Special Problem 1: Ant-Chase

September 3, 2002

Three ants move on a table-top. Initially, they are positioned at the vertices of an equilateral triangle with sides of length 1 cm. Ant A is attracted to Ant B, who is attracted to Ant C, who is attracted to Ant A. Each of the ants heads toward the ant of her choice, at a constant speed 1 cm/sec. The ants are quick in perception but not very smart. So at every moment, each ant is heading toward the instantaneous position of her target at that time.

- Do the ants ever meet?
- If so, how long does it take? (7)
- (Optional) What is the path they follow?
- (Optional) Write a little program to plot it.
- Generalize to 4 ants. (3)

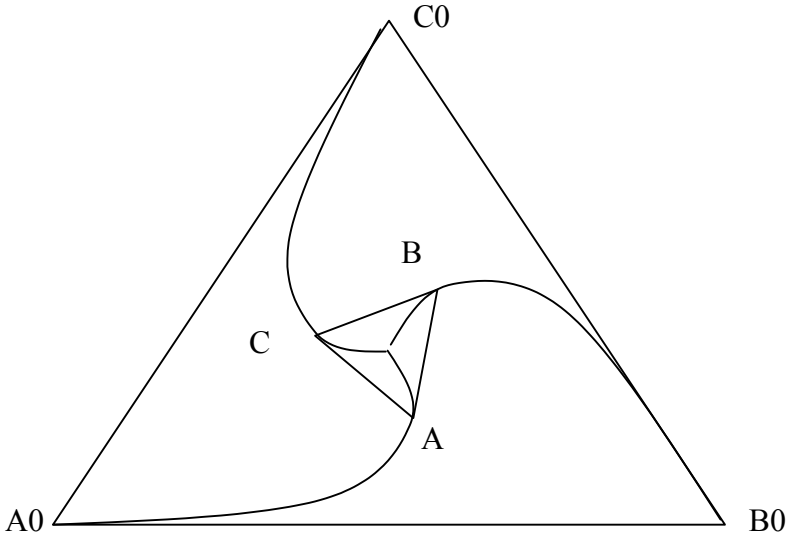
Hint: If you use vectors and symmetry, no elaborate calculation is required (for the non-optional parts).

Due: September 17

Clues for Special Problem 1: Ant-Chase

Clue #1:

When you start a physics problem, almost all the time, the first thing you will do is to draw a picture. The following is a picture you can draw when you see the ant-chase problem.

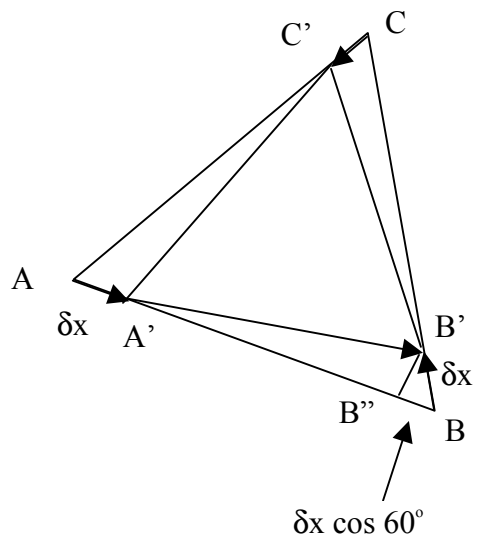


The three ants started at the three vertices (A0, B0 and C0) of the equilateral triangle. They run towards their targets. Clearly the paths must be some curves. The three curves should have the same shape because of symmetry. At any moment, the lines connecting the ants should form another equilateral triangle, i.e., ABC.

The triangle formed by the ants will rotate and shrink its size. Once the sides of the triangle shrink down to 0, the three ants meet.

Clue #2:

Now the question is what is the rate that the triangle shrinks?
It is always a good idea to draw a picture:



Consider ant A and ant B. At one time, ant A starts moving toward B and B towards C. After a very short time δt, they all moved a small distance δx. Now their positions are A' and B'. The equilateral triangle now is A'B'C'.

What is the length of A'B'? It is about the same as A'B". (B'B" is perpendicular to AB.) So the change rate of triangle side length $\delta L/\delta t$ is:

$\delta L/\delta t = (\delta x + \delta x \cos 60^o)/ \delta t$

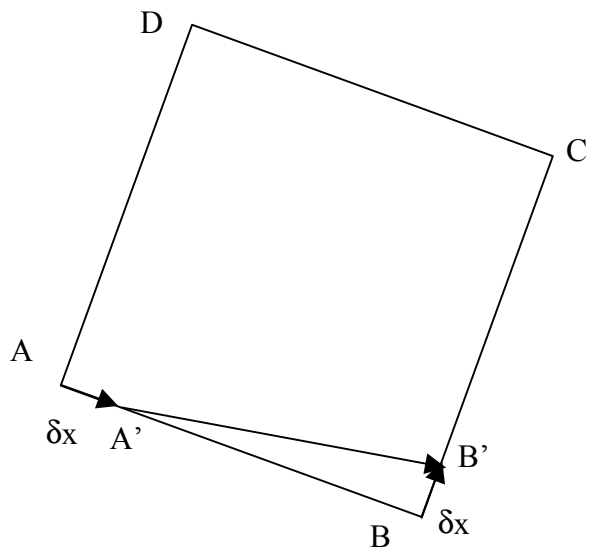
Since $\delta x / \delta t = 1$ cm/s, which is the speed of the ant. So the shrink speed of the triangle sides is: (1+0.5) cm/s =1.5 cm/s.

How many seconds does it take for a 1 cm triangle to shrink down to 0 at shrinking rate 1.5 cm/s?

<u>Possible mistake</u>: The length of A'B' does not approximately equal to the length of A'B! The "small" difference of B"B is $O(\delta x)$, not $O(\delta x^2)$.

Four ants case:

Just draw a picture:



This time:

$\delta L/\delta t = (\delta x + \delta x \cos 90^{o})/ \delta t = \delta x/\delta t = 1$ cm/s

In other words, the length of A'B' can be approximated with the length of A'B.