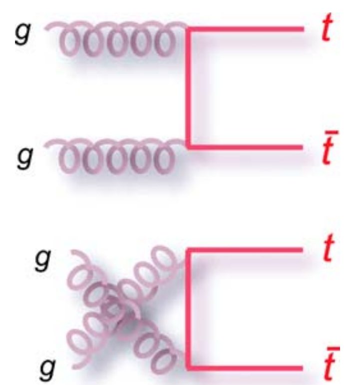
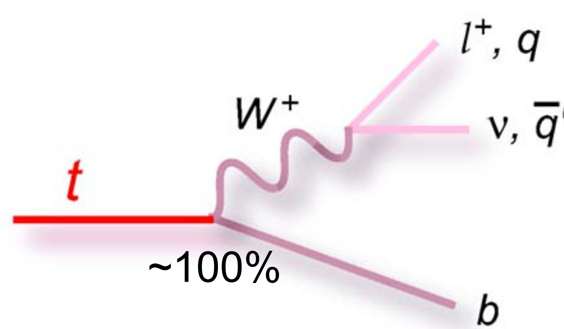
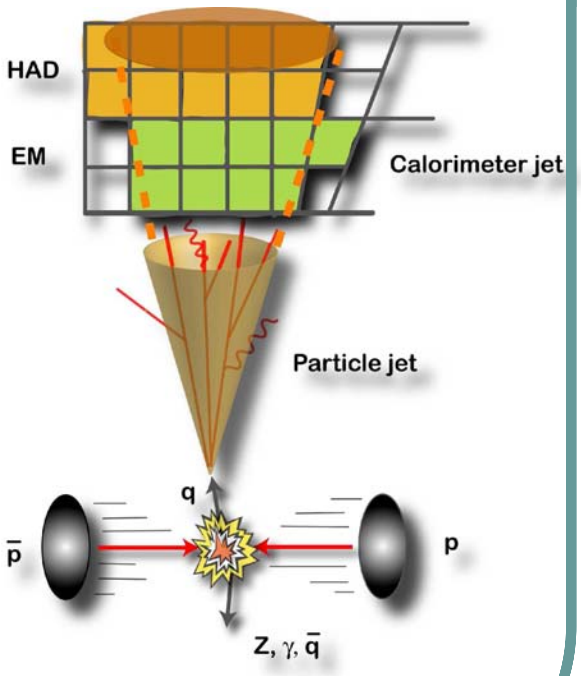




~100%

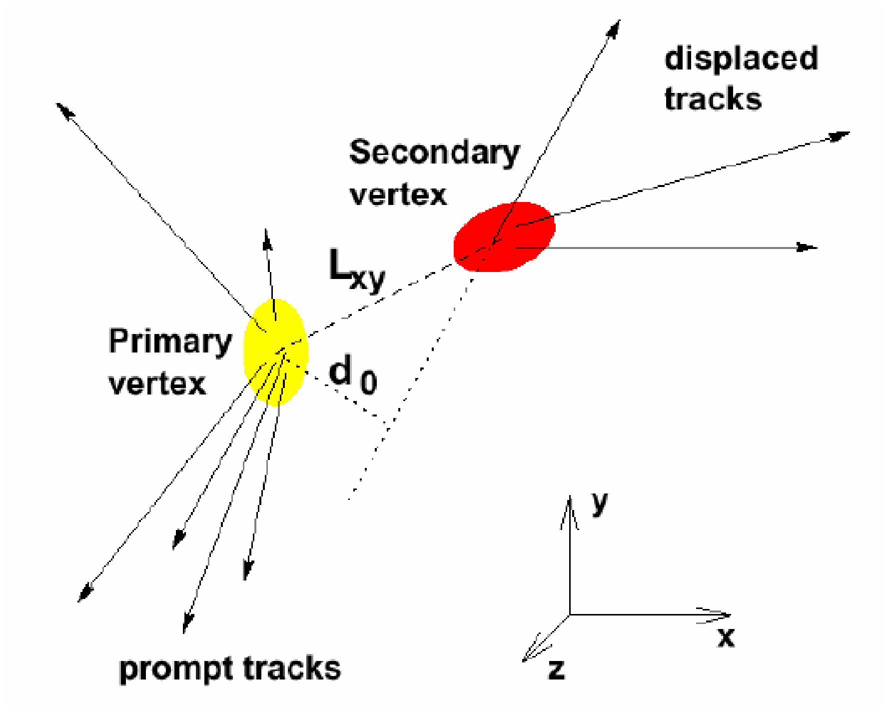
Measurement challenges

- **Top events rare**: $\sigma_{t\bar{t}} / \sigma_{p\bar{p}} \simeq 10^{-10}$, **event selection** important
- **Background processes** can mimic $t\bar{t}$, contaminate sample
- **Combinatorics**: Detector can ID e,μ, but which jets came from which parton?
- **Neutrinos not detected**: missing kinematic information
- **Jet reconstruction**
  - Measurement requires knowledge of quark momenta
  - Large Jet Energy Scale (JES) uncertainties in jet reconstruction (right)
- Define Δ_{JES} = JES correction
  - Δ_{JES} constrained via invariant mass of 2 jets from W boson
  - Measure Δ_{JES} simultaneously
  - Large systematic JES uncertainty replaced by smaller statistical Δ_{JES} uncertainty



Event selection

- Use Secondary Vertex b-tagging to identify b-jets
  - reduces background fraction and # jet-parton assigments (6 for single-tag, 2 for double tag)
- Event selection tuned to the lepton+jets decay signature
- Standard CDF lepton+jets selection:
  - *e* or *μ*, E_T>20 GeV, $|\eta|$<1.1
  - Missing E_T>20 GeV
  - Exactly 4 jets E_T>20 GeV, $|\eta|$<2.0
  - ≥ 1 b-tag

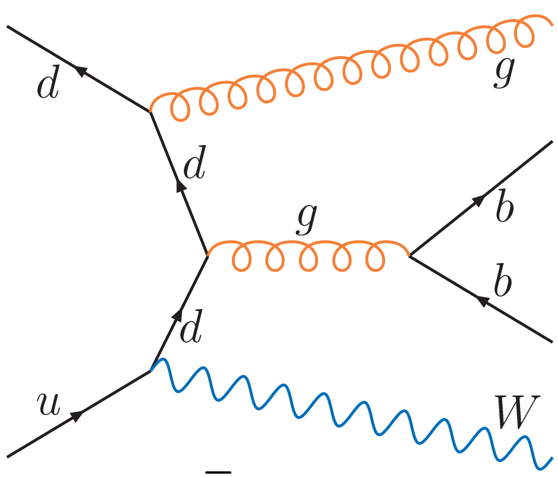


b-tag efficiency ~40%

- Analysis uses data up to August 2008 (3.2 fb^{-1})

Sample composition

- Background estimation
  - Partly data-based estimate
  - Also uses theoretical cross-sections and MC events
- Expected total $t\bar{t}$ fraction 76%
- W+jets dominant background



$Wb\bar{b}$ + 2 light jets

sample	# of events	% of total	% of bkg
$t\bar{t}$ signal	425.0 ± 58.9	76.0%	-
W+jets	92.6 ± 15.9	16.6%	69.0%
non-W	25.0 ± 12.5	4.5%	18.7%
single top quark	6.6 ± 0.4	1.2%	4.9%
diboson	6.0 ± 0.6	1.1%	4.5%
Z+jets	3.9 ± 0.5	0.7%	2.9%
total	559.2 ± 67.0	100%	-
Observed	578		

- Numbers used to create pseudo-experiments and calibrate method
- Fully-simulated events generated using PYTHIA, ALPGEN, MadEvent

Matrix Element method overview

- Calculate event probability density function (p.d.f.) based on theoretical LO matrix elements

- For parton-level quantities $\vec{y}$ produced by a process with matrix element $\mathcal{M}$:

p.d.f. $P(\vec{y}) = \frac{1}{\sigma}\frac{d\sigma}{dy}$ with differential cross-section $d\sigma \propto |\mathcal{M}|^2 \frac{d\Phi}{\Phi}$

- Map $\vec{y}$ to measured quantities ($\vec{x}$) using a Transfer Function (TF)
  - Integrate $P(\vec{y})$ over TF, PDFs and other unknown quantities:

(Parton Distribution Functions)

$$P(\vec{x}) = \frac{1}{\sigma} \sum_{\text{assign}} \int d\sigma W(\vec{x}, \vec{y})\, f(\tilde{q}_1)\, f(\tilde{q}_1)\, d\tilde{q}_1 d\tilde{q}_2$$

measured quantities

Average over possible jet-parton assignments

Integrate over parton-level quantities

Transfer Function

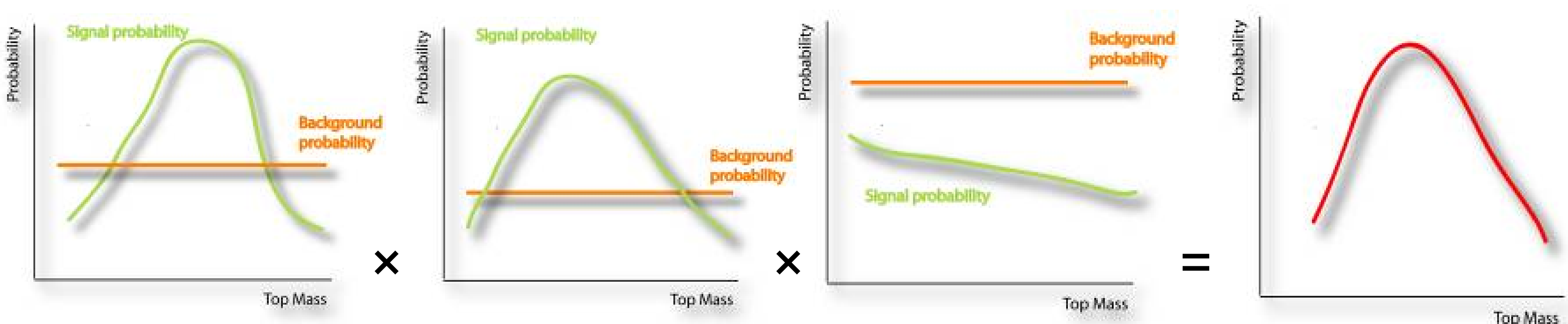
PDFs

Incoming parton momentum fractions

Matrix Element method overview

- "Signal" events: $P_{t\bar{t}}$ takes LO $q\bar{q} \to t\bar{t} \to W^{+}bW^{-}\bar{b}$ matrix element
- Dominant background comes from W+jets production (69%)
  - represent all background events with a single p.d.f., $P_{\mathrm{W+jets}}$
  - $P_{\mathrm{W+jets}}$ matrix element taken from sum of 1286 W+4p amplitudes (VECBOS)
- Combine signal and background p.d.f.s using signal fraction ν_{sig}
- $m_t, \Delta_{\mathrm{JES}}, \nu_{\mathrm{sig}}$ measured from maximisation of the likelihood function:

$$L(m_t, \Delta_{\mathrm{JES}}, \nu_{\mathrm{sig}}) \propto \prod_{i=1}^{Nevents} (\nu_{\mathrm{sig}} P_{t\bar{t},i}(m_t, \Delta_{\mathrm{JES}}) + (1-\nu_{\mathrm{sig}})\, P_{\mathrm{W+jets},i})$$



Signal event **Signal event** **Background event** **Likelihood**

Transfer Function

- Probability that parton-level quantity '*y*' resulted in measured quantity '*x*'
  - accounts for detector resolution, jet reconstruction
  - also accounts for parton showers and hadronisation
- Taken from fit to PYTHIA Monte-Carlo $t\bar{t}$ events (with known '*x*' and '*y*')
- Functional form:

$$W\left(\vec{x},\vec{y};\Delta_{\mathrm{JES}}\right)=\delta^3\left(p_l^y-p_l^x\right)W_A\left(\mathbf{\Omega}^{\mathbf{x}},\mathbf{\Omega}^{\mathbf{y}}\right)\prod_{i=1}^{4\text{ jets}}\frac{1}{E_{jet,i}\,p_{jet,i}}W_E\left(E_{jet,i},E_{parton,i};\Delta_{\mathrm{JES}}\right)$$

lepton assumed well measured — jet angle TF — jet energy TF

  - Allows for JES correction Δ_{JES}
- Define JES-corrected jet energies using $E'_{jet}=E_{jet}-\Delta_{\mathrm{JES}}\cdot\sigma_{jet}$
- σ_{jet} is the total uncertainty on the jet's energy
  - estimated for each jet based on CDF JES calibration
  - plot on next slide

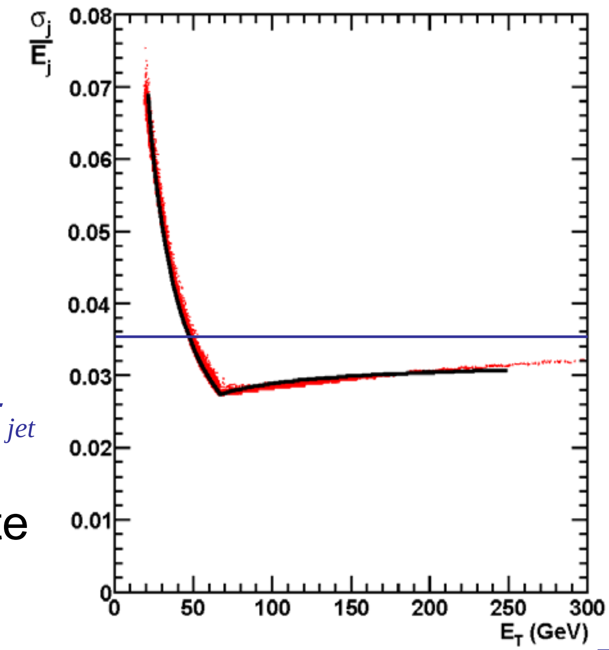
σ_{jet} based JES correction

- Energies of jets with low E_T less well known
- Makes sense to allow their energies to vary more with Δ_{JES}

$$E'_{jet} = E_{jet} - \Delta_{\text{JES}} \cdot \sigma_{jet}$$

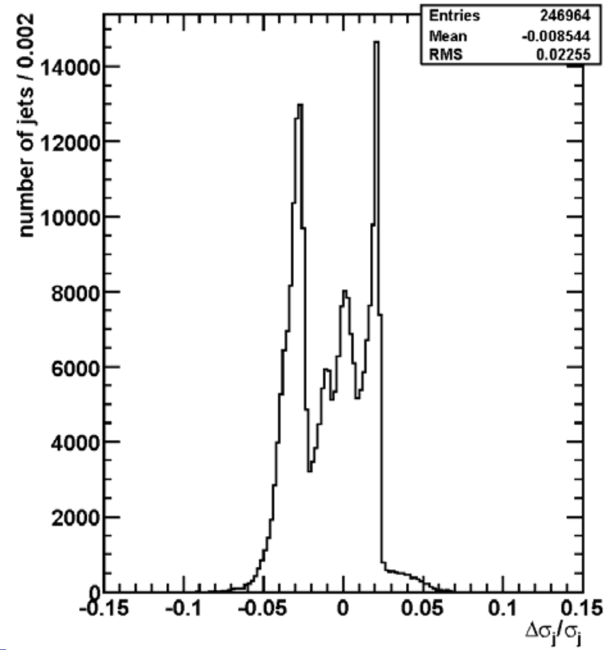
- Analytic energy TF normalisation not possible using exact σ_{jet} for each jet
- Make a fit and calculate approximate σ_{jet}
  - σ_{jet}/E_{jet} distribution (red points) can be approximated by a 1D function of jet E_T (black line)

σ_{jet}/E_{jet} vs E_T



"scale factor" $\sigma_{jet} \sim \text{constant} \cdot E_{jet}$ plotted in blue for comparison

$\Delta\sigma_{jet} / \sigma_{jet}$



- Difference $\Delta\sigma_{jet}$ between approx σ_{jet} and true σ_{jet} is small (<5%)

Jet energy Transfer Function

- TF describes parton-jet mapping for events well-described by signal p.d.f. $P_{t\bar{t}}$
- Use ‘good’ signal events only in fit
  - 4 measured jets well matched to partons (ΔR<0.4)
  - Expected lepton + jets decay chain

- Parameterise jet energy TF using double-Gaussian

$$W_E\left(E_{parton}-E'_{jet}\right)=\frac{1}{N\left(E_{parton},\Delta_{\mathrm{JES}}\right)}\left(e^{-\frac{1}{2}\left(\frac{E_{parton}-E'_{jet}-p_1}{p_2}\right)^2}+\left|p_3\right|e^{-\frac{1}{2}\left(\frac{E_{parton}-E'_{jet}-p_4}{p_5}\right)^2}\right)$$

$$p_i=a_i+b_iE_{parton}$$

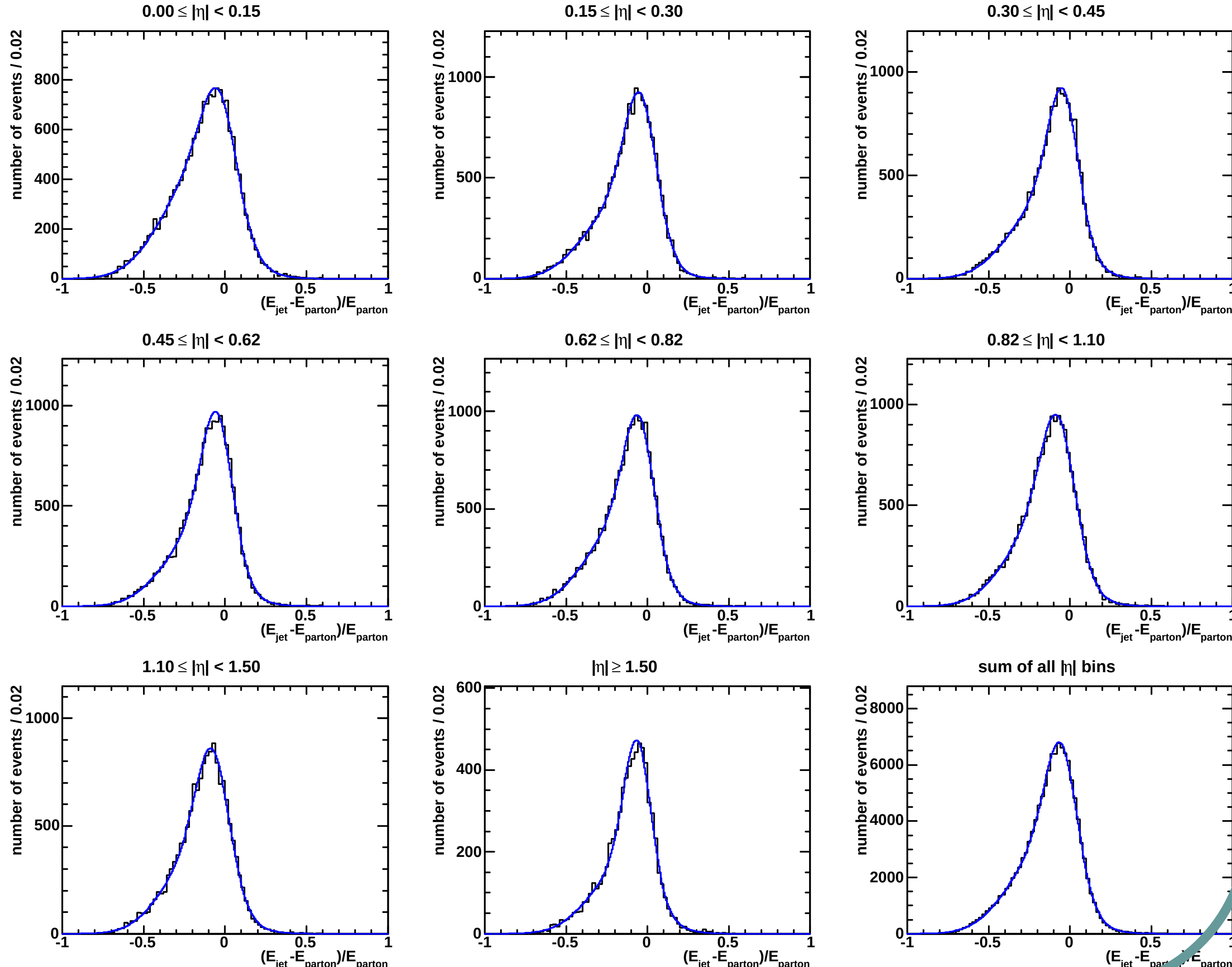
- describes mapping between parton energies E_{parton} and Δ_{JES} -corrected jet energies E'_{jet}
  - parameterisation independent of Δ_{JES}

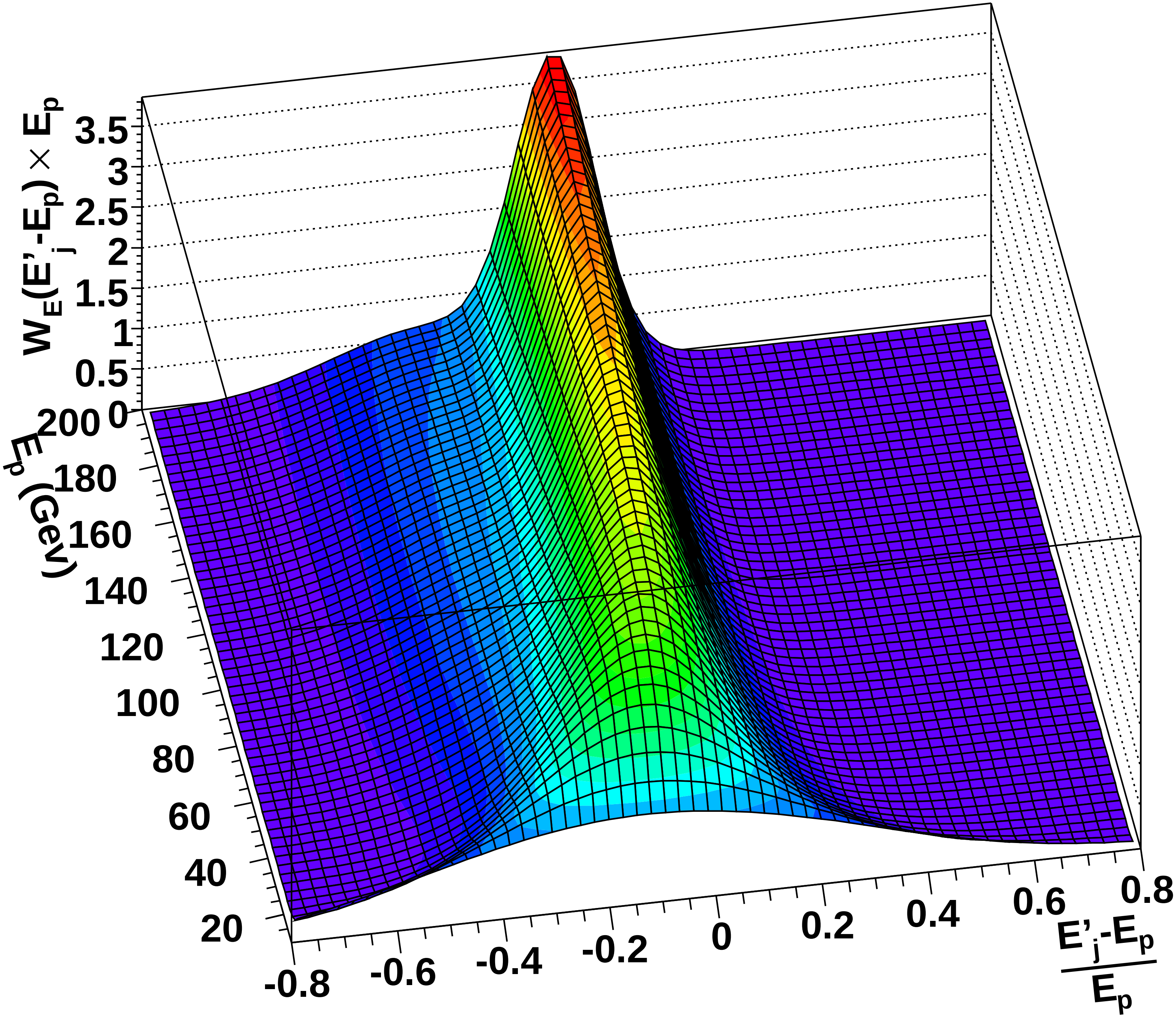
- Fit is made in 8 |η| bins to model changing calorimeter response

- Fit separately for b and light quarks

Jet energy Transfer Function

- b-jet TF bins
- Histograms show MC distributions of $\left(E_j - E_p\right)/E_p$
- Blue line is fitted TF
- Note these are projections of full 2D TF (next slide)





$W_E(E'_j-E_p) \times E_p$
0
0.5
1
1.5
2
2.5
3
3.5
E_p (Gev)
20
40
60
80
100
120
140
160
180
200
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
$\frac{E'_j-E_p}{E_p}$

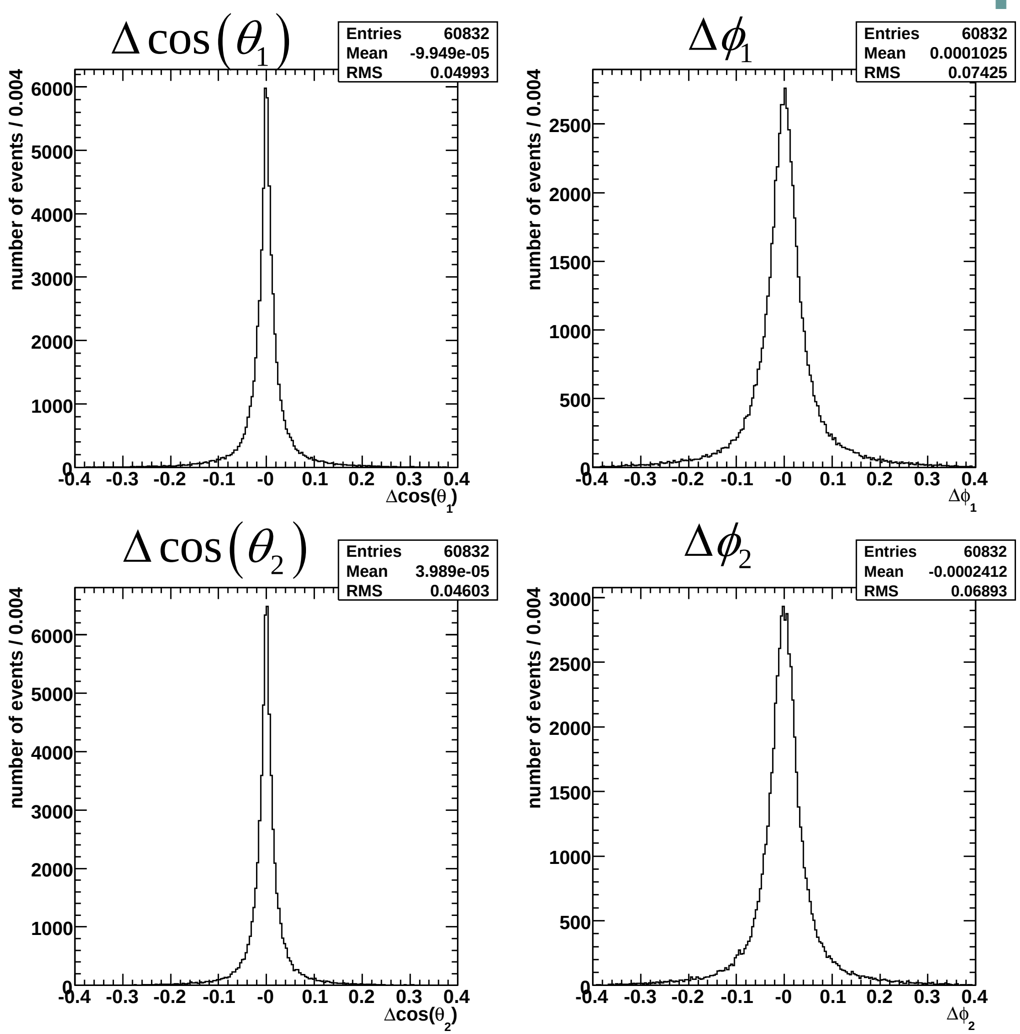
Jet angular TF: Motivation

- Previously assumed TF

$$W_A\left(\Omega_x,\Omega_y\right)=\delta^2\left(\Omega_x-\Omega_y\right)$$

- Appeared to be a reasonable assumption
  - Angular resolution distributions have mean=0 and small RMS
- However, effect not necessarily visible in $\theta_1,\phi_1,\theta_2,\phi_2$
- Noticed that measured angles gave lower m_W than parton-level angles
- $m_W^2\simeq 2E_1E_2\left(1-\cos\left(\alpha_{12}\right)\right)$
- look at cos(α_{12}) resolution

Angular resolution of the two jets from hadronic W decay

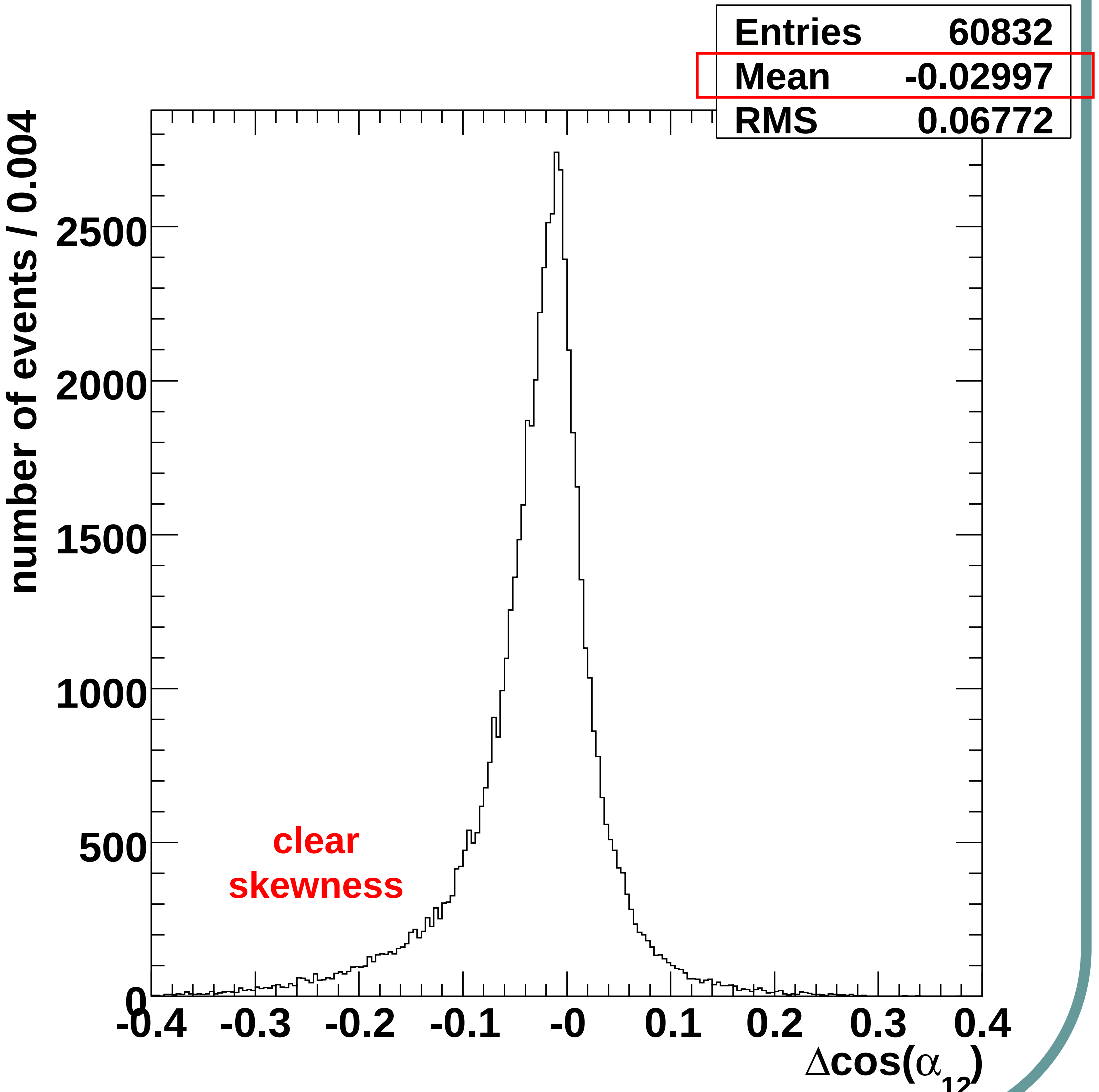


Correlation in measured jet angles

- α_{12} is the angle between the two hadronic-side W jets
- Non-zero mean and significant skewness seen in $\cos(\alpha_{12})$ resolution plot

$$\Delta\cos(\alpha_{12}) = \cos(\alpha_{12})_{parton} - \cos(\alpha_{12})_{jet}$$

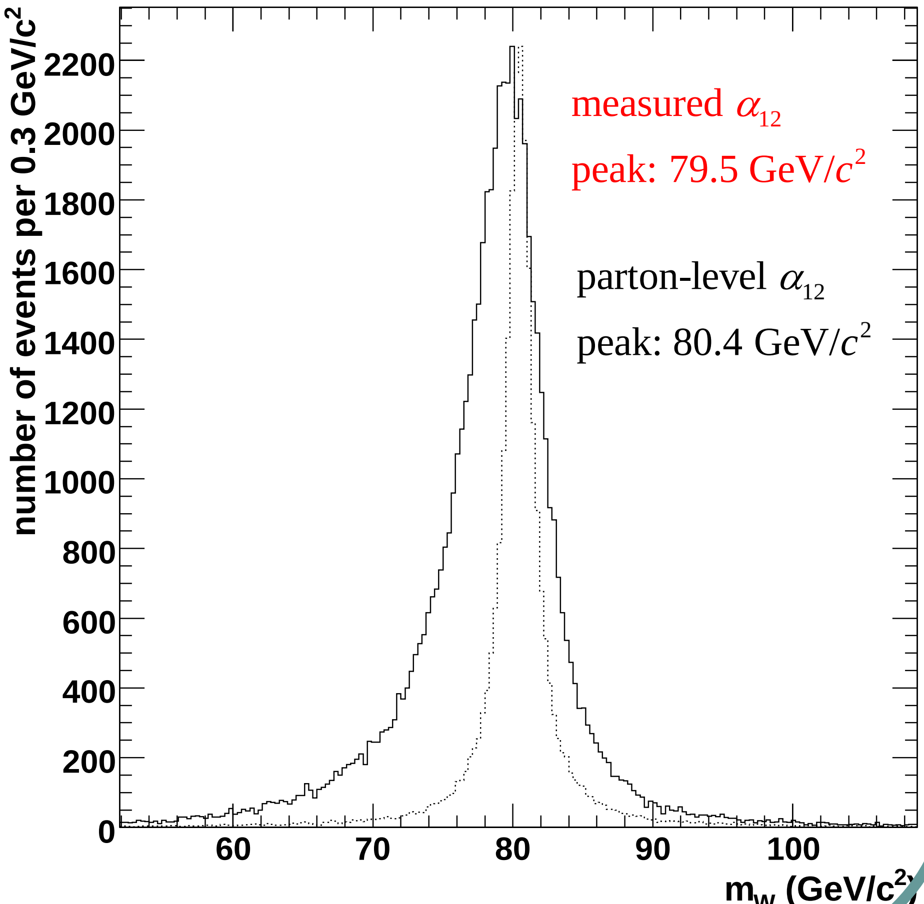
- Correlation between measured jet directions
  - Measured jets appear closer together than parent partons



Effect on W mass

- Shift in $\cos(\alpha_{12})$ directly affects measured W mass
  - -1 GeV in peak (~1%)
  - Clear skewness
- Plot uses parton-level jet energies
  - Solid line for measured angles
  - Dashed is for parton angles
- Δ_{JES} measurement relies on the apparent W mass
  - $E'_{jet} = E_{jet} - \Delta_{\mathrm{JES}} \cdot \sigma_{jet}$
  - Low apparent m_W would cause negative bias in Δ_{JES} (to increase the jet energies)
  - Resulting bias in m_t (~1%)
- Construct angular TF in $\cos(\alpha_{12})$

$$m_W \simeq \sqrt{2E_1^{parton} E_2^{parton} \left(1 - \cos\left(\alpha_{12}\right)\right)}$$

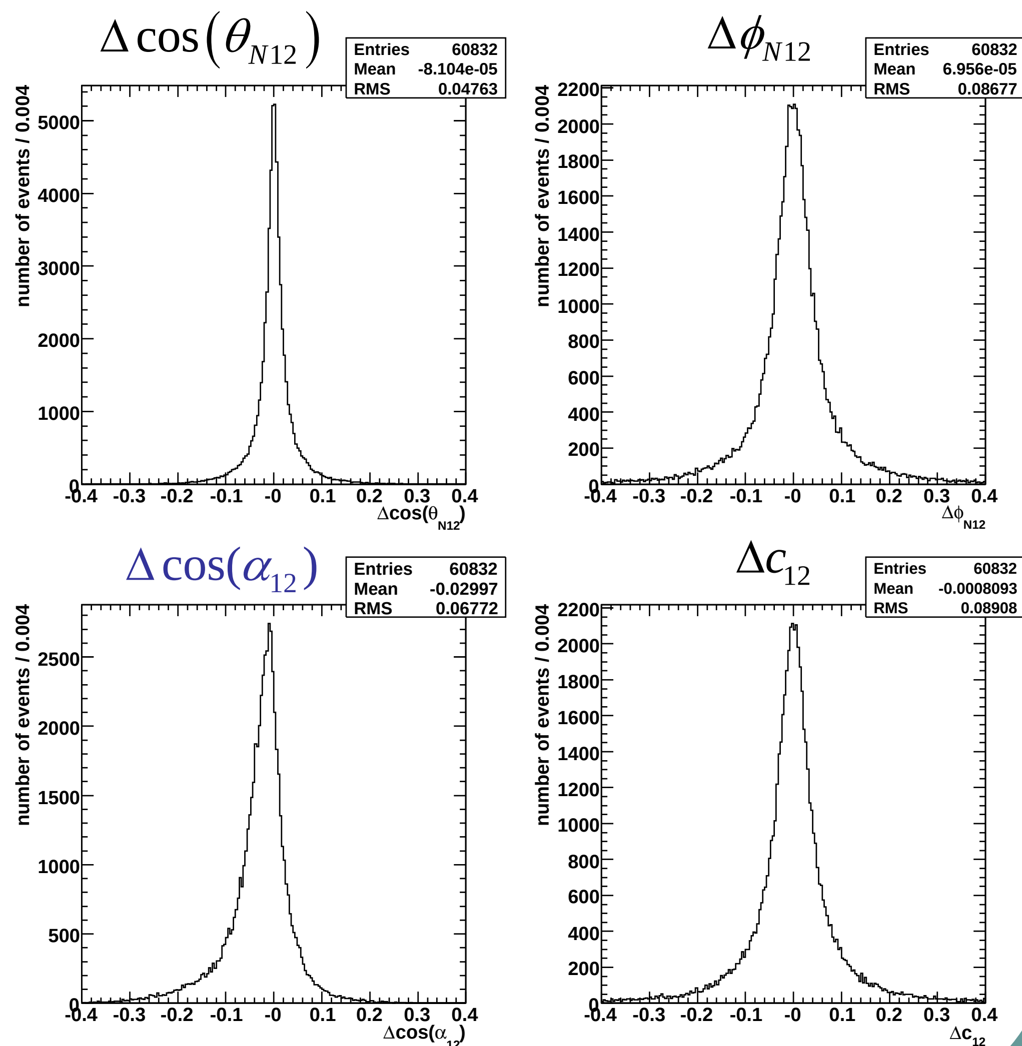


Change of variables

- Need a jet angle coordinate system in which α_{12} is a variable
- θ_{N12}, ϕ_{N12} are the angles of the normal to the two jets
- ϕ_1', ϕ_2' are the angles of the jets in the plane perpendicular to the normal
- Then $\alpha_{12} = \phi_2' - \phi_1'$
- Also define $c_{12} = \dfrac{\phi_2' + \phi_1'}{2}$
- Transform from measured variables $\theta_1, \phi_1, \theta_2, \phi_2$ to new variables
- Jacobian determinant $\mathrm{J}\left(\dfrac{\cos(\theta_1)\phi_1 \cos(\theta_2)\phi_2}{\cos(\theta_{N12})\phi_{N12}\cos(\alpha_{12})c_{12}}\right) = -1$

Change of variables

- 4 new variables represent the 4 measured angles of the two hadronic-side W jets
- Remaining 3 distributions are symmetric and have mean 0
- Use delta-function approx for TF for these 3 variables
- Parameterize shape for $\Delta\cos(\alpha_{12})$



Parameterization

- Parameterise shape with a Skew-Cauchy distribution plus 2 Gaussians

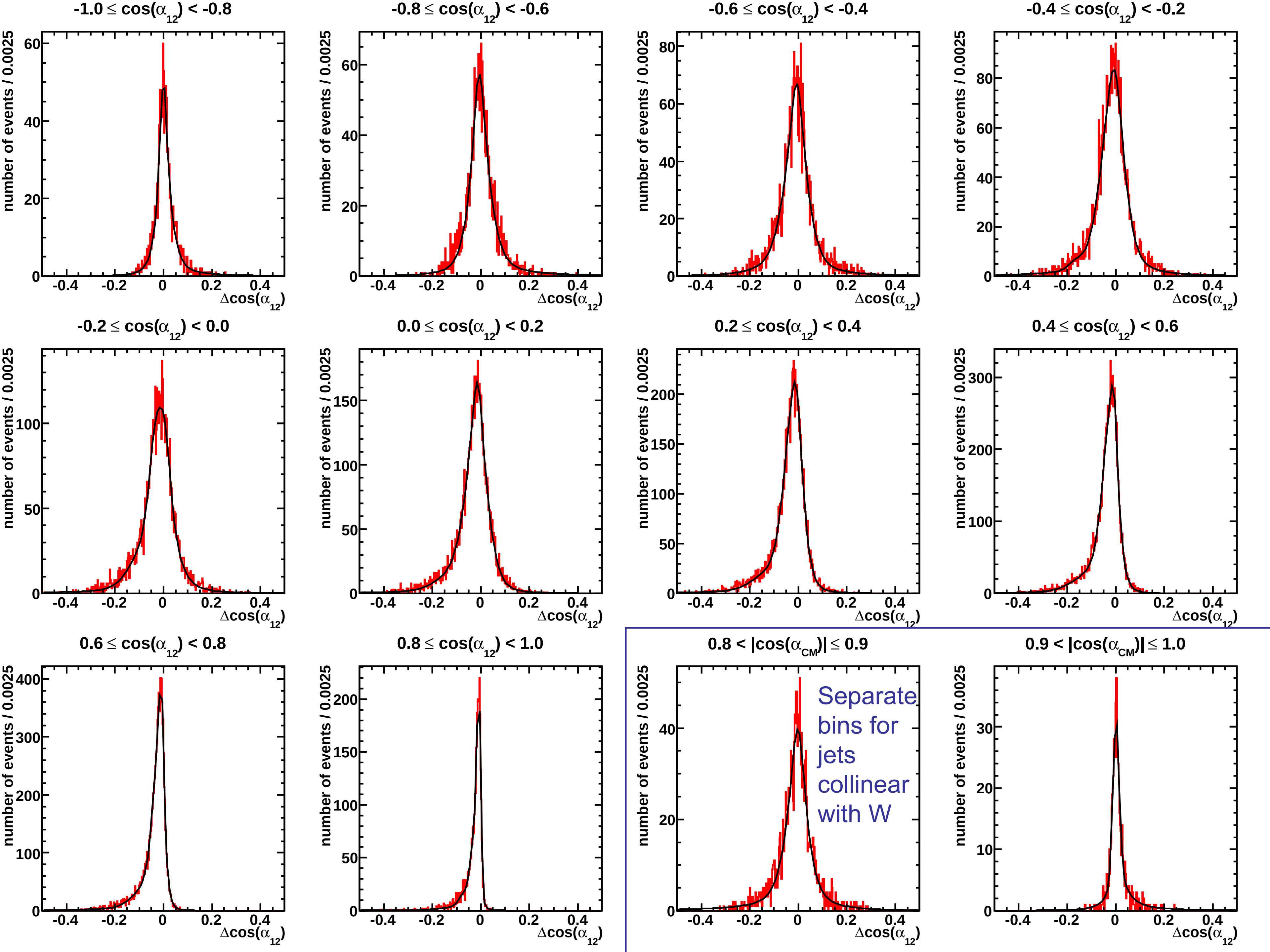
$$W\left(\Delta\cos(\alpha_{12})\right) = W\left(\cos\left(\alpha_{12}^{parton}\right) - \cos\left(\alpha_{12}^{jet}\right)\right)$$

Second Gaussian omitted in bins with few events

$$W\left(\Delta\cos\alpha_{12}\right) = a_o \frac{\left(1+\frac{2\arctan\left(\left(\Delta\cos\alpha_{12}-a_1\right)a_3\right)}{\pi}\right)}{\pi a_2\left(1+\frac{\left(\Delta\cos\alpha_{12}-a_1\right)^2}{a_2{}^2}\right)} + a_4\frac{\exp\left(-\frac{\left(\Delta\cos\alpha_{12}-a_5\right)^2}{2a_6{}^2}\right)}{\sqrt{2\pi}a_6} + a_7\frac{\exp\left(-\frac{\left(\Delta\cos\alpha_{12}-a_8\right)^2}{2a_9{}^2}\right)}{\sqrt{2\pi}a_9}$$

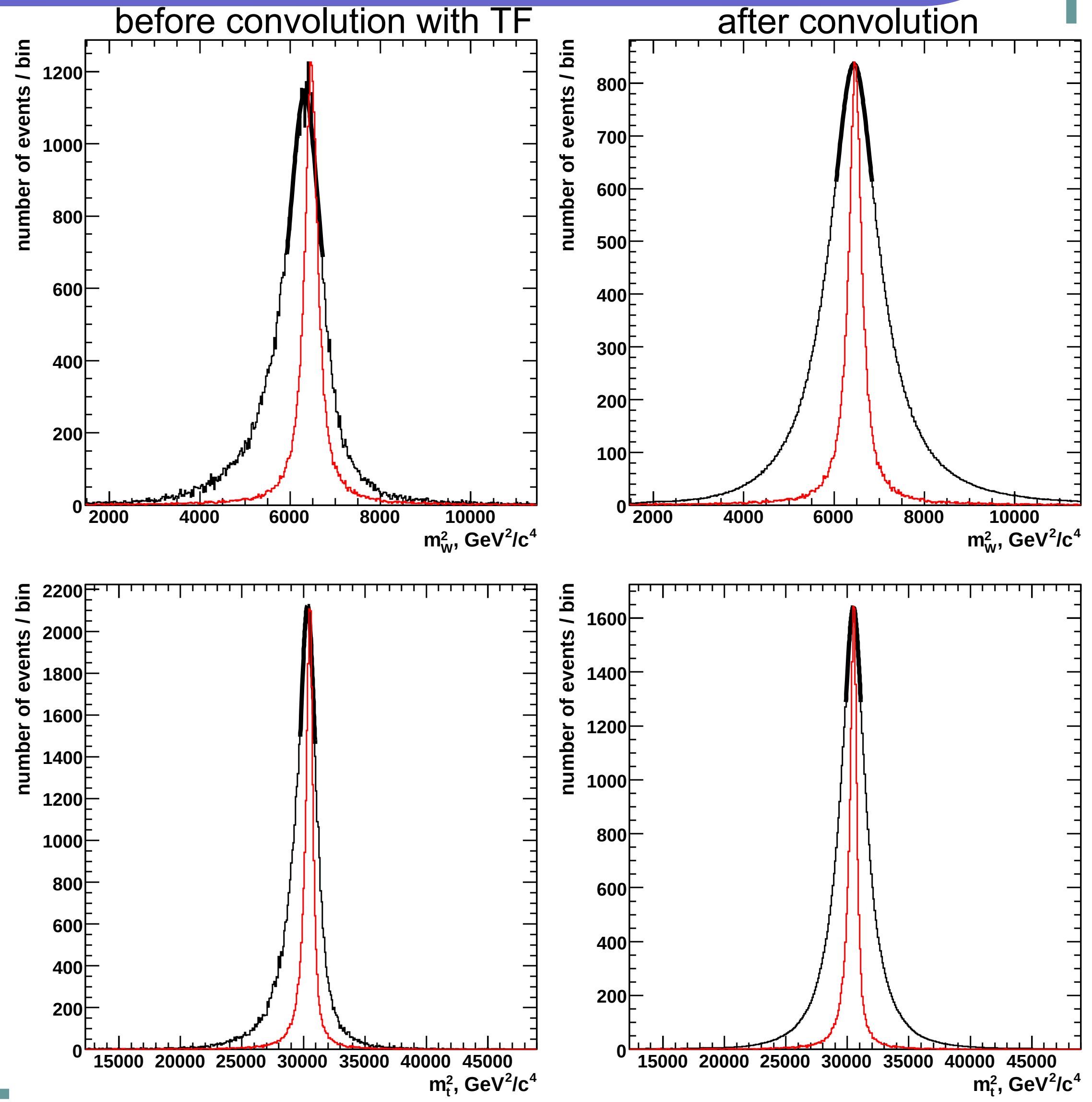
- Shape of distribution depends primarily on 2 measured quantities
  - $\cos\left(\alpha_{12}^{jet}\right)$
  - $\left|\cos\left(\theta_{CM}\right)\right|$
    - θ_{CM} is the angle of the jets with respect to the W direction in the W CM frame

- 12 bins, in $\cos\left(\alpha_{12}^{jet}\right)$ and $\left|\cos\left(\theta_{CM}\right)\right|$ (next slide)


-1.0 ≤ cos(α12) < -0.8
-0.8 ≤ cos(α12) < -0.6
-0.6 ≤ cos(α12) < -0.4
-0.4 ≤ cos(α12) < -0.2
-0.2 ≤ cos(α12) < 0.0
0.0 ≤ cos(α12) < 0.2
0.2 ≤ cos(α12) < 0.4
0.4 ≤ cos(α12) < 0.6
0.6 ≤ cos(α12) < 0.8
0.8 ≤ cos(α12) < 1.0
0.8 < |cos(αCM)| ≤ 0.9
0.9 < |cos(αCM)| ≤ 1.0
number of events / 0.0025
Δcos(α12)
Separate bins for jets collinear with W

Angular TF

- Similar correlation in the angle between hadronic-side W and b, α_{Wb}
  - much smaller effect
  - similar parameterization
- Use delta-function TF for all remaining angles (CPU limitations)
- Shape of m_W and m_t distributions unbiased after convolution with TF
  - —— measured angles
  - —— parton-level angles
- Angular TF also describes most important angular resolution effects (ones which affect hadronic-side m_W and m_t)



$P_{t\bar{t}}$ calculation details

- $$P_{t\bar{t}}(\vec{x};m_t,\Delta_{\text{JES}}) = \frac{1}{\text{Acc}(m_t,\Delta_{\text{JES}})\sigma(m_t)} \sum_{\text{assign}} \int d\sigma W(\vec{x},\vec{y},\Delta_{\text{JES}})\, f(\tilde{q}_1)\, f(\tilde{q}_1)\, d\tilde{q}_1 d\tilde{q}_2$$

  Acceptance normalisation term (next slide)

  18 integration variables (6 outgoing particles' 3-momentum)

  2 further integrations for PDFs

- 20 total integrations reduced by
  - 4-momentum conservation
  - Delta-functions in TF
  - Change of variables from jet energies to top and W squared masses, with narrow-width approx for top Breit-Wigners
- Overall 5-dimensional integration over $m^2_{W,h}, m^2_{W,l}, E_{\text{jet1}}, \cos(\alpha_{12}), \cos(\alpha_{Wb})$
- Evaluated using VEGAS adaptive MC algorithm (importance sampling)
  - ~3 CPU hours per event

$P_{t\bar{t}}$ normalisation

- Event acceptance effects represented by acceptance function $\mathrm{Acc}(\vec{x})$
  - fraction of events with quantities x that get detected and selected
  - assume $\mathrm{Acc}(\vec{x})$ depends only on *x*
- Normalisation: require $\int \mathrm{Acc}(\vec{x}) P_{t\bar{t}}(\vec{x}) d\vec{x} = 1$
- $P(\vec{x})$ depends on m_t and Δ_{JES} so $\overline{\mathrm{Acc}}(m_t, \Delta_{\mathrm{JES}}) = \int \mathrm{Acc}(\vec{x}) P(\vec{x}) d\vec{x}$
- Monte-Carlo integration using PYTHIA events:

$$\overline{\mathrm{Acc}}(m_t, \Delta_{\mathrm{JES}}) \simeq \sum_i^{\mathrm{events}} \mathrm{Acc}(\vec{x}_i) \frac{1}{n_{\mathrm{events}}}$$

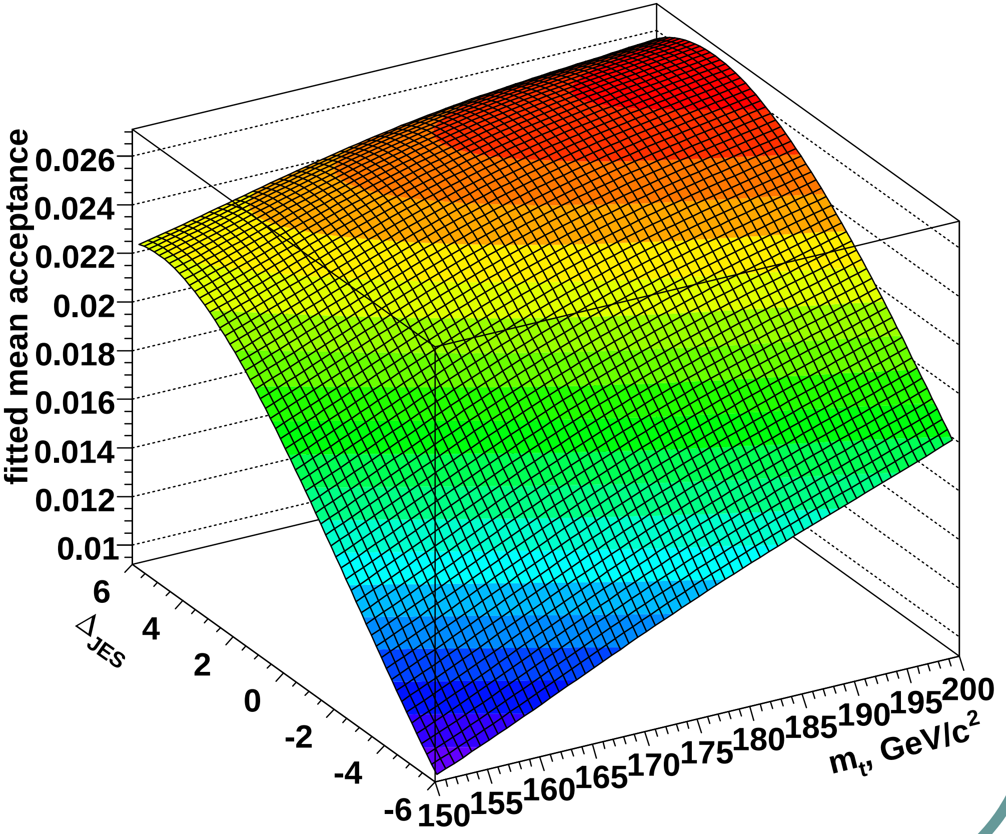
- $\overline{Acc}(m_t, \Delta_{\mathrm{JES}})$ is taken from the mean acceptance of PYTHIA events
- Note that since $\mathrm{Acc}(\vec{x})$ is independent of m_t and Δ_{JES} it factors out of the Likelihood

Acceptance Function

- Mean acceptance function $\overline{\mathrm{Acc}}(m_t, \Delta_{\mathrm{JES}}) \simeq \dfrac{\text{Number passing all cuts}}{\text{Total generated events}}$

- Take mean acceptance for 44 mass samples at 25 Δ_{JES} points

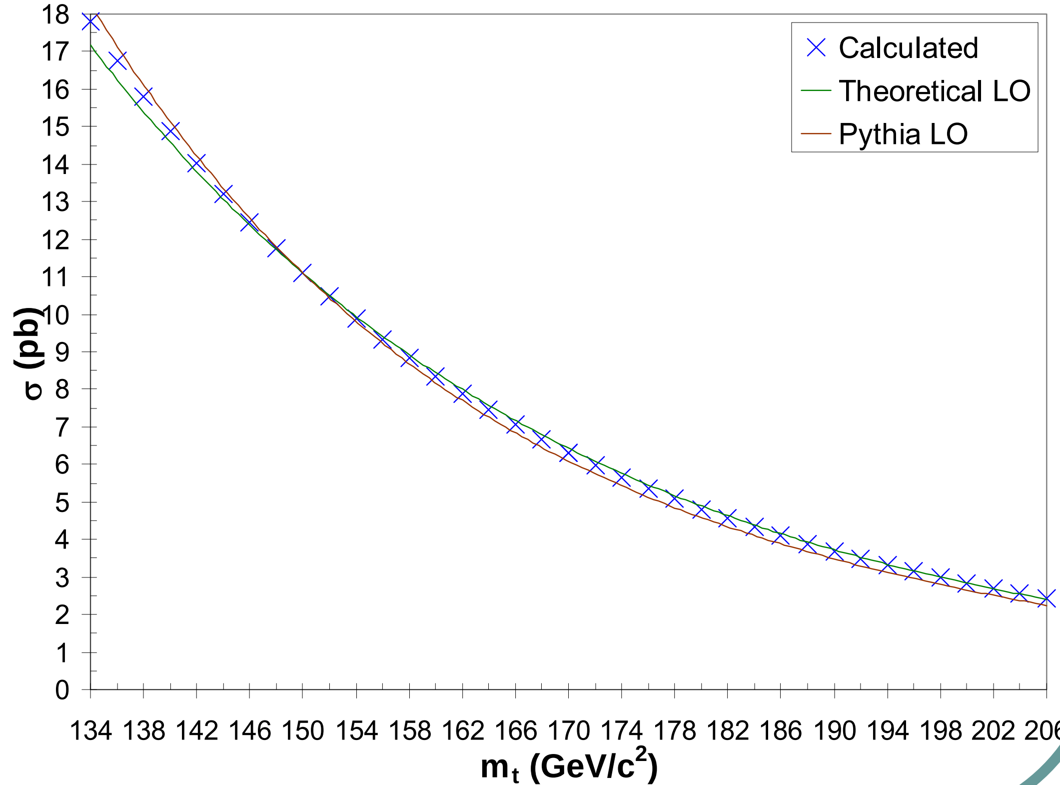
- Fit with 2D fourth-order polynomial in m_t and Δ_{JES}



Cross-section normalisation

- $\sigma(m_t)$ calculated using similar integration to $P_{t\bar{t}}$
  - $\sigma(m_t) = \int d\sigma \;\; f(\tilde{q}_1) f(\tilde{q}_1) d\tilde{q}_1 d\tilde{q}_2$
  - 14-dimensional integration (~20 CPU hours for each m_t point)

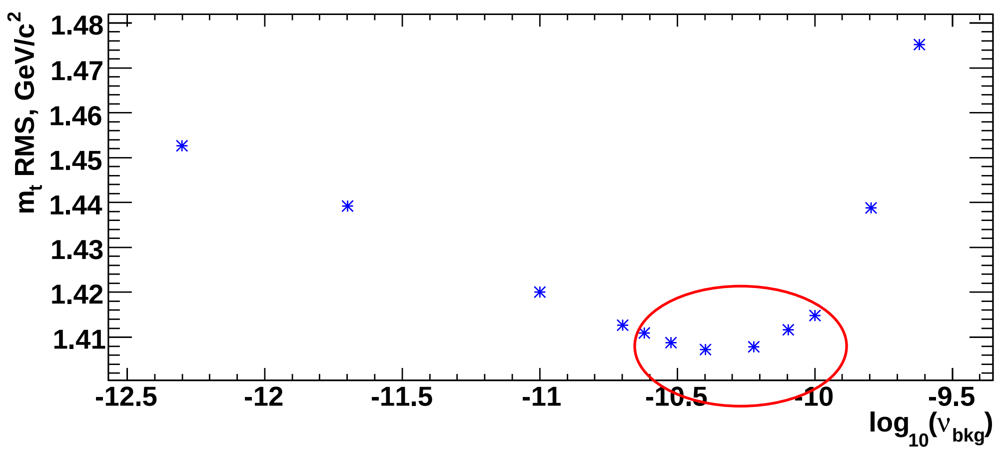
- Previously $\sigma(m_t)$ was taken from Pythia cross-section
- Now fully consistent with $P_{t\bar{t}}$
- Also provides a test of code (compare to theoretical $\sigma(m_t)$)



$P_{\text{W+jets}}$ details

- W+jets matrix element much more computationally intensive (many diagrams)
- Customised MC integration technique with further approximations:
  - Narrow-width W Breit-Wigner to constrain neutrino p_z
  - Integrate only over the 4 jet energy TFs, and fix $\Delta_{\text{JES}} = 0$
  - $P_{\text{W+jets}}$ thus has no dependence on the likelihood parameters m_t, Δ_{JES}

- $P_{\text{W+jets}}$ normalisation is just a constant (but not calculable)
  - Require relative normalisation constant between signal and background prob
- Constant, ν_{bkg} , chosen to minimize
  - expected measurement statistical uncertainty
  - bias caused by background events

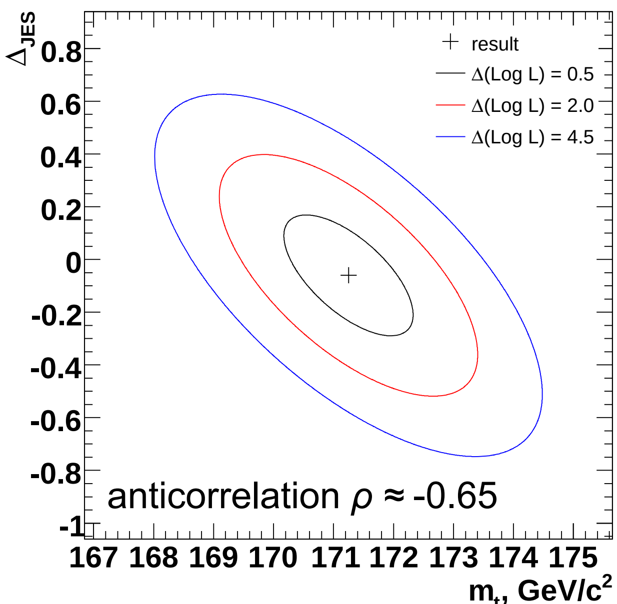


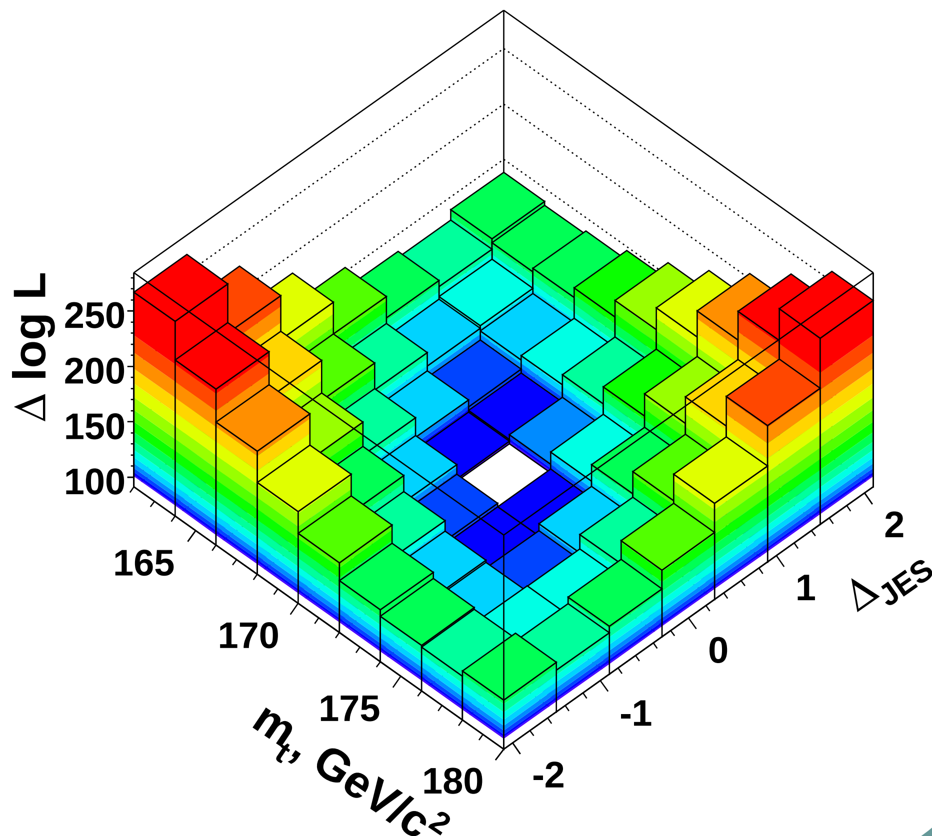
Likelihood

- $m_t, \Delta_{\text{JES}}, \nu_{\text{sig}}$ measured from maximisation of the likelihood function

$$L(m_t, \Delta_{\text{JES}}, \nu_{\text{sig}}) \propto \prod_{i=1}^{Nevents} (\nu_{\text{sig}} P_{t\bar{t},i}(m_t, \Delta_{\text{JES}}) + (1-\nu_{\text{sig}})\, P_{\text{W+jets},i})$$

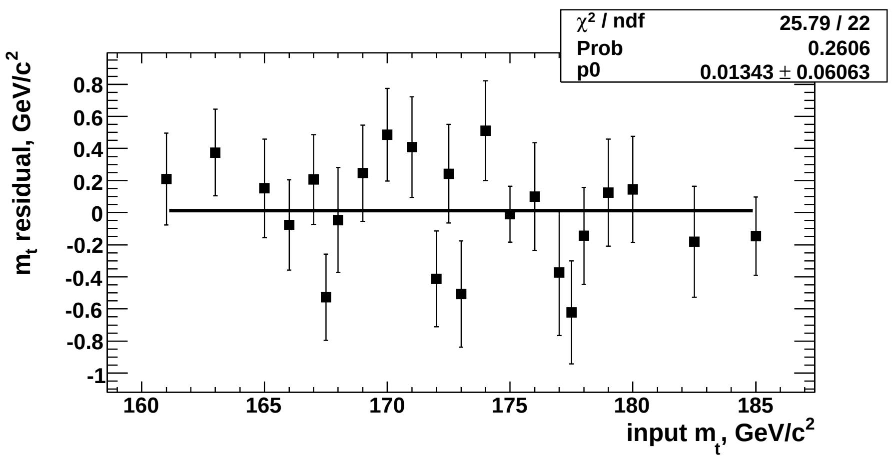
- First take profile Log L at max ν_{sig} (using MINUIT)
- Parabolic fit for m_t, Δ_{JES}



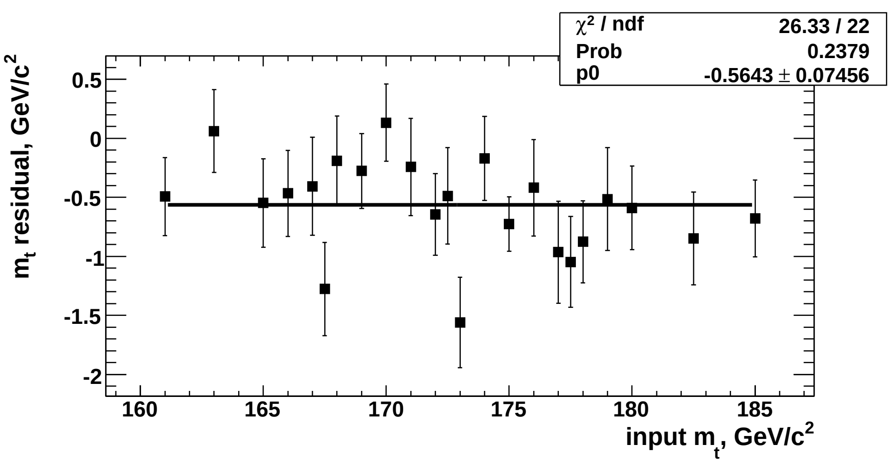


Linearity tests

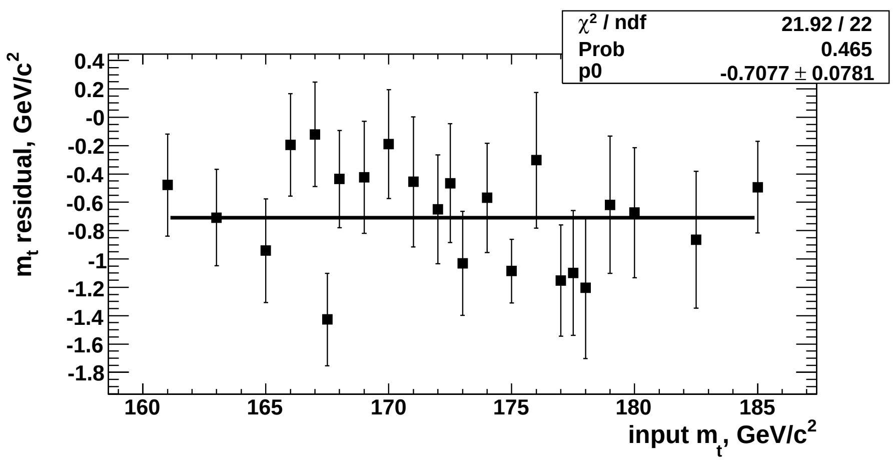
- Check method response over range of m_t
- First use "good" signal Pythia events only
  - expected lepton+jets decay chain
  - correct jet-parton assignments
- Second plot, average over allowed jet-parton assignments
- Third plot uses all signal events
- Measured m_t shifts down from ideal case



Good signal, correct combo



Good signal, average over assignments

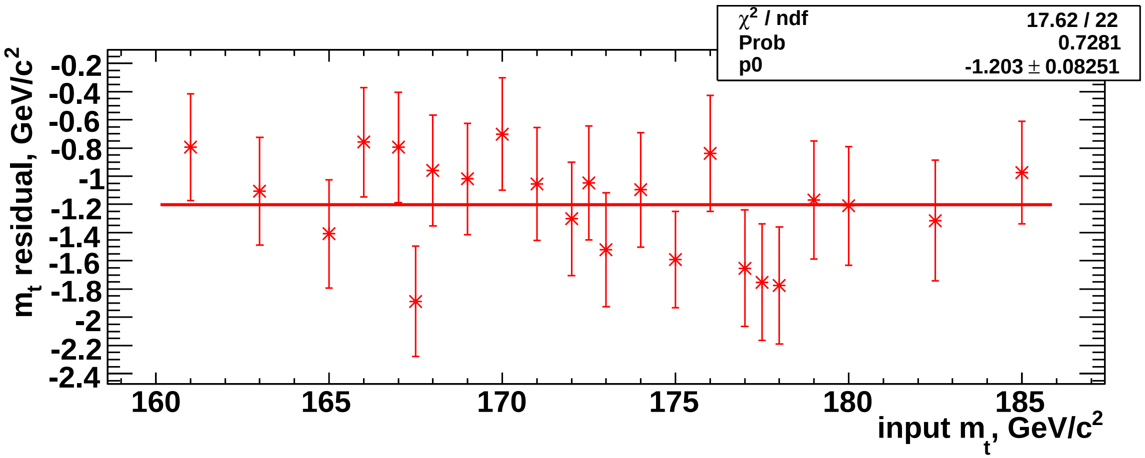


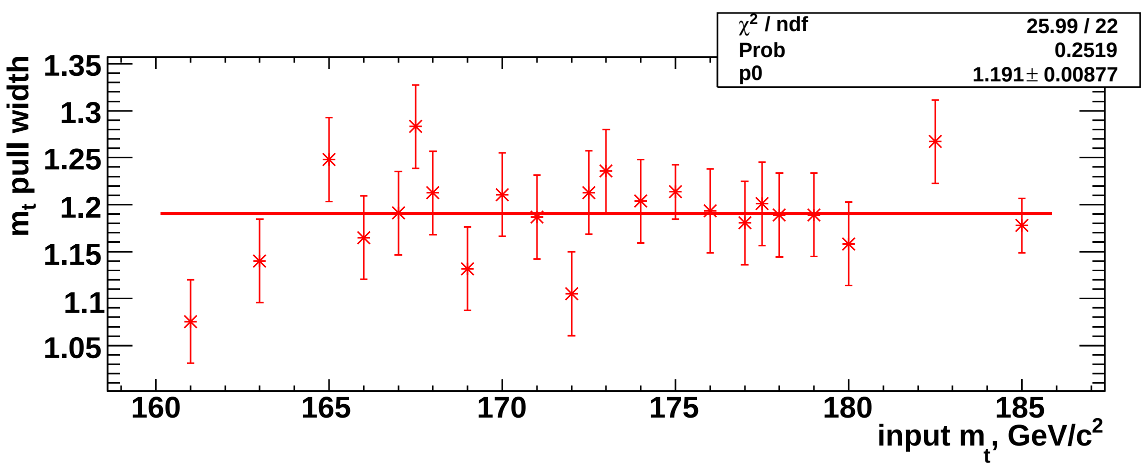
All signal, average over assignments

Pseudoexperiments

- Pseudoexperiments using full simulated background
- Background events cause further negative shift in measured m_t

Events	Δm_t (GeV/c^2)
Good signal, correct assign	0
Good signal, average assign	-0.6
All signal, average assign	-0.7
All signal + background	-1.2

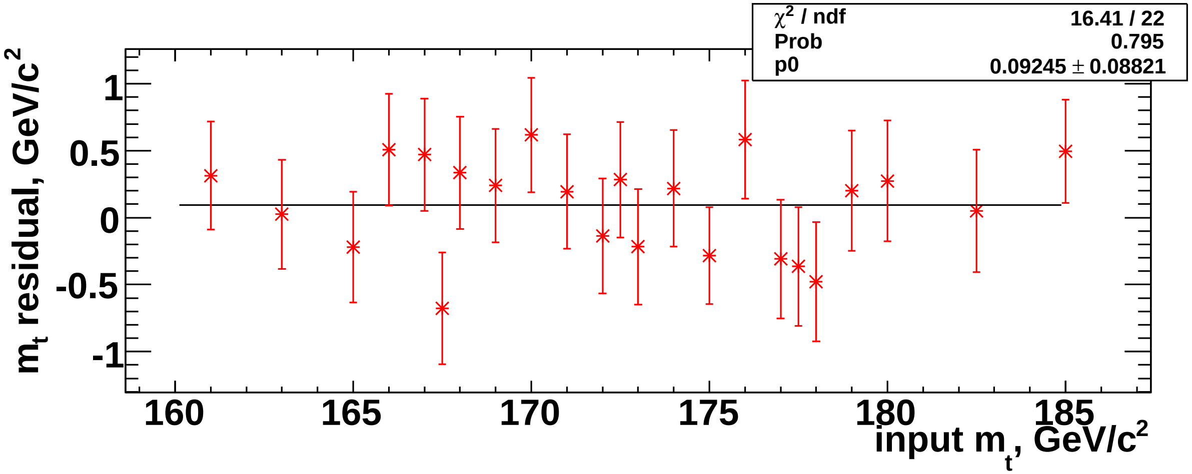




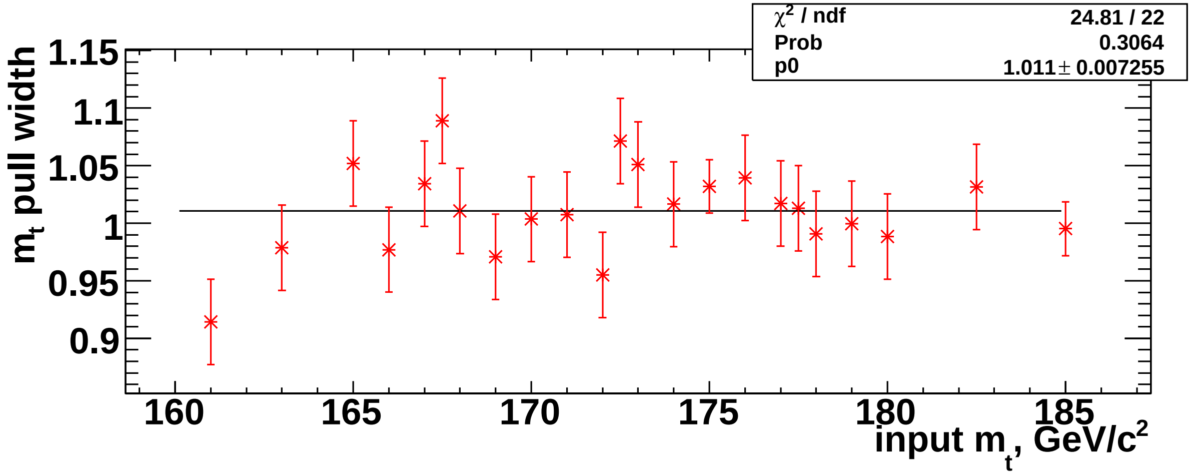
- Calibrate based on measured signal fraction ν_{sig}
    - Calibration based on measured Δ_{JES} is also required

Linearity

- Check performance of calibrated method using PYTHIA samples at various top masses, along with full simulated background



Measured m_t consistent across large range



Pull width also constant and consistent with 1

Systematic uncertainties

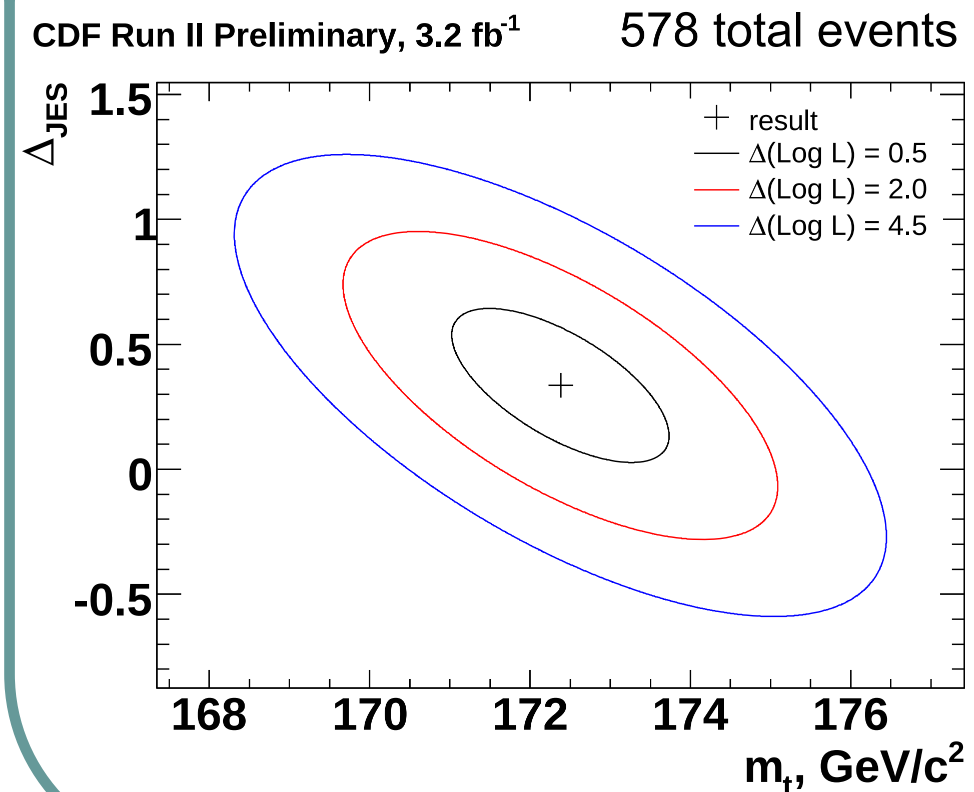
- Systematic uncertainties mainly due to uncertainties in MC event simulation
- Evaluated using standard CDF prescriptions
  - May be some double counting
  - Some effects only roughly described (eg colour reconnection)
- Future uncertainty on m_t dominated by systematics

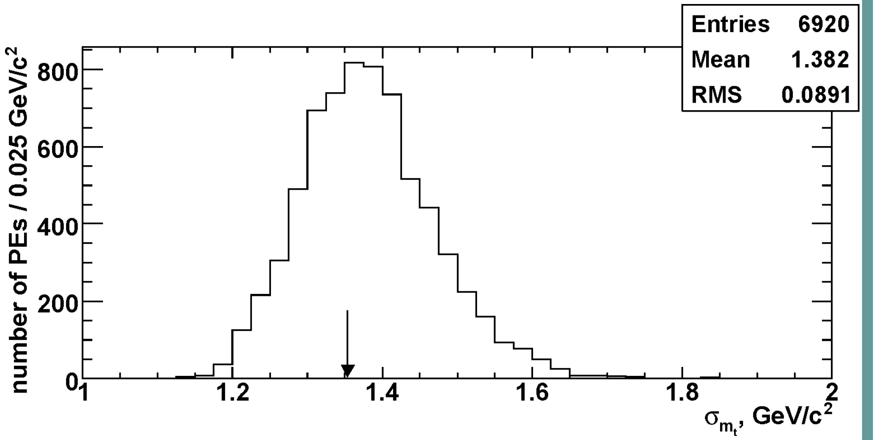
systematic uncertainties

Systematic source (Lepton+jets, ME)	**Systematic uncertainty (GeV/c^2)**
MC generator	0.70
Residual JES	0.65
Colour reconnection	0.56
Background	0.45
b-jet energy	0.39
ISR and FSR	0.23
Multiple hadron interactions	0.22
PDFs	0.13
Lepton P_T uncertainty	0.12
Method calibration	0.12
Total	**1.31**

Result

m_t = 172.4 ± 1.4 (stat+Δ_{JES}) ± 1.3 (syst) GeV/c^2
= 172.4 ± 1.9 (total) GeV/c^2





40% of pseudoexperiments have lower stat uncertainty

Summary

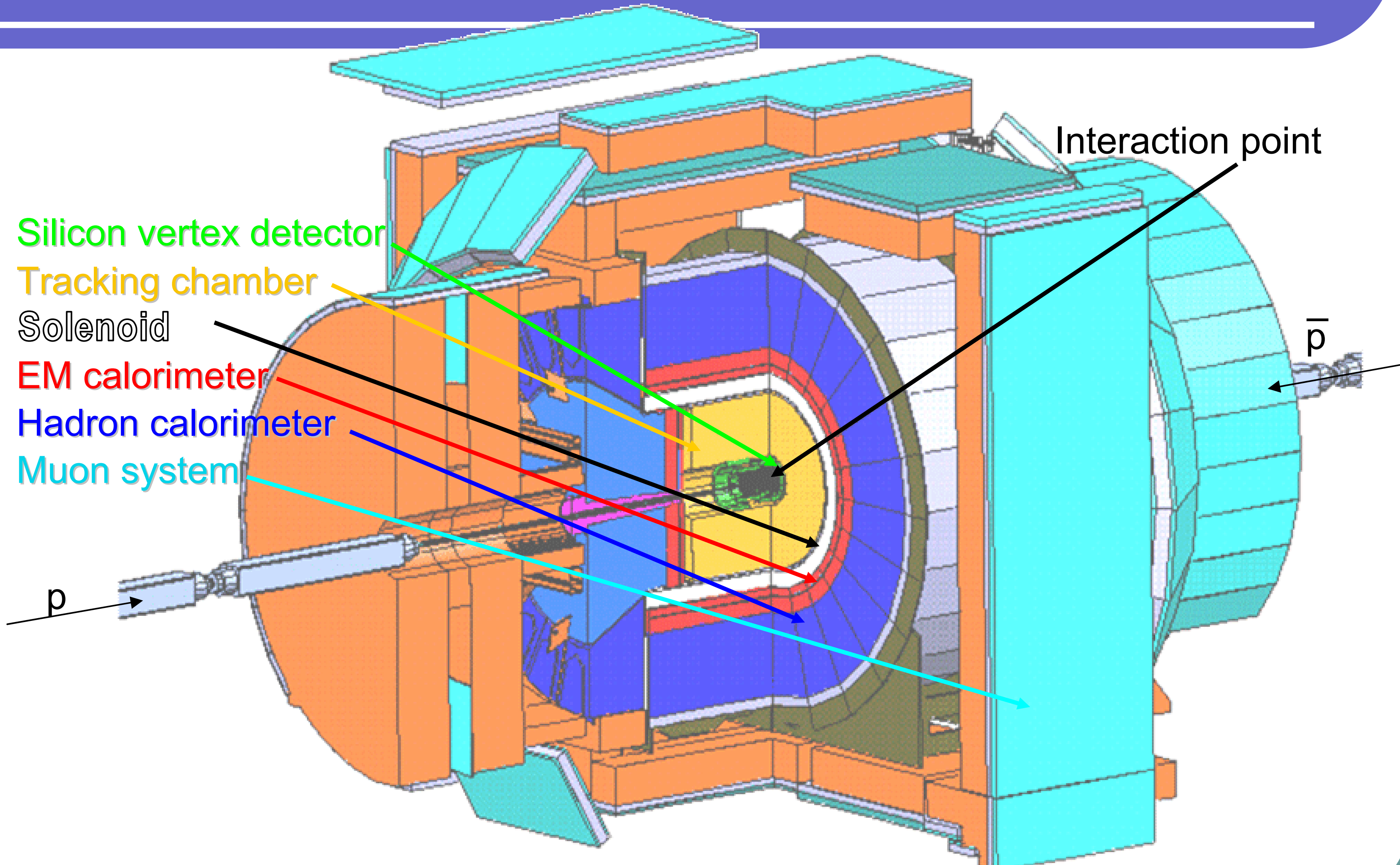
- Precision top quark mass measurement using 3.2 fb^{-1} CDF data
- Matrix element based unbinned maximum likelihood technique
  - p.d.f.s for both signal and background events
  - in-situ Δ_{JES} calibration
- Transfer Function model improved
  - Jet angular correlations not previously described
- $\boldsymbol{m_t = 172.4 \pm 1.9}$ **GeV/c^2**
- Higgs mass limits (in combination with other precision EW variables)
  - This m_t result: $m_H = 84^{+36}_{-26}$ GeV/c^2

    $m_H < 156$ GeV/c^2 (95% CL)
  - World Average: $m_H = 87^{+35}_{-26}$ GeV/c^2

    $m_H < 157$ GeV/c^2 (95% CL)

Backup

CDF detector



Energy TF Normalisation

Each of the jet Energy Transfer Functions (ETFs) have the form

$$W_E^i(E_j' - E_p) = W_E^i(\delta) = \frac{1}{\sqrt{2\pi}(p_2 + p_3 p_5)} \left[e^{\frac{-(\delta - p_1)^2}{2p_2^2}} + p_3 \cdot e^{\frac{-(\delta - p_4)^2}{2p_5^2}} \right]$$

where $E_j' = E_j - \Delta_{\text{JES}} \cdot \sigma_j$ the normalisation, from the requirement that

$$\frac{1}{N_{W_E}} \int_0^\infty W_E(E_j' - E_p) dE_j = 1,$$ is given by

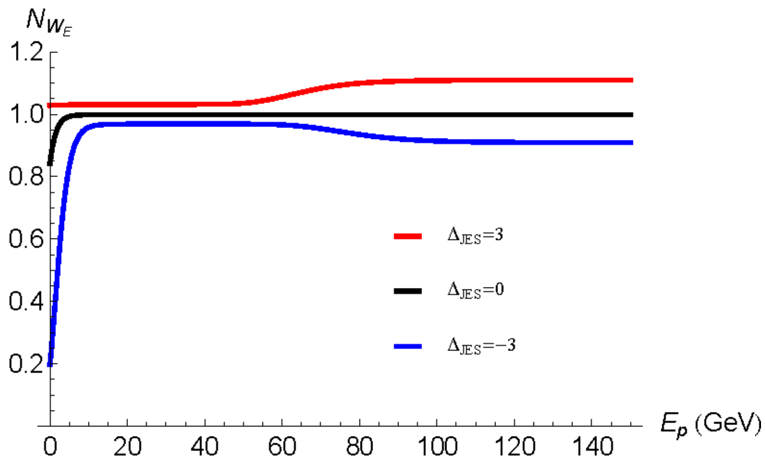
$$N_{W_E} = \int_{C_0}^\infty W_E(E_j' - E_p) \frac{dE_j}{dE_j'} dE_j',$$

where $C_0 = -\Delta_{\text{JES}} \cdot \sigma_j(0) = -\Delta_{\text{JES}} \cdot l_1/s$ from the change of variables.

Since σ_j is defined in two parts

$$\sigma_j \equiv H(67 - E_j s)\,(k_1 E_j + l_1/s) + H(E_j s - 67)\,(k_2 E_j + l_2/s)$$
$$s \equiv \sin(\theta_j),$$

the integral is also split into two parts: $C_0 \leq E_j' \leq C$ and $C \leq E_j' \leq \infty$, where C represents the cut-off point between the two functions at $E_T = 67$ GeV, expressed in terms of E_j'



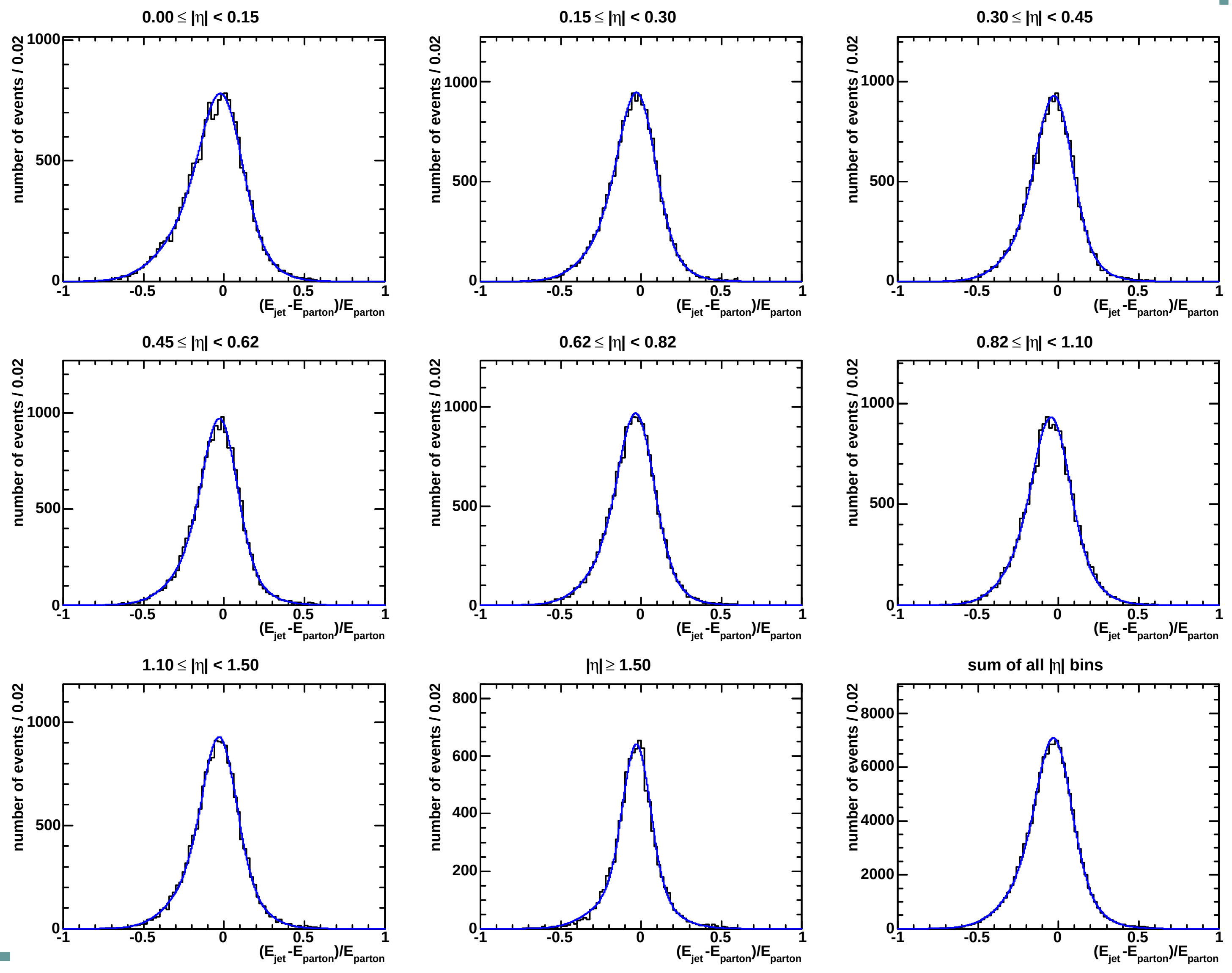
$$N_{W_E} = \int_{C_0}^{C} W_E(E_j' - E_p) \frac{1}{1 - k_1 \Delta_{\text{JES}}} dE_j' + \int_{C}^{\infty} W_E(E_j' - E_p) \frac{1}{1 - k_2 \Delta_{\text{JES}}} dE_j'.$$

The result has several error functions (Erf):

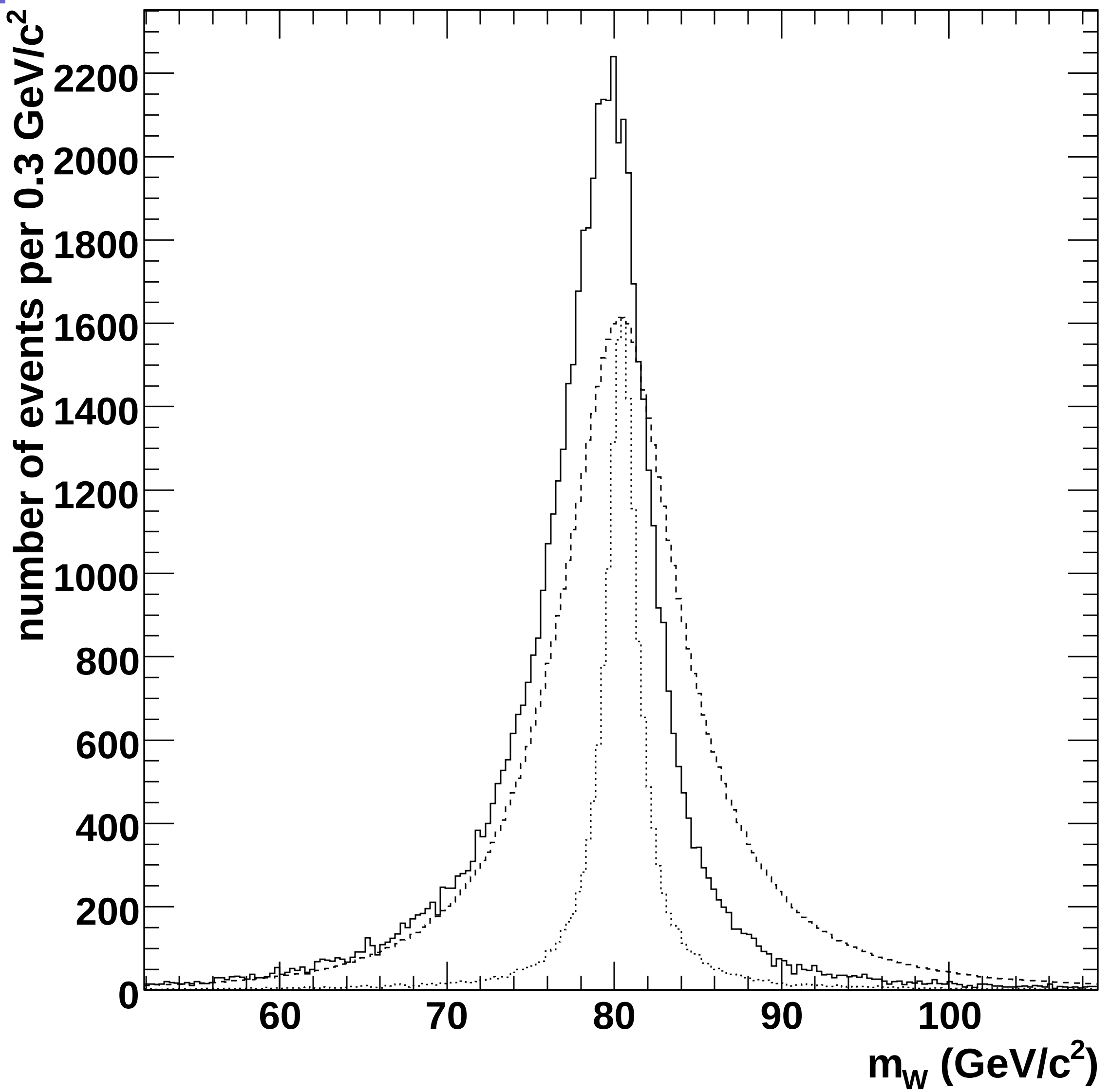
$$N_{W_E} = \frac{p_2 \text{Erf}\left(\frac{E_p - C_0 + p_1}{\sqrt{2}p_2}\right) - p_2 \text{Erf}\left(\frac{E_p - C + p_1}{\sqrt{2}p_2}\right) + p_3 p_5 \text{Erf}\left(\frac{E_p - C_0 + p_4}{\sqrt{2}p_5}\right) - p_3 p_5 \text{Erf}\left(\frac{E_p - C + p_4}{\sqrt{2}p_5}\right)}{2(1 - k_1 \Delta_{\text{JES}})(p_2 + p_3 p_5)}$$
$$+ \frac{p_2 + p_3 p_5 + p_2 \text{Erf}\left(\frac{E_p - C + p_1}{\sqrt{2}p_2}\right) + p_3 p_5 \text{Erf}\left(\frac{E_p - C + p_4}{\sqrt{2}p_5}\right)}{2(1 - k_2 \Delta_{\text{JES}})(p_2 + p_3 p_5)}.$$

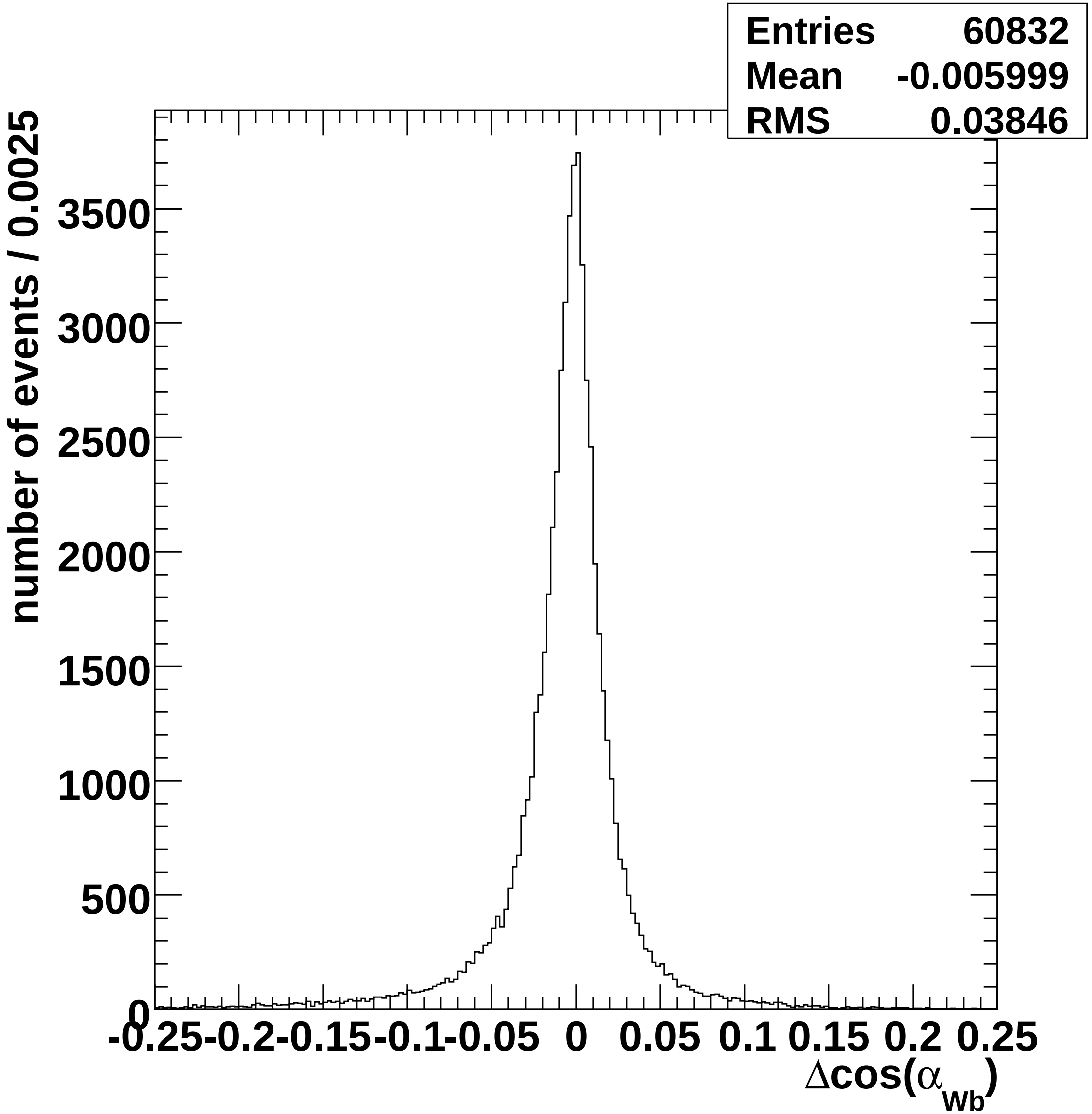
Energy TF

- Light jet TF bins
- Histograms show MC distributions of $\left(E_j - E_p\right)/E_p$
- Blue line is fitted TF



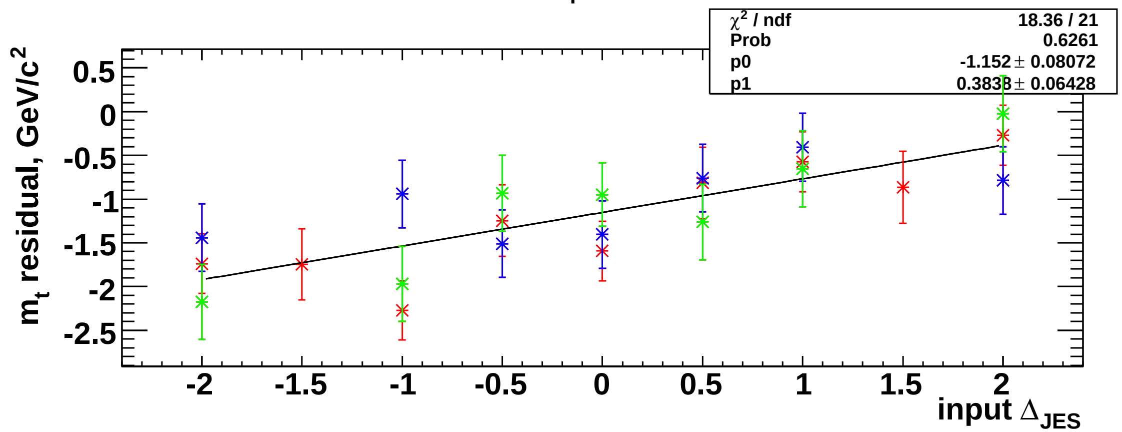
m_W


number of events per 0.3 GeV/c²
0
200
400
600
800
1000
1200
1400
1600
1800
2000
2200
60
70
80
90
100
m_W (GeV/c²)


Entries 60832
Mean -0.005999
RMS 0.03846
number of events / 0.0025
0
500
1000
1500
2000
2500
3000
3500
-0.25
-0.2
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
0.2
0.25
$\Delta\cos(\alpha_{Wb})$

JES Linearity

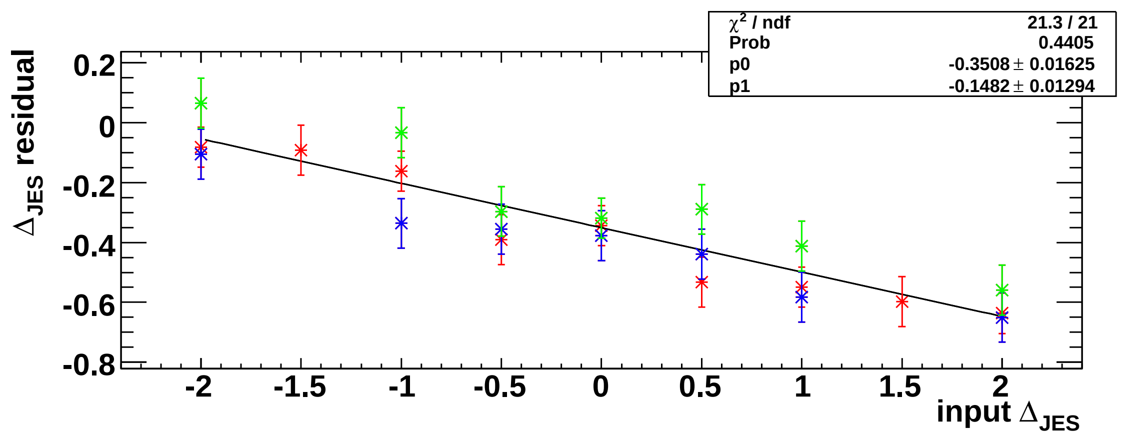
- Mass and Δ_{JES} residuals both show dependence on input Δ_{JES}
  - Thought to be due to biased TF fit due to E_T cut



m_t=165 GeV

m_t=175 GeV

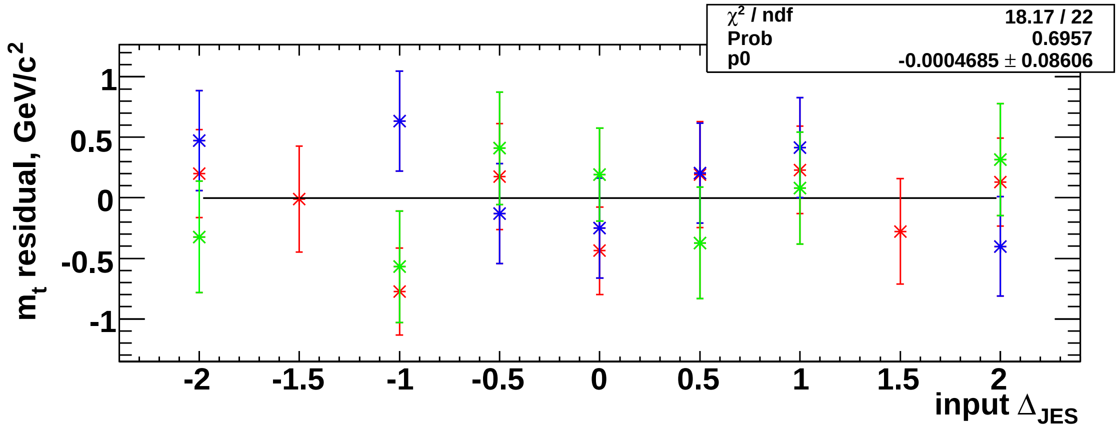
m_t=185 GeV



JES Linearity Calibration

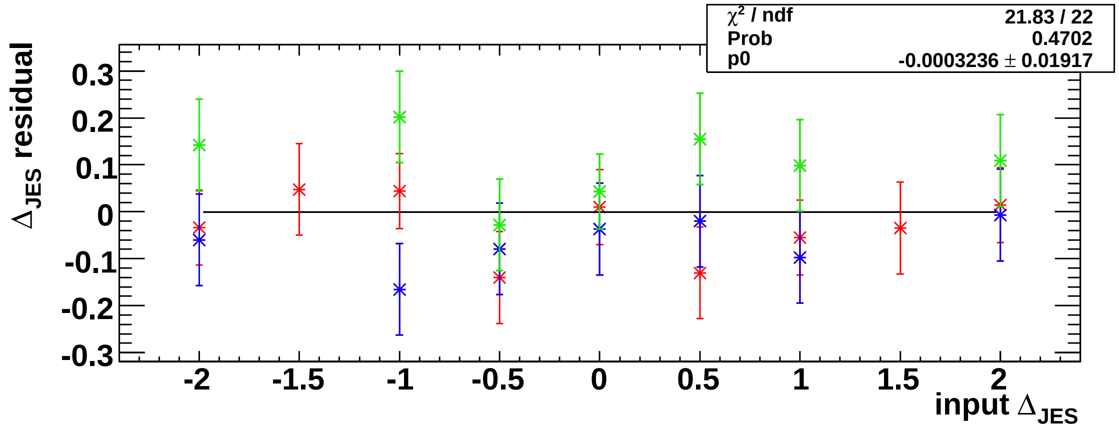
- Apply calibration functions:

$$\Delta_{\mathrm{JES}corr} = \left(\Delta_{\mathrm{JES}meas} + 0.351\right)/\left(1 - 0.148\right)$$



m_t=165 GeV
m_t=175 GeV
m_t=185 GeV

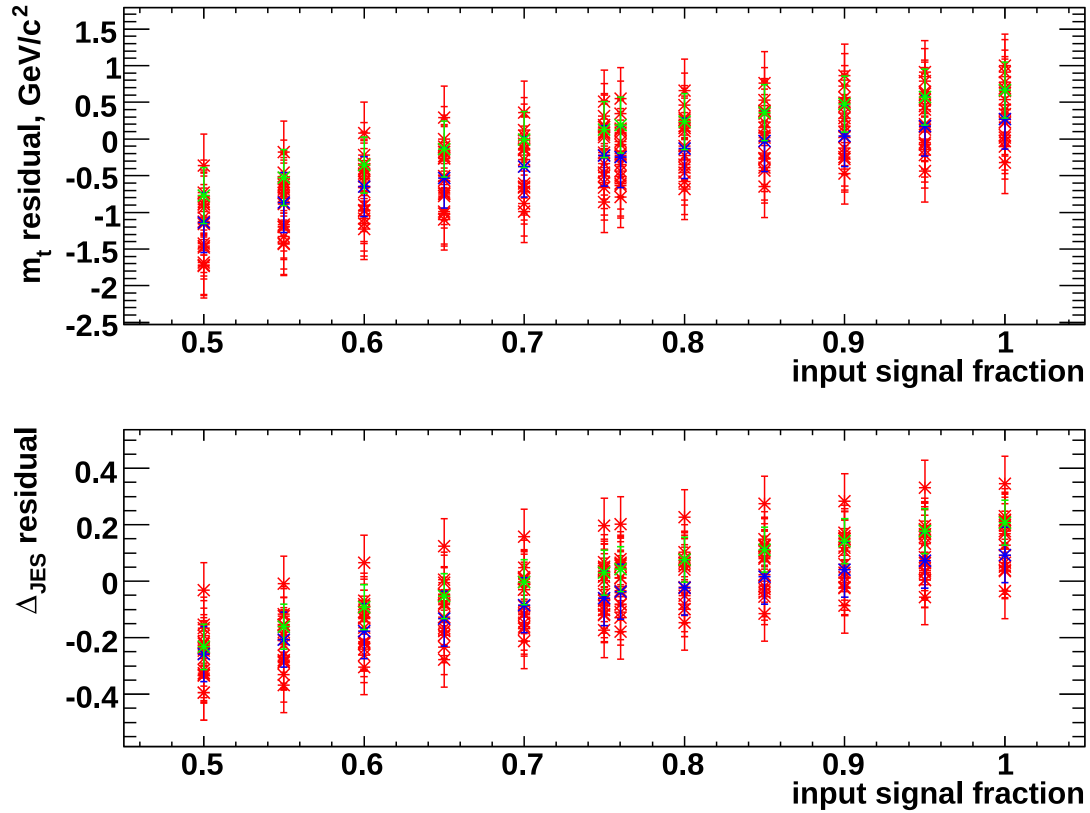
$$m_{corr} = m_{meas} + 1.152 - 0.384 \cdot \Delta_{\mathrm{JES}corr}$$



- Note, corrections increase pull widths

Signal fraction calibration

- Note, all subsequent plots are post Δ_{JES}-calibration
- Test method response to signal fraction
- Create PEs with different mean signal fractions
- Bias increases as more background events are added
- ttop65 and ttop85 show very similar dependence
  - Other 15 mass points also plotted in red



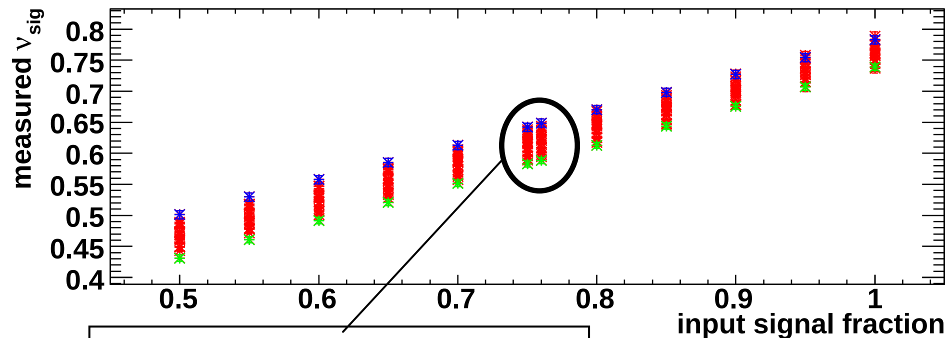
15 mass points 167.5 – 182.5 GeV

m_t=165 GeV

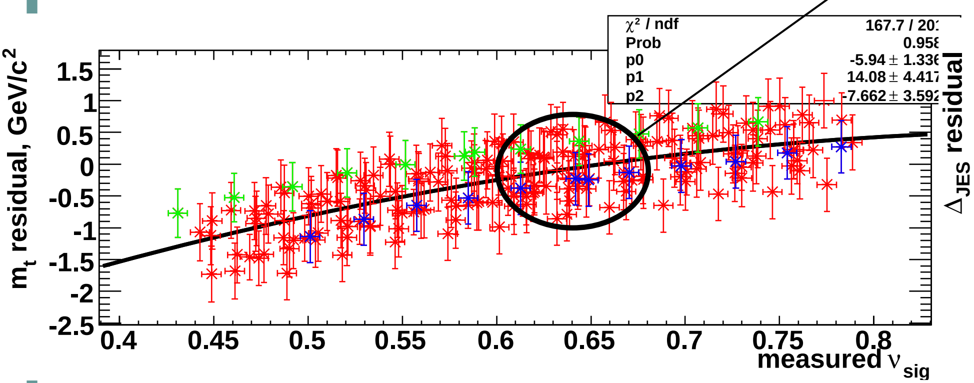
m_t=185 GeV

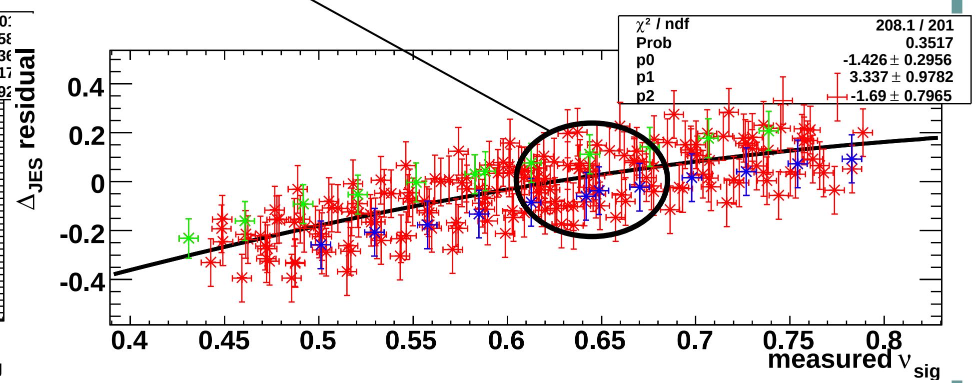
Signal fraction calibration

- Parameterise correction wrt v_{sig} (fitted signal fraction)
  - v_{sig} closely related to input signal fraction



Expected v_{sig} range

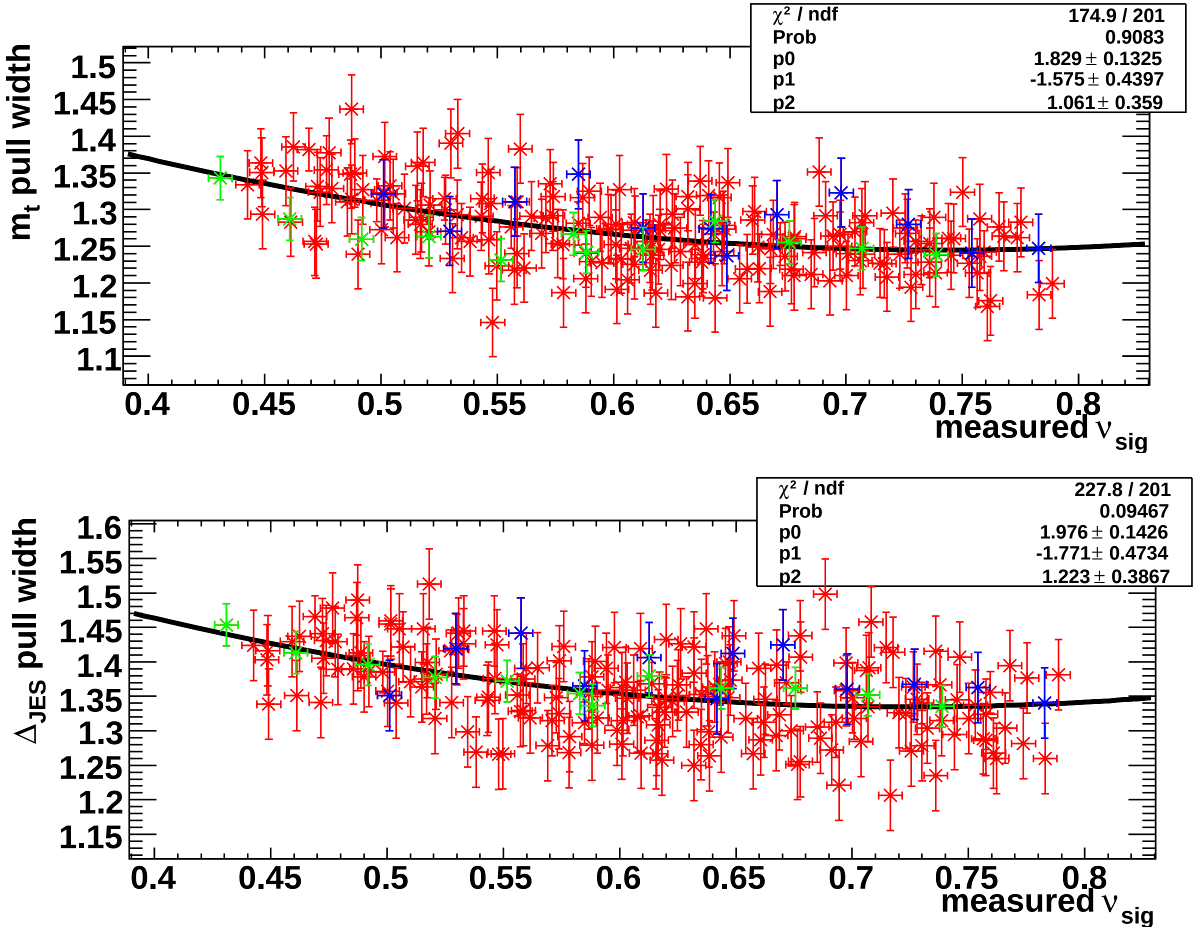




15 mass points 167.5 – 182.5 GeV m_t=165 GeV m_t=185 GeV

Signal fraction calibration

- Pull width also requires calibration



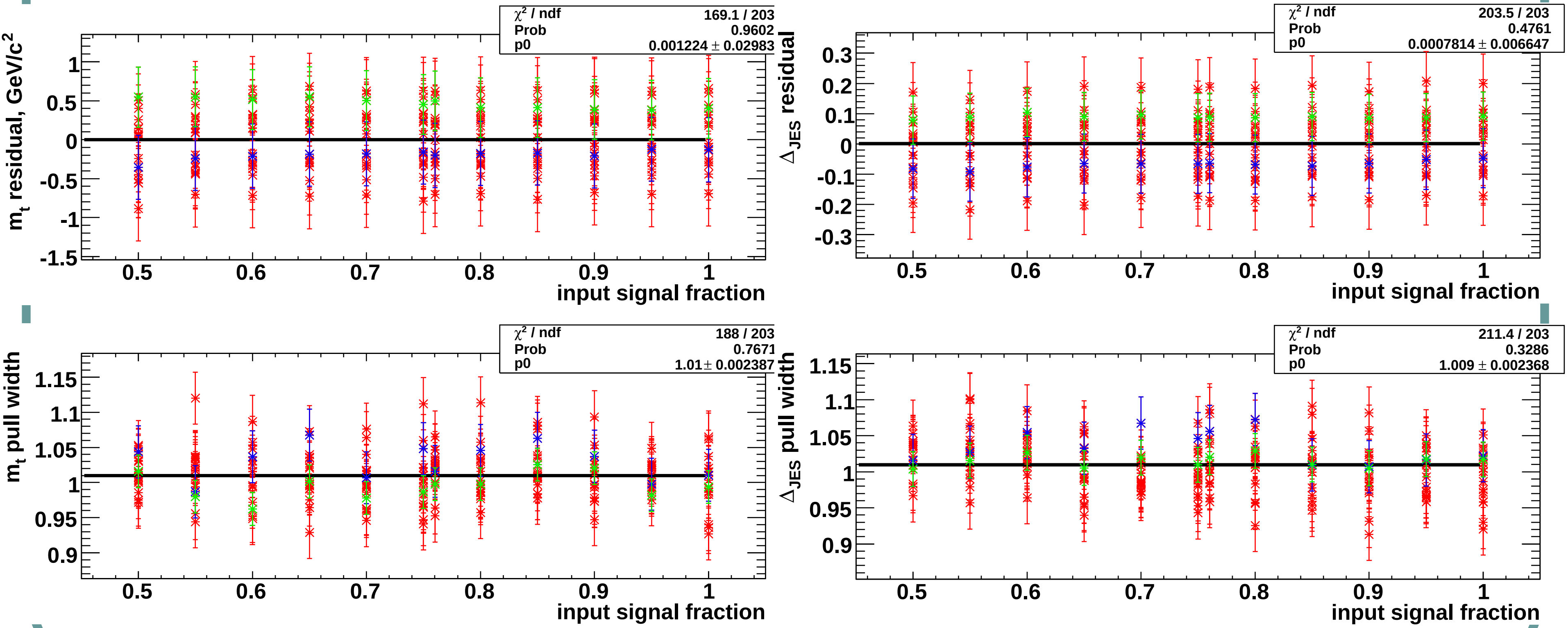
15 mass points 167.5 – 182.5 GeV m_t=165 GeV m_t=185 GeV

- Plots post-calibration
- Signal-fraction dependence removed

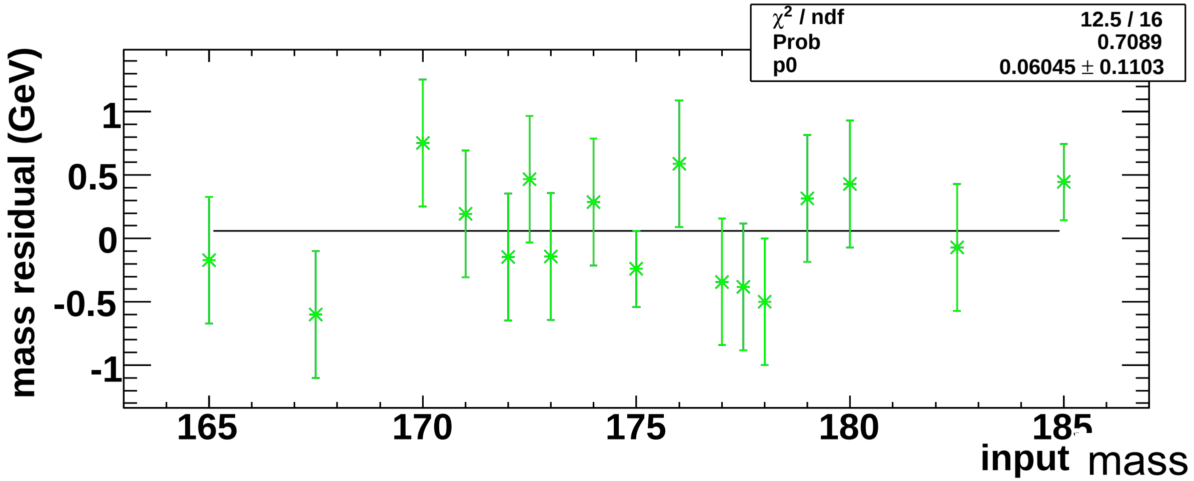
m_t=165 GeV

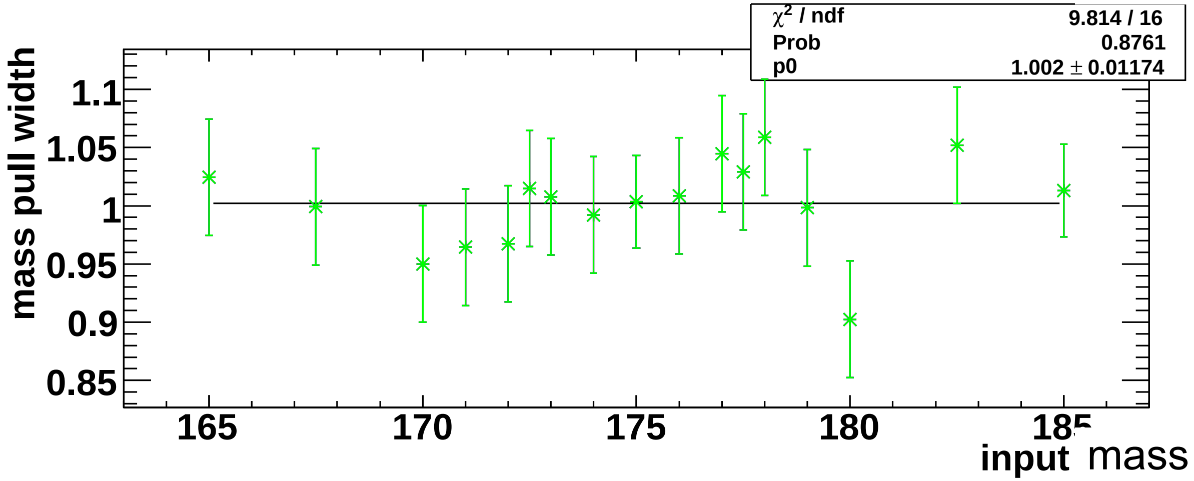
m_t=185 GeV

All other m_t

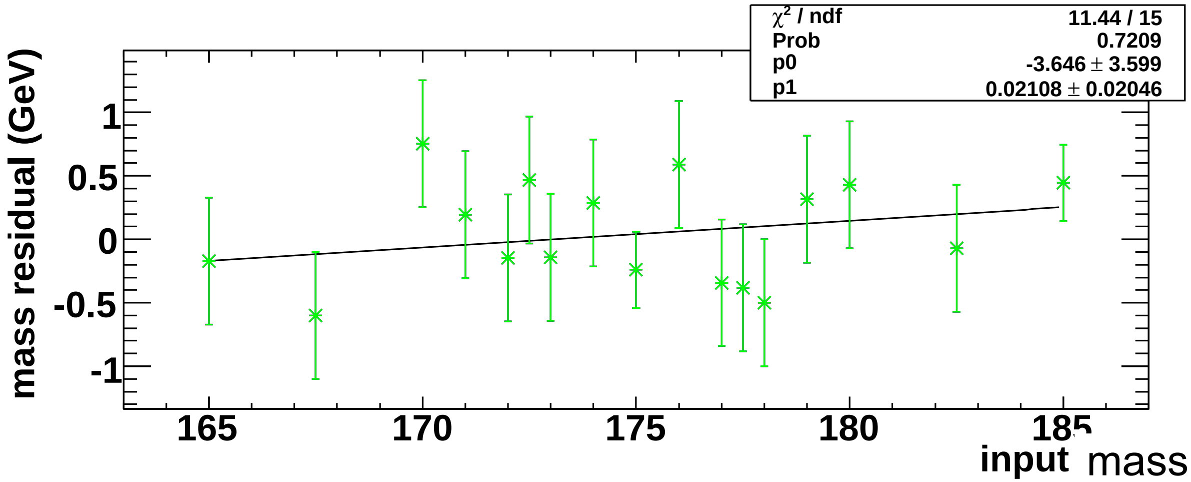


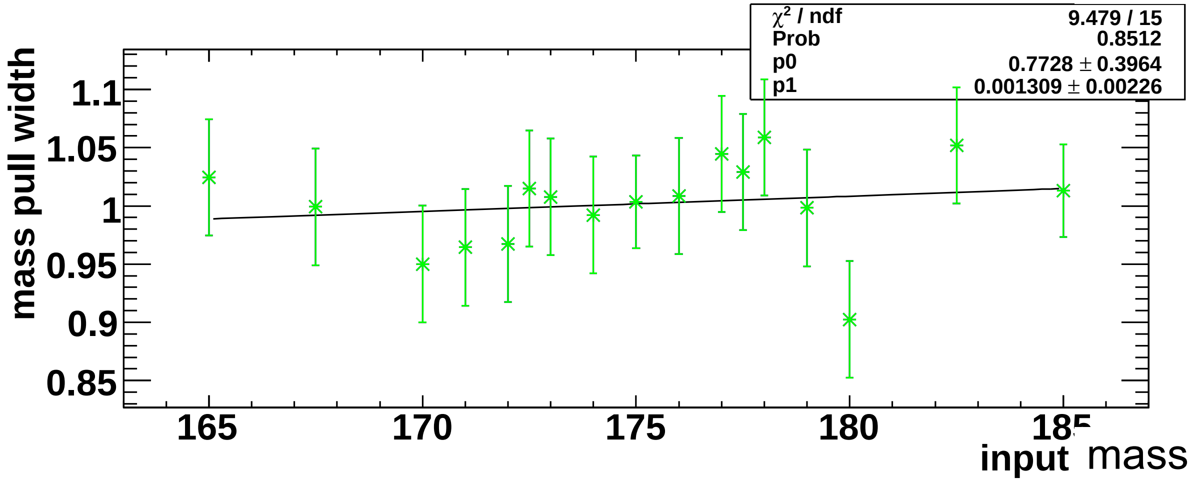
Linearity: mass, post-calibration



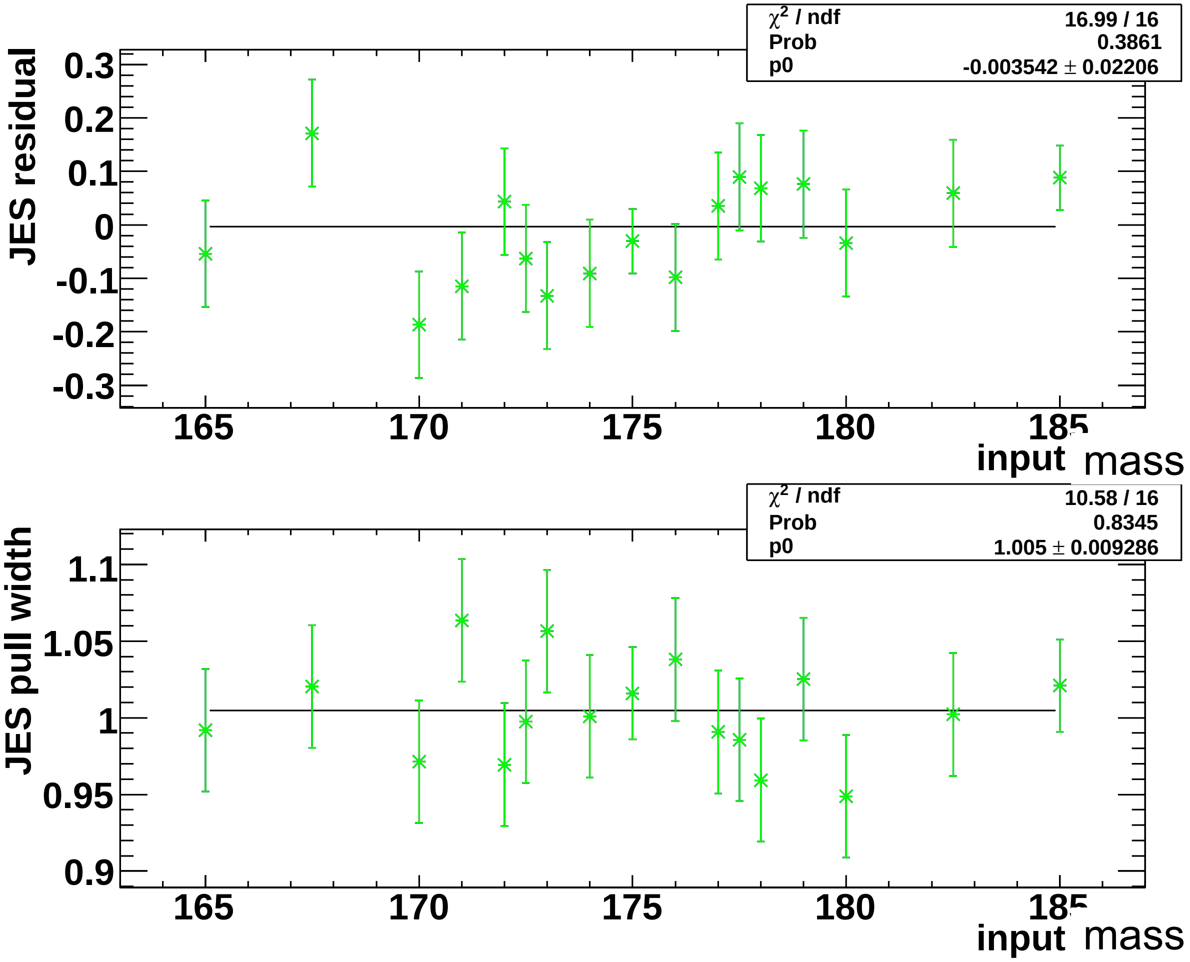


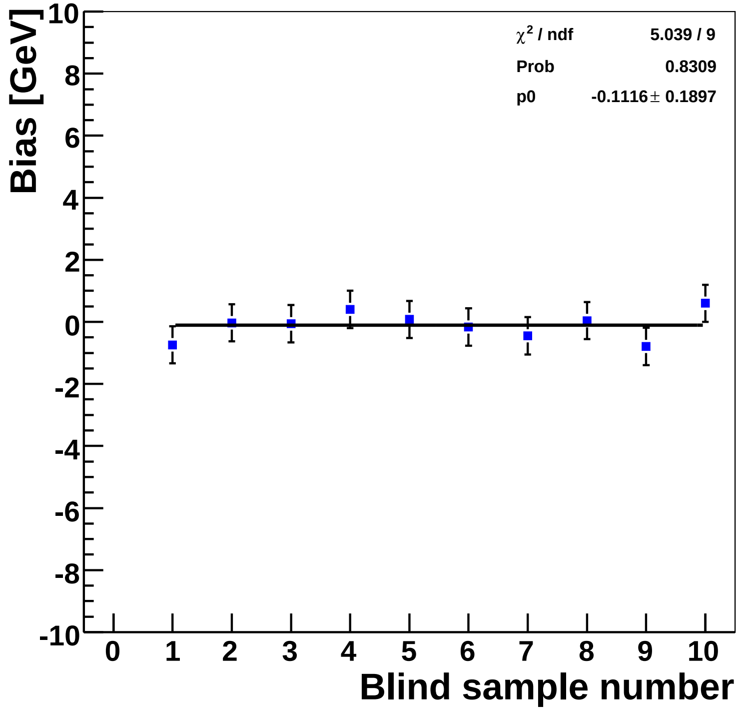
Linearity: mass, post-calibration

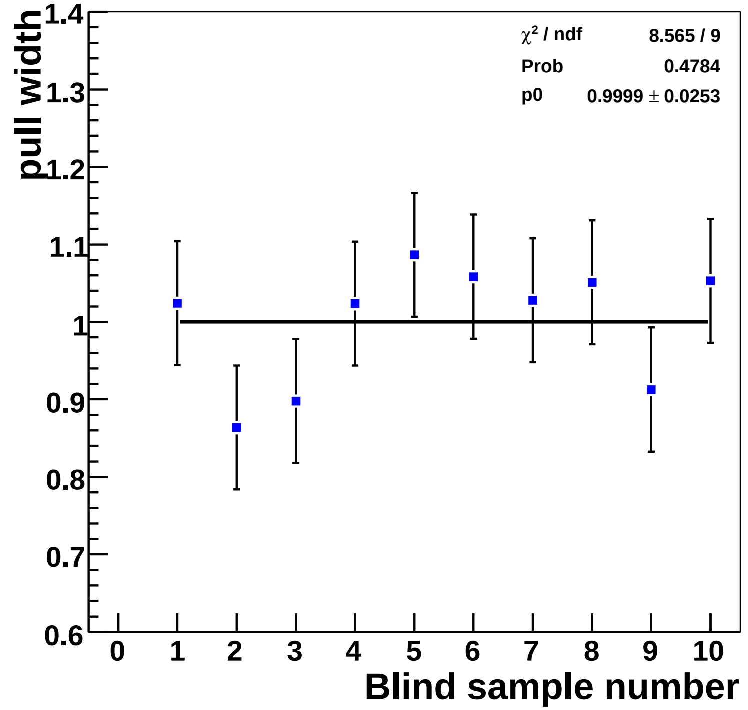


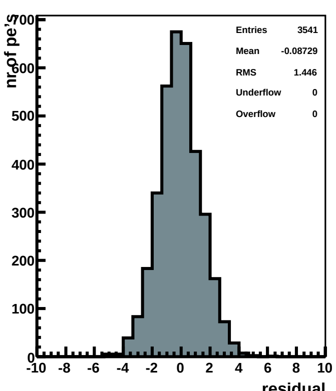


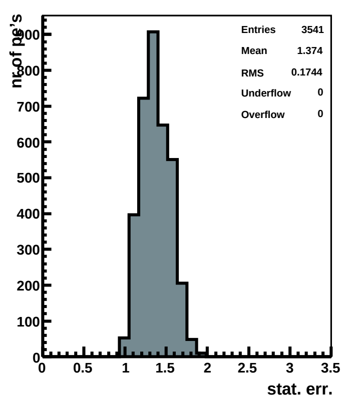
Linearity: JES, post-calibration

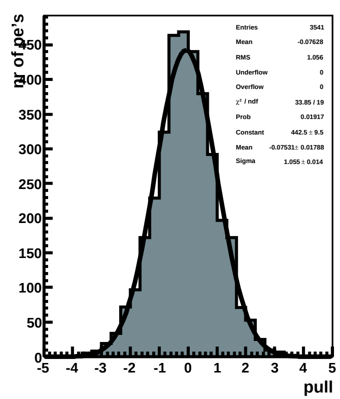



Bias [GeV]
χ² / ndf 5.039 / 9
Prob 0.8309
p0 -0.1116 ± 0.1897
Blind sample number


pull width
χ² / ndf 8.565 / 9
Prob 0.4784
p0 0.9999 ± 0.0253
Blind sample number


nr of pe's
Entries 3541
Mean -0.08729
RMS 1.446
Underflow 0
Overflow 0
residual


nr of pe's
Entries 3541
Mean 1.374
RMS 0.1744
Underflow 0
Overflow 0
stat. err.


nr of pe's
Entries 3541
Mean -0.07628
RMS 1.056
Underflow 0
Overflow 0
χ² / ndf 33.85 / 19
Prob 0.01917
Constant 442.5 ± 9.5
Mean -0.07531 ± 0.01788
Sigma 1.055 ± 0.014
pull

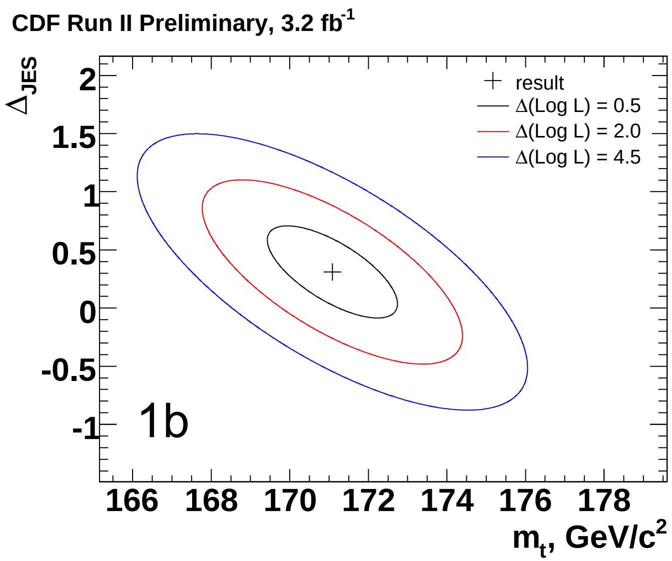
systematics

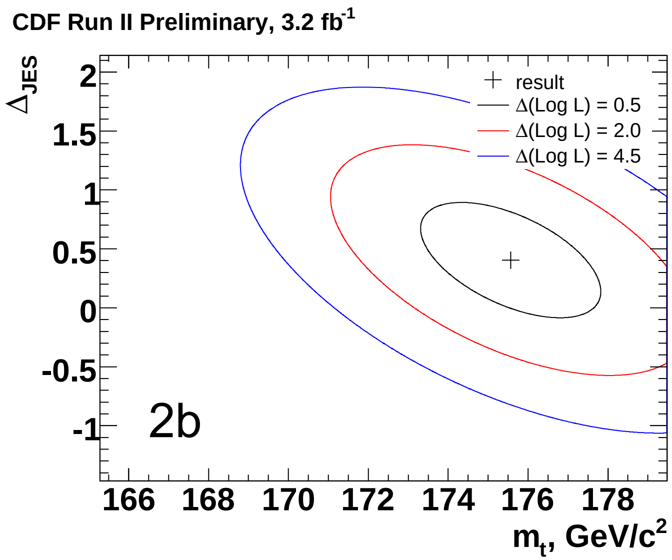
MC Generator	Method is calibrated using signal MC from Pythia generator. Systematic taken as difference in result between Pythia and the Herwig generator.
Residual JES	Systematics associated with each level of the JES jet corrections, summed in quadrature
Colour Reconnection	Difference between two Pythia MC samples, tune Apro (no CR) and tune ACRpro (includes CR)
b-jet Energy	b jet energies varied by ± 1% in MC
Background	Vary background composition and fraction
ISR/FSR	Difference in result in MC with more or less I&FSR
Multiple Hadron Interaction	Systematic associated with mismodelling of luminosity profile in MC
PDFs	Difference in MC using difference PDFs
Lepton Energy	Electrons and muons shifted ± 1 sigma in MC
Method Calibration	Uncertainty associated with method calibration

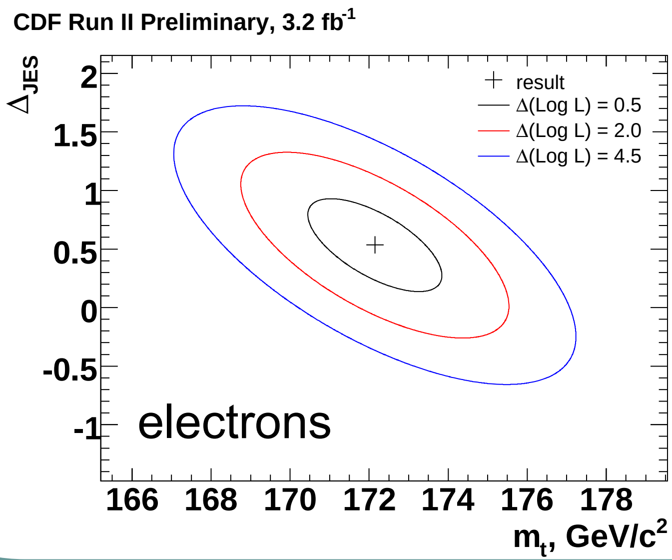
Improvements

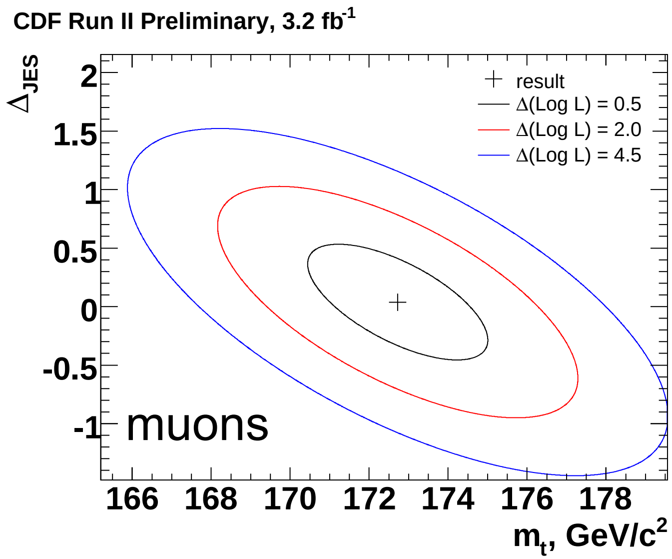
- P_{W+jets}
  - More precise evaluation, integration
  - More p.d.f.s (P_{QCD}, $P_{bad\ signal}$?)
- Split 1, 2 b-tag samples (different background composition)
- Assumption of seperable energy TF, angular TF?
  - Angular correlation effect could affect energies also
- Δ_{JES}-slope (caused by ETF bias?)
- $t\bar{t}$ transverse momentum integration
- Add loose muons, taus
- Systematics

Subsamples


CDF Run II Preliminary, 3.2 fb^{-1}
Δ_{JES}
result
Δ(Log L) = 0.5
Δ(Log L) = 2.0
Δ(Log L) = 4.5
1b
m_t, GeV/c^2


CDF Run II Preliminary, 3.2 fb^{-1}
Δ_{JES}
result
Δ(Log L) = 0.5
Δ(Log L) = 2.0
Δ(Log L) = 4.5
2b
m_t, GeV/c^2


CDF Run II Preliminary, 3.2 fb^{-1}
Δ_{JES}
result
Δ(Log L) = 0.5
Δ(Log L) = 2.0
Δ(Log L) = 4.5
electrons
m_t, GeV/c^2


CDF Run II Preliminary, 3.2 fb^{-1}
Δ_{JES}
result
Δ(Log L) = 0.5
Δ(Log L) = 2.0
Δ(Log L) = 4.5
muons
m_t, GeV/c^2

Flux factor

- Expression for P_{ttbar} (MEAT v2):

$$P(\vec{x}; m_t, JES) = \frac{1}{\sigma(m_t)} \int \sum_{soln}^{4} |M|^2 \frac{f(\tilde{q}_1)}{|q_1|} \frac{f(\tilde{q}_2)}{|q_2|} \prod_{j=1}^{4} \left(W(x, y; JES) \right) d\tilde{\Phi}$$

- “Mistake” in the MEAT v2 code: Flux factor missing
  - However, it is present in the notes
  - It was thought to be contained in the PDFs
- Significant effect on m_t
  - Upwards shift of ~4GeV
- Flux factor restored in MEAT v3

Changes: Energy TFs

- Mistake in MEATv2, TF is normalised in E'_{jet} :

$$\int W\left(E_{parton} - E'_{jet}\right)dE'_{jet} = 1$$

- But p.d.f. must be normalised wrt measured quantities (E_{jet})
  - Require $\int W\left(E_{parton} - E'_{jet}\right)dE_{jet} = 1$

Change of variables from E'_{jet} to E_{jet}

- For scale factor *JES*, $E'_{jet} = JES \cdot E_{jet}$:

$$\int W\left(E_{parton} - JES \cdot E_{jet}\right)dE_{jet} = \underbrace{\int W\left(E_{parton} - JES \cdot E_{jet}\right)d\left(JES \cdot E_{jet}\right)}_{=1} \frac{dE_{jet}}{\underbrace{d\left(JES \cdot E_{jet}\right)}_{=\frac{1}{JES}}} = \frac{1}{JES}$$

- TFs require normalisation factor JES^4 (4 jets)
- Missing in MEAT v2
  - Cause of the *JES* bias (*JES*~0.95) in MEAT v2
- Corrected in MEAT v3

Cancelled out high mass caused by missing flux factor